# Unpolarized SIDIS Cross Section from a ${ }^{3}$ He Target 

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics
in the Graduate School of Duke University

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## Abstract

The unpolarized semi-inclusive deep-inelastic scattering (SIDIS) differential cross sections in ${ }^{3} \mathrm{He}\left(e, e^{\prime} \pi^{ \pm}\right) X$ have been measured for the first time in Jefferson Lab experiment E06-010 performed with a $5.9 \mathrm{GeV} e^{-}$beam on a ${ }^{3} \mathrm{He}$ target. The experiment focuses on the valence quark region, covering a kinematic range $0.12<x_{b j}<0.45$, $1<Q^{2}<4(\mathrm{GeV} / \mathrm{c})^{2}, 0.45<z_{h}<0.65$, and $0.05<P_{t}<0.55 \mathrm{GeV} / \mathrm{c}$. The extracted SIDIS differential cross sections of $\pi^{ \pm}$production are compared with existing phenomenological models while the ${ }^{3} \mathrm{He}$ nucleus approximated as two protons and one neutron in a plane wave picture, in multi-dimensional bins. Within the experimental uncertainties, the azimuthal modulations of the cross sections are found to be consistent with zero.

In this dissertation, the studies for the unpolarized SIDIS differential cross sections are presented. The dissertation will start with the introduction on the physics related to SIDIS, then the experiment E06-010 will be described, followed by the data analysis. The results of the unpolarized SIDIS differential cross sections will be shown afterwards with discussions.

In addition to the work on the unpolarized SIDIS, the author also updated the approximated formalism for radiative effects (REs) for inclusive scattering channels (lifted the energy peaking approximation of the formalism). This updated formalism and a detailed discussion of the approximations in different formalisms of REs are presented in the appendix.

To my parents.

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## List of Abbreviations and Symbols

Symbols

$\vec{k} \quad$ three-vector of $k$.
$\alpha \quad$ Electromagnetic fine structure constant.
$k_{\perp} \quad$ Intrinsic transverse momentum of a parton in a hadron.
$p_{\perp} \quad$ Transverse momentum of the fragmenting hadron with respect to a parton.
$\phi_{h} \quad$ The azimuthal angle between the lepton plane and the hadron plane in a semi-inclusive scattering process.

## Abbreviations

ADC Analog to digital converter.
BCC Bin-centering correction.
BCM Beam current monitor.
BPM Beam position monitor.
CEBAF Continuous electron beam accelerator facility.
CT Coincidence time.
DAQ Data acquisition.
DIS Deep-inelastic scattering.
DXS Differential cross section.
EPR Electron paramagnetic resonance.
ERM Equivalent radiator method.

HRS High resolution spectrometer.
NMR Nuclear magnetic resonance.
OPE Operator product expansion.
PID Particle identification.
QED Quantum electrodynamics.
QCD Quantum chromodynamics.
RMS Root mean square.
RP Reversed polarity.
SIDIS Semi-inclusive deep-inelastic scattering.
ppm Part per million.

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## Introduction

The strong interaction, one of the four fundamental interactions in nature, is described by Quantum Chromodynamics (QCD) in the framework of the Yang-Mills gauge theory, as a part of the standard model of particle physics $[11,12]$. An important feature of QCD is the asymptotic freedom, which means the strong coupling constant $\alpha_{S}$ decreases with increasing energy scale [13, 14]. At very high energy, $\alpha_{S}$ becomes so small that the quarks behave almost like free particles. This feature has been verified by the high-energy scattering experimental data [15], and the 2004 Nobel Prize of physics was awarded to Gross, Politzer and Wilczek for the discovery of asymptotic freedom in the theory. A plot of $\alpha_{S}$ as a function of the energy scale $(Q)$ is presented in Fig. 1.1.

In the low energy regime, QCD confinement emerges, which means the quarks and gluons are confined inside bound states, forming hadrons such as nucleons and mesons. In this region, $\alpha_{S}$ becomes large, and the perturbative expansion is invalidated. Although some nonperturbative methods, such as the lattice gauge theory and the Dyson-Schwinger equation approach, were developed during last decades, it is still a challenging task to calculate all hadronic properties from the first principles


Figure 1.1: The strong coupling constant $\alpha_{S}$ as a function of the energy scale $Q$. The respective degree of QCD perturbation theory used in the extraction of $\alpha_{S}$ is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; N3LO: next-to-NNLO. The figure is from [1].
of QCD. The understanding of QCD in the confinement regime can be enhanced via the study of the nucleon structure.

Nucleons (protons and neutrons), are building blocks of more than $99 \%$ of the visible universe, and the study of their structure is a rich subject, e.g., probing the charge, magnetic-moment, quark-momentum and quark-spin distributions inside the nucleon, and studying the nucleon spin and mass compositions. In the following part of this chapter, we first introduce several important studies of the nucleon structure as milestones of this subject in history.

Nucleons were thought as point-like particles until a sizable anomalous magnetic moment of the proton was measured by Stern et al. in 1933 [16]. This discovery was the first evidence for a proton not being point-like, and a Nobel Prize was awarded
for it in 1943.
In 1955, Hofstadter et al. measured the electromagnetic form factors of the proton via the elastic electron-proton scattering process with a 1 GeV electron beam [17]. A Nobel Prize was awarded for this study in 1961. In the elastic scattering, the nucleon stays intact other than a finite recoil, and the charge and current distributions inside the nucleon can be studied. A graphical illustration for probing the nucleon structure at this level is presented in Fig. 1.2.


Figure 1.2: Probing the nucleon structure in the elastic channel. The blue lines with arrows represent the incoming and outgoing leptons (electrons). The violet wavy line represents the virtual photon. The partially transparent blue disk represents the nucleon being probed.

In 1964, Gell-Mann et al. introduced the concept of quarks and postulated that a proton consists of three quarks [18] for the study of hadron spectroscopy and classification. A spin- $\frac{3}{2}$ baryon $\Omega^{-}$was predicted as a member of the ground-state decuplet in the classification model, and was later discovered at Brookhaven National Laboratory [19]. A Nobel Prize was awarded for this model in 1969.

In 1969, Friedman, Kendall, Taylor, and their collaborators carried out the first deep-inelastic scattering (DIS) experiment with 7 to 17 GeV beam electrons and a liquid hydrogen (proton) target [20, 21]. This study led to decades of DIS experiments probing many aspects of the nucleon structure. In the DIS process, the target nucleon
is shattered into fragments, and its substructures at the parton (quark and gluon) level can be probed. A Nobel Prize was awarded to Friedman, Kendall and Taylor for their pioneering study of the DIS process in 1990. A graphical illustration for this level of probing is presented in Fig. 1.3.


Figure 1.3: Probing the nucleon structure in the DIS process. The blue lines with arrows represent the incoming and outgoing leptons (electrons). The violet wavy line represent the virtual photon. The black empty circle represents the nucleon (proton) being probed. The blue, green and red solid disks represent the valence quarks in the nucleon. The small black solid disks represent the sea quarks. The curly lines represent the interaction mediated by the gluons.

In the late 1980s, the European Muon Collaboration (EMC) carried out a DIS experiment with polarized beam muons at 100,120 and 200 GeV , and a solid ammonia target in which the protons were polarized by the dynamic nuclear polarization (DNP). In this experiment, the quark-helicity contribution to the proton spin was measured for the first time, and was found to be only $6 \%$ with a large uncertainty [22, 23]. This discovery shocked the physics community, since it severely deviated from the picture of the naive quark model in which the proton spin is due to the quark spins only. With more theoretical considerations and more experimental data, the current value of the fraction of quark helicity contributions to the nucleon spin
is around $30 \%$ with significantly improved accuracy [24, 25, 26]. The contradiction between the naive quark model and the polarized DIS measurement is known as the "proton spin crisis" [27, 28].

While recent studies indicate that the contribution from the gluon helicities may be sizable [29], there is still a large unidentified component in the nucleon spin. It was pointed out in the early 1990s by Ma that the quark orbital angular momentum (OAM) is non-negligible due to the mechanism of Melosh-Wigner rotations [30, 31, 32 ].

In order to understand the nucleon spin, different spin sum rules have been proposed, in which the nucleon spin is decomposed into the angular momenta of quarks and gluons. The most intuitive decomposition of the nucleon spin is the Jaffe-Manohar sum rule [33], expressed as

$$
\begin{equation*}
\boldsymbol{J}=\boldsymbol{S}_{q}+\boldsymbol{L}_{q}+\boldsymbol{S}_{g}+\boldsymbol{L}_{g} \tag{1.1}
\end{equation*}
$$

where $\boldsymbol{J}$ is the nucleon angular momentum, $\boldsymbol{S}_{q}$ is the spin angular momentum of the quarks, $\boldsymbol{L}_{q}$ is the OAM of the quarks, $\boldsymbol{S}_{g}$ is the spin angular momentum of the gluons, and $\boldsymbol{L}_{g}$ is the OAM of the gluons. While the Jaffe-Manohar sum rule provides a full decomposition and the operator in each term satisfies the commutation relation of the angular momentum algebra $\mathrm{SU}(2)$, all the operators except $\boldsymbol{S}_{q}$ are not obviously gauge independent, and thus may not be directly measurable.

Considering the gauge-invariance, Ji proposed a different sum rule in 1997 (Ji's sum rule) [34]:

$$
\begin{equation*}
\boldsymbol{J}=\boldsymbol{S}_{q}+\boldsymbol{L}_{q}+\boldsymbol{J}_{g} \tag{1.2}
\end{equation*}
$$

where $\boldsymbol{J}, \boldsymbol{S}_{q}$, and $\boldsymbol{L}_{q}$ are the nucleon angular momentum, the spin angular momentum of the quarks, and the OAM of the quarks, repectively. The $\boldsymbol{J}_{g}$ term represents the total angular momentum of the gluons, which is not decomposed into a spin term and
an OAM term. While every term in Ji's sum rule is gauge-invariant, the operators $\boldsymbol{L}_{q}$ and $\boldsymbol{J}_{g}$ do not satisfy the commutation relation of the angular momentum algebra.

Apart from the two discussed above, many other versions of nucleon-spin decomposition were derived in recent years. All the decomposition versions are generally categorized into the canonical decomposition, e.g. Jaffe-Manohar sum rule, and the kinetic decomposition, e.g. Ji's sum rule. The main difference between the two categories is the definition of the OAM terms. It is important to connect and compare each term to experimental observables. To discuss about the observables, we first express the state vector $|\psi\rangle$ of a nucleon as

$$
\begin{equation*}
|\psi\rangle=\sum_{i} A_{i}\left|\psi^{i}: N, S_{q}, L_{q}, S_{g}, L_{g}\right\rangle, \tag{1.3}
\end{equation*}
$$

where $A_{i}$ is the amplitude of the $i$ th state $\left|\psi^{i}: N, S_{q}, L_{q}, S_{g}, L_{g}\right\rangle$. The quantities $S_{a}$ and $L_{a}$ denote the spin and OAM of the partons in a state, where $a=q(g)$ represents the quantities of the quarks (gluons). Using Eq. (1.1) as an example, the total spin of the nucleon can be decomposed as

$$
\begin{equation*}
\frac{1}{2}=\langle\psi| J_{z}|\psi\rangle=\langle\psi| S_{q, z}|\psi\rangle+\langle\psi| L_{q, z}|\psi\rangle+\langle\psi| S_{g, z}|\psi\rangle+\langle\psi| L_{g, z}|\psi\rangle \tag{1.4}
\end{equation*}
$$

where the nucleon spin is in the $z$ direction, and the subscript $z$ represents the $z$ component of an operator. The contribution from the quark spin can be expressed as

$$
\begin{equation*}
\langle\psi| S_{q, z}|\psi\rangle=\sum_{i}\left|A_{i}\right|^{2}\left\langle\psi^{i}\right| S_{q, z}\left|\psi^{i}\right\rangle . \tag{1.5}
\end{equation*}
$$

This contribution is directly related to the polarized collinear parton distribution functions (PDFs):

$$
\begin{equation*}
\langle\psi| S_{q, z}|\psi\rangle=\frac{1}{2} \int_{0}^{1} d x \Delta \Sigma(x)=\frac{1}{2} \sum_{j} \int_{0}^{1} d x\left[f_{j}^{+}(x)-f_{j}^{-}(x)\right], \tag{1.6}
\end{equation*}
$$

where $f_{j}^{+}\left(f_{j}^{-}\right)$represents the collinear PDF of quarks with flavor $j(j=u, d, s$, $\bar{u}, \bar{d}, \bar{s}$ ) and the same (opposite) helicity as the nucleon's. Currently, $|\psi\rangle$ cannot be directly measured, but some quantifies related to $|\psi\rangle$, such as the PDFs, can be accessed experimentally. To illustrate the experimental measurability, it is useful to take a look into the distribution functions describing the nucleon structure.

A unified framework of parton distributions in nucleons is provided by the Wigner distribution [35, 36], which is a five-dimensional quantum phase space distribution. The Wigner distribution is a function of the longitudinal momentum fraction $x$, the intrinsic transverse momentum $k_{T}$, and the intrinsic transverse coordinate $b_{T}$. It is related to the generalized transverse-momentum-dependent parton distribution (GTMD) via a transverse Fourier transformation. The transverse-momentumdependent (TMD) PDF is obtained by integrating the Wigner distribution over $b_{T}$, and the impact parameter dependent distribution (IPD) is obtained by integrating over $k_{T}$. The IPD is related to the generalized parton distribution (GPD) via a transverse Fourier transformation. The TMD PDFs and the GPDs provide threedimensional descriptions of nucleons, and they play important roles in understanding nucleon spin structures. The collinear (one-dimensional) PDF is obtained by integrating out all the transverse variables. Fig. 1.4 gives a graphical illustration of the distribution functions introduced above.

The five-dimensional quantities (the Wigner distribution and the GTMD) provides full information of a parton inside a nucleon, but is not accessible experimentally. The three-dimensional distributions can be probed experimentally, and provide important information of the nucleon structure, namely, the GPDs can be measured via the deeply virtual Compton scattering (DVCS) and exclusive meson production [37], and the TMD PDFs can be probed via the semi-inclusive DIS (SIDIS) process and the Drell-Yan process [38]. The distributions containing less information than


Figure 1.4: From Wigner distribution to charge: descriptions of the nucleon structure.
the three-dimensional ones can also be measured, namely, the form factors can be measured via the elastic lepton-hadron scattering, and the collinear PDFs can be measured via the inclusive DIS process.

In the next chapter, various ways of probing the nucleon structure will be discussed. The formalism of the inclusive lepton-hadron scattering processes and the SIDIS process will be presented. More details regarding the TMD PDFs, and the motivation of the study in this dissertation will be also presented.

## Physics Motivation

The most direct approach to investigate the hadron/nucleon structure is to carry out an anatomy with certain types of experimental probes. The electromagnetic probes include the lepton-hadron scattering, and the Drell-Yan process [38]. In the lepton-hadron scattering, the lepton scatters from the hadron (or its constituents) by exchanging one or more virtual photons. In the Drell-Yan process, a quark and an antiquark from two hadrons annihilate electromagnetically, and produce a leptonantilepton pair. The final state particles in both of these processes carry the information of the hadron structure. There are also strong-interaction and weak-interaction probes in different processes, which are used to study various features of the hadrons. In this dissertation, we focus on the lepton-hadron scattering processes.

Different aspects of the hadron/nucleon structure can be studied by the leptonhadron scattering with virtual photons carrying different energies. At low lepton beam energies, the virtual photons carry low energies, and the elastic scattering process dominates. The distributions of the charge and magnetic moment inside the nucleon can be studied in this process.

With higher beam energies, the inelastic processes become dominant. In these processes, the target nucleon is either excited to a resonance state or shattered into fragments, and the particles in the final state are no longer the same as in the initial state. The DIS process provides the access to the quark and gluon distributions inside the nucleon and has significantly enhanced our understanding of the nucleon structure in the past decades.

In the inclusive DIS process, the collinear PDFs can be studied via the measurements of the structure functions $F_{1}, F_{2}, g_{1}$ and $g_{2}$. In the SIDIS process, the information of the 3D momentum distributions and the momentum-spin correlations of the partons inside the nucleon can be accessed via the study of the TMD PDFs. These studies will enhance our understanding of the nucleon spin composition and the QCD dynamics.

In this chapter, the inclusive and the semi-inclusive scattering processes and the related theoretical formalisms will be presented first. Then a discussion of the TMD PDFs and the TMD factorization will be presented. The simple quark-parton model widely used in the SIDIS phenomenology and a review of the typical phenomenological studies will be presented afterwards. Lastly, the motivation of the study in this dissertation will be discussed.

### 2.1 Inclusive scattering processes

In the inclusive scattering process, an incoming lepton scatters from a target nucleus or a beam ion and only the scattered lepton is detected. This process can be expressed as

$$
\begin{equation*}
L(l)+N(P) \rightarrow L^{\prime}\left(l^{\prime}\right)+X\left(P_{X}\right), \tag{2.1}
\end{equation*}
$$

where $L$ is the incoming lepton, $N$ is the target nucleus/nucleon or the beam ion, $L^{\prime}$ is the scattered lepton being detected, and $X$ represents the final state particles not
being detected. The variables in the parentheses are the corresponding four-momenta of the particles.

### 2.1.1 Inclusive scattering on a structureless particle

The differential cross section of an electron scattering from an unpolarized structureless spin- $\frac{1}{2}$ particle with mass $M$ can be expressed with the one-photon-exchange approximation as

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{\alpha^{2}}{4 p^{2} \beta^{2} \sin ^{4} \frac{\theta}{2}}\left(1-\beta^{2} \sin ^{2} \frac{\theta}{2}+\delta\right), \tag{2.2}
\end{equation*}
$$

where $\alpha$ is the electromagnetic fine structure constant, $\beta$ is the speed of the incoming electron, $p$ is the momentum of the incoming electron, and $\theta$ is the polar angle of the scattered electron. The $\delta$ term is defined as

$$
\begin{equation*}
\delta=\frac{1}{2 E^{2} M^{2}} M\left(\sqrt{M^{2}+p^{2}}-M\right)\left(2 E^{2}-m_{e}^{2}\right) \tag{2.3}
\end{equation*}
$$

where $E$ is the energy of the incoming electron, and $m_{e}$ is the mass of the electron. This term vanishes in the limit of $M \rightarrow \infty$. All the kinematical variables are defined in the target rest frame. In many cases, the Mott cross section is defined in the limits of $M \rightarrow \infty$ and $m_{e} \rightarrow 0$ as

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{\mathrm{Mott}}=\frac{\alpha^{2} \cos ^{2} \frac{\theta}{2}}{4 E^{2} \sin ^{4} \frac{\theta}{2}} \tag{2.4}
\end{equation*}
$$

### 2.1.2 Inclusive elastic scattering

The differential cross section of the elastic electron-proton (ep) scattering is expressed as

$$
\begin{align*}
\left(\frac{d \sigma}{d \Omega}\right)_{\text {Elastic }} & =\frac{\alpha^{2}}{4 E^{2} \sin ^{4} \frac{\theta}{2}} \frac{E^{\prime}}{E}\left(\frac{G_{E}^{2}+\tau G_{M}^{2}}{1+\tau} \cos ^{2} \frac{\theta}{2}+2 \tau G_{M}^{2} \sin ^{2} \frac{\theta}{2}\right) \\
& =\left(\frac{d \sigma}{d \Omega}\right)_{\text {Mott }} \frac{E^{\prime}}{E}\left(\frac{G_{E}^{2}+\tau G_{M}^{2}}{1+\tau}+2 \tau G_{M}^{2} \tan ^{2} \frac{\theta}{2}\right), \tag{2.5}
\end{align*}
$$

where $\tau=Q^{2} / 4 M^{2}, E^{\prime}$ is the energy of the scattered electron, $M$ is the proton mass, and $Q^{2}=-\left(l-l^{\prime}\right)^{2}=4 E E^{\prime} \sin ^{2} \frac{\theta}{2}$ is the transferred four-momentum squared. Using energy-momentum conservation in the elastic process, $E^{\prime}$ can be expressed as

$$
\begin{equation*}
E^{\prime}=\frac{E}{1+2 \frac{E}{M} \sin ^{2} \frac{\theta}{2}} . \tag{2.6}
\end{equation*}
$$

The quantities $G_{E}\left(Q^{2}\right)$ and $G_{M}\left(Q^{2}\right)$ are the electromagnetic form factors, which reflect the electric charge and the magnetic moment distributions inside the proton. It is observed that when a particle with internal structures is probed by the elastic scattering, non-vanishing form factors will enter the amplitude and give additional factors to the Mott cross section.

The form factors $G_{E}\left(Q^{2}\right)$ and $G_{M}\left(Q^{2}\right)$ are functions of $Q^{2}$, and are usually interpreted as the Fourier transformation of the electric charge and magnetic moment distributions inside the nucleon in the Breit frame. More details of the interpretations of the form factors can be found in [39].

### 2.1.3 Inclusive inelastic scattering

The variables needed in the description of the inclusive inelastic scattering processes are listed below.

- The momentum-transfer squared is $Q^{2}=-q^{2}$, where $q=l-l^{\prime}$ is the momentumtransfer four-vector.
- The Bjorken scaling factor is defined as $x_{b j}=Q^{2} /(2 P \cdot q)$ and represents the fraction of the nucleon's longitudinal momentum carried by the parton.
- The invariant mass of $X$ is defined as $W=\sqrt{(P+q)^{2}}$.

In the modest $Q^{2}$ region, the target nucleon may be excited to a resonance state with invariant mass $W \approx M_{\text {resonance }}$. With large enough $Q^{2}$, the $W$ spectrum becomes a smooth distribution instead of the resonance peaks. The DIS refers to the
inelastic scattering process with large $Q^{2}$ and $W$ in the smooth distribution region (usually $W>2 \mathrm{GeV} / c^{2}$ ). There is no longer a simple expression for the energy of the scattered lepton as in the elastic scattering process. The unpolarized inelastic inclusive differential cross section is expressed as

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega d E^{\prime}}\right)_{\text {inelastic }}=\left(\frac{d \sigma}{d \Omega}\right)_{\text {Mott }}\left[W_{2}\left(\nu, Q^{2}\right)+2 W_{1}\left(\nu, Q^{2}\right) \tan ^{2} \frac{\theta}{2}\right], \tag{2.7}
\end{equation*}
$$

where $\nu=E-E^{\prime}$ is the transferred energy in the target rest frame, $W_{1}$ and $W_{2}$ are the structure functions. In the literature, it is also expressed in a different way:

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega d E^{\prime}}\right)_{\text {inelastic }}=\left(\frac{d \sigma}{d \Omega}\right)_{\mathrm{Mott}}\left[\frac{2}{M} F_{1}\left(x_{b j}, Q^{2}\right) \tan ^{2} \frac{\theta}{2}+\frac{1}{\nu} F_{2}\left(x_{b j}, Q^{2}\right)\right], \tag{2.8}
\end{equation*}
$$

where the structure functions are $F_{1}\left(x_{b j}, Q^{2}\right)=M W_{1}\left(\nu, Q^{2}\right)$ and $F_{2}\left(x_{b j}, Q^{2}\right)=$ $\nu W_{2}\left(\nu, Q^{2}\right)$.

The differences between the differential cross sections with opposite beam or target polarizations ( $\Delta \sigma_{b}$ or $\Delta \sigma_{t}$ ) are determined by the polarized structure functions $g_{1}$ and $g_{2}$ while the contribution from $F_{1}$ and $F_{2}$ are cancelled. The $\Delta \sigma_{b}$ and $\Delta \sigma_{t}$ can be calculated from the product of the antisymmetric parts of the leptonic tensor and the hadronic tensor as in [40].

The antisymmetric part of the leptonic tensor is

$$
\begin{equation*}
L_{\mu \nu}^{(A)}=m \epsilon_{\mu \nu \alpha \beta} s^{\alpha} q^{\beta} \tag{2.9}
\end{equation*}
$$

where $m$ is the electron mass, $s^{\alpha}$ is the beam-spin four-vector, $q^{\beta}$ is the momentumtransfer four-vector, and $\epsilon_{\mu \nu \alpha \beta}$ is the totally antisymmetric tensor. The antisymmetric part of the hadronic tensor is

$$
\begin{equation*}
W_{\mu \nu}^{(A)}=2 M \epsilon_{\mu \nu \alpha \beta} q^{\alpha}\left[M S^{\beta} G_{1}+\left(P \cdot q S^{\beta}-S \cdot q P^{\beta}\right) \frac{G_{2}}{M}\right] \tag{2.10}
\end{equation*}
$$

where $M$ is the target mass, $S^{\beta}$ is the target-spin four-vector, and $P$ is the target four-momentum. The polarized structure functions are $G_{1}=g_{1} /\left(M^{2} \nu\right)$ and $G_{2}=$ $g_{2} /\left(M \nu^{2}\right)$.

The difference between the polarized differential cross sections with opposite target polarizations $\left(\Delta \sigma_{t}\right)$ is calculated in [40], and the one with opposite beam polarizations ( $\Delta \sigma_{b}$ ) can be calculated in a similar way as

$$
\begin{align*}
\Delta \sigma_{b}= & \frac{8 \alpha^{2} m E^{\prime}}{Q^{4} E}\left[M G_{1}\left(q \cdot S q \cdot s+Q^{2} S \cdot s\right)\right. \\
& \left.+G_{2}\left(Q^{2} / M\right)(P \cdot q S \cdot s-q \cdot S P \cdot s)\right] \tag{2.11}
\end{align*}
$$

where $s(S)$ is the beam (target) polarization four-vector. In the target rest frame, the four-vectors in Eq. (2.11) can be expressed explicitly as

$$
\begin{align*}
k^{\mu} & =E(1,0,0,1) \\
k^{\prime \mu} & =E^{\prime}(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
q^{\mu} & =k^{\mu}-k^{\prime \mu} \\
s^{\mu} & =\frac{E}{m}(1,0,0,1) \\
S^{\mu} & =(0, \sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha) \tag{2.12}
\end{align*}
$$

where the definition of the angles are shown in Fig. 2.1.
Substituting Eq. (2.12) into Eq. (2.11), the following expression can be obtained:

$$
\begin{align*}
\Delta \sigma_{b}= & -\frac{4 \alpha^{2} E^{\prime}}{Q^{2} E}\left[M G_{1}\left(E \cos \alpha+E^{\prime} \cos \Theta\right)\right. \\
& \left.+2 E E^{\prime} G_{2}(\cos \Theta-\cos \alpha)\right] \tag{2.13}
\end{align*}
$$

where $\Theta$, the relative angle between $\hat{k}$ and $\hat{S}$, is defined as

$$
\begin{align*}
\cos \Theta & =\sin \theta \cos \phi \sin \alpha \cos \beta+\sin \theta \sin \phi \sin \alpha \sin \beta+\cos \theta \cos \alpha \\
& =\sin \theta \sin \alpha \cos \phi_{\text {rela }}+\cos \theta \cos \alpha \tag{2.14}
\end{align*}
$$



Figure 2.1: Definition of the angles.
where $\phi_{\text {rela }}=\phi-\beta$ is the azimuthal angle of the scattered electron with respect to the target polarization.

Experimental data with the parallel-target-polarization ( $\alpha=0^{\circ}$ ) and the perpendicular-target-polarization $\left(\alpha=90^{\circ}\right)$ are usually combined to access $g_{1}$ and $g_{2}$. For $\alpha=0^{\circ}$, $\Delta \sigma_{b}$ is expressed as

$$
\begin{equation*}
\Delta \sigma_{\|}=-\frac{4 \alpha^{2} E^{\prime}}{Q^{2} E M \nu}\left[g_{1}\left(E+E^{\prime} \cos \theta\right)-2 x_{b j} M g_{2}\right] \tag{2.15}
\end{equation*}
$$

and for $\alpha=90^{\circ}$, it is expressed as

$$
\begin{equation*}
\Delta \sigma_{\perp}=-\frac{4 \alpha^{2} E^{\prime 2}}{Q^{2} E M \nu} \sin \theta \cos \phi_{\text {rela }}\left[g_{1}+\frac{2 E}{\nu} g_{2}\right] \tag{2.16}
\end{equation*}
$$

It should be noted that there are negative signs in $\Delta \sigma_{\|}$and $\Delta \sigma_{\perp}$ as one subtracts anti-parallel beam polarization $s^{\mu}=-\frac{E}{m}(1,0,0,1)$ from parallel beam polarization $s^{\mu}=\frac{E}{m}(1,0,0,1)$ while the target polarization $S^{\mu}=(0, \sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha)$ is fixed. As one can readily see from Eq. (2.11), flipping the target spin direction by
$S^{\prime}=-S$ will change the signs of $\Delta \sigma_{\| \mid}$and $\Delta \sigma_{\perp}$. In the parallel target polarization $\left(\alpha=0^{\circ}\right)$, the target spin is flipped by changing $\alpha$ to $180^{\circ}$. In the perpendicular target polarization $\left(\alpha=90^{\circ}\right)$, the target spin is flipped by a $180^{\circ}$ change of $\phi_{\text {rela }}$.

In the simple quark-parton model, the structure functions $F_{1}\left(x_{b j}, Q^{2}\right), F_{2}\left(x_{b j}, Q^{2}\right)$ and $g_{1}\left(x_{b j}, Q^{2}\right)$ can be expressed in terms of the PDFs as

$$
\begin{align*}
F_{1} & =\frac{1}{2} \sum_{q} e_{q}^{2}\left(f_{1}^{q}+f_{1}^{\bar{q}}\right),  \tag{2.17}\\
F_{2} & =x_{b j} \sum_{q} e_{q}^{2}\left(f_{1}^{q}+f_{1}^{\bar{q}}\right),  \tag{2.18}\\
g_{1} & =\frac{1}{2} \sum_{q} e_{q}^{2}\left(g_{1 L}^{q}+g_{1 L}^{\bar{q}}\right), \tag{2.19}
\end{align*}
$$

where $f_{1}$ is the unpolarized PDF and $g_{1 L}^{q}=f_{1}^{q \uparrow}-f_{1}^{q \downarrow}$ is the longitudinally polarized PDF. The twist-2 contribution to the structure function $g_{2}$ can be obtained through the Wandzura-Wilczek relation [41]:

$$
\begin{equation*}
g_{2}=-g_{1}+\int_{x_{b j}}^{1} \frac{d y}{y} g_{1}(y) . \tag{2.20}
\end{equation*}
$$

At the Bjorken limit $Q^{2} \rightarrow \infty$ and $\nu \rightarrow \infty$, the structure functions $F_{1}$ and $F_{2}$ depend only on $x_{b j}$. This feature is known as the Bjorken scaling [42]. In general, the structure functions and the PDFs also depend on $Q^{2}$. With the QCD factorization theorem, the PDFs at different factorization scales (usually chosen as $Q^{2}$ ) are connected via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [43, 44, 45, 46]. By fitting the world data in a broad kinematic range, the PDFs have been extracted and saved in global databases such as unpolarized PDF sets CTEQ [47] and MSTR [48] and polarized PDF sets DSSV [49] and GRSV [50]. The $Q^{2}$ dependence of the PDFs in the databases follows the DGLAP evolution equations.

### 2.2 Semi-inclusive scattering process

The semi-inclusive scattering process with the DIS kinematics is known as SIDIS, and will be discussed in detail. The SIDIS process provides access to the TMD PDFs and paves the way to a more detailed understanding of the nucleon structure. The SIDIS process, in which a leading hadron is detected in addition to the detection of the scattered lepton, can be expressed as

$$
\begin{equation*}
L(l)+N(P) \rightarrow L^{\prime}\left(l^{\prime}\right)+H\left(P_{h}\right)+X\left(P_{X}\right), \tag{2.21}
\end{equation*}
$$

where $L$ is the incoming lepton, $N$ is the target nucleon, $L^{\prime}$ is the scattered lepton being detected, $H$ is the detected hadron, and $X$ represents the final state particles not being detected. The variables in the parentheses are the four-momenta. A graphic illustration of the SIDIS process is presented in Fig. 2.2.


Figure 2.2: The SIDIS process and the angle definitions. The figure is from [2].

The variables needed to describe the SIDIS process are listed below.

- The variables related to the incoming and scattered leptons are: $q=l-l^{\prime}$, $Q^{2}=-q^{2}, x_{b j}=Q^{2} /(2 P \cdot q), y=(P \cdot q) /(P \cdot l)$, and $W=\sqrt{(P+q)^{2}}$.
- The ratio of the energy carried by the detected hadron $H$ and by the virtual photon is defined as

$$
\begin{equation*}
z_{h}=\left(P \cdot P_{h}\right) /(P \cdot q) \tag{2.22}
\end{equation*}
$$

- The transverse momentum of the detected hadron with respect to the virtual photon is defined as

$$
\begin{equation*}
P_{t}=\frac{\vec{q} \cdot \overrightarrow{P_{h}}}{|\vec{q}|} . \tag{2.23}
\end{equation*}
$$

- The missing mass $W^{\prime}$ is defined as

$$
\begin{equation*}
W^{\prime}=\sqrt{\left(q+P-P_{h}\right)^{2}}=\sqrt{P_{X}^{2}} \tag{2.24}
\end{equation*}
$$

- The azimuthal angle $\phi_{h}$ between the lepton plane and the hadron plane is defined following the "Trento Conventions" [2], as in Fig. 2.2. In the target rest frame, this convention is expressed as the following:

$$
\begin{align*}
\cos \phi_{h} & =\frac{\hat{q} \times \vec{l}}{|\hat{q} \times \vec{l}|} \cdot \frac{\hat{q} \times \vec{P}_{h}}{\left|\hat{q} \times \vec{P}_{h}\right|}, \\
\sin \phi_{h} & =\frac{\left(\vec{l} \times \vec{P}_{h}\right) \cdot \hat{q}}{|\hat{q} \times \vec{l}|\left|\hat{q} \times \vec{P}_{h}\right|}, \tag{2.25}
\end{align*}
$$

where $\hat{q}=\vec{q} /|\vec{q}|$.

- The azimuthal angle $\phi_{S}$ between the lepton plane and the spin vector is illustrated in Fig. 2.2.

The SIDIS differential cross section can be expressed as a contraction between the leptonic tensor $L_{\mu \nu}$ and the hadronic tensor $W^{\mu \nu}$ as [51]

$$
\begin{equation*}
\frac{d \sigma}{d x_{b j} d y d z_{h} d \phi_{S} d P_{t}^{2} d \phi_{h}}=\frac{\alpha^{2} y}{8 z_{h} Q^{4}} 2 M W^{\mu \nu} L_{\mu \nu} \tag{2.26}
\end{equation*}
$$

The leptonic tensor is defined as [51]

$$
\begin{equation*}
L_{\mu \nu}=2\left(l_{\mu} l_{\nu}^{\prime}+l_{\mu}^{\prime} l_{\nu}-l \cdot l^{\prime} g_{\mu \nu}\right)+2 \lambda_{e} \epsilon_{\mu \nu \rho \sigma} l^{\rho} l^{\sigma \prime} \tag{2.27}
\end{equation*}
$$

The hadronic tensor is defined as [51]

$$
\begin{align*}
W^{\mu \nu}= & \frac{1}{(2 \pi)^{3}} \frac{1}{2 M} \sum_{X} \int \frac{d^{3} \mathbf{P}_{\mathbf{x}}}{2 P_{X}^{0}} \delta^{(4)}\left(q+P-P_{X}-P_{h}\right) \\
& \times\langle P| J^{\mu}(0)|h, X\rangle\langle h, X| J^{\nu}(0)|P\rangle, \tag{2.28}
\end{align*}
$$

where $J^{\mu}(\xi)$ is the electromagnetic current divided by the elementary charge and a sum is implied over the polarizations of all hadrons in the final state. The hadronic tensor, at the leading order of the $1 / Q$ expansion, can be expressed as a convolution of the correlation functions $\Phi$ for the quark distributions, and $\Delta$ for the quark fragmentation [51]

$$
\begin{align*}
W^{\mu \nu}= & \frac{z_{h}}{M} \sum_{q} e_{q}^{2} x_{b j} \int d^{2} \mathbf{k}_{\perp} d^{2} \mathbf{p}_{\perp} \delta^{(2)}\left(\mathbf{p}_{\perp}+z_{h} \mathbf{k}_{\perp}-\mathbf{P}_{\mathbf{t}}\right) \\
& \times \operatorname{Tr}\left[\Phi\left(\mathrm{x}_{\mathrm{bj}}, \mathrm{k}_{\perp}\right) \gamma^{\mu} \Delta\left(\mathrm{z}_{\mathrm{h}}, \mathrm{p}_{\perp}\right) \gamma^{\nu}\right] \tag{2.29}
\end{align*}
$$

where $k_{\perp}$ is the intrinsic transverse momentum of the parton, and $p_{\perp}$ is the transverse momentum of the fragmenting hadron with respect to the parton. In general, $\Phi$ can be expressed as [51]

$$
\begin{equation*}
\Phi\left(x_{b j}, k_{\perp}\right)=\sum_{i} A_{i}\left(x_{b j}, k_{\perp}\right) \hat{O}_{i} \tag{2.30}
\end{equation*}
$$

where $A_{i}\left(x_{b j}, k_{\perp}\right)$ represents the factor for the $i$ th operator $\hat{O}_{i}$. Each operator $\hat{O}_{i}$ has a unique Lorentzian structure and spin dependence, and the $i$ th type of the TMD PDFs is defined as $A_{i}\left(x_{b j}, k_{\perp}\right)$ times a coefficient based on a certain convention. Similarly, the TMD FFs are defined in the expansion of $\Delta$. More details regarding the definitions of the TMD PDFs and the TMD FFs, and the explicit forms of the operators can be found in [51].

The differential cross section of the SIDIS process can be expressed in terms of 18 structure functions, which are categorized based on the beam and target polarizations, and the azimuthal angular dependence [51]:

$$
\begin{align*}
\frac{d \sigma}{d x_{b j} d y d z_{h} d \phi_{S} d P_{t}^{2} d \phi_{h}}= & \frac{\alpha^{2}}{2 Q^{2} x_{b j} y} \frac{y^{2}}{2(1-\epsilon)} \\
& \times\left\{F_{U U, T}+\epsilon F_{U U, L}+\sqrt{2 \epsilon(1+\epsilon)} \cos \phi_{h} F_{U U}^{\cos \phi_{h}}\right. \\
& +\epsilon \cos 2 \phi_{h} F_{U U}^{\cos 2 \phi_{h}}+\lambda_{e} \sqrt{2 \epsilon(1-\epsilon)} \sin \phi_{h} F_{L U}^{\sin \phi_{h}} \\
& +S_{\|}\left(\sqrt{2 \epsilon(1+\epsilon)} \sin \phi_{h} F_{U L}^{\sin \phi_{h}}+\epsilon \sin 2 \phi_{h} F_{U L}^{\sin 2 \phi_{h}}\right) \\
& +S_{\|} \lambda_{e}\left(\sqrt{1-\epsilon^{2}} F_{L L}+\sqrt{2 \epsilon(1-\epsilon)} \cos \phi_{h} F_{L L}^{\cos \phi_{h}}\right) \\
& +\left|\vec{S}_{\perp}\right|\left[\sin \left(\phi_{h}-\phi_{S}\right)\left(F_{U T, T}^{\sin \left(\phi_{h}-\phi_{S}\right)}+\epsilon F_{U T, L}^{\sin \left(\phi_{h}-\phi_{S}\right)}\right)\right. \\
& +\epsilon \sin \left(\phi_{h}+\phi_{S}\right) F_{U T}^{\sin \left(\phi_{h}+\phi_{S}\right)}+\epsilon \sin \left(3 \phi_{h}-\phi_{S}\right) F_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)} \\
& +\sqrt{2 \epsilon(1+\epsilon)} \sin \phi_{S} F_{U T}^{\sin \phi_{S}} \\
& \left.+\sqrt{2 \epsilon(1+\epsilon)} \sin \left(2 \phi_{h}-\phi_{S}\right) F_{U T}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}\right] \\
& +\left|\vec{S}_{\perp}\right| \lambda_{e}\left[\sqrt{1-\epsilon^{2}} \cos \left(\phi_{h}-\phi_{S}\right) F_{L T}^{\cos \left(\phi_{h}-\phi_{S}\right)}\right. \\
& +\sqrt{2 \epsilon(1-\epsilon)} \cos \phi_{S} F_{U T}^{\cos \phi_{S}} \\
& \left.++\sqrt{2 \epsilon(1-\epsilon)} \cos \left(2 \phi_{h}-\phi_{S}\right) F_{L T}^{\cos \left(2 \phi_{h}-\phi_{S}\right)}\right\} \tag{2.31}
\end{align*}
$$

where $\epsilon=(1-y) /\left(1-y+y^{2} / 2\right)$, and $\lambda_{e}$ is the lepton beam helicity. The quantities $S_{\|}$and $S_{\perp}$ are defined by decomposing the covariant target spin vector $S$ as in the following expressions:

$$
\begin{align*}
S_{\|} & =\frac{S \cdot q}{P \cdot q} \frac{M}{\sqrt{1+\gamma^{2}}}  \tag{2.32}\\
S_{\perp}^{\mu} & =S^{\mu}-S_{\|} \frac{P^{\mu}-q^{\mu} M^{2} / P \cdot q}{M \sqrt{1+\gamma^{2}}} \tag{2.33}
\end{align*}
$$

where $\gamma=2 M x_{b j} / Q$.

### 2.3 TMD PDFs and TMD factorization

The TMD PDFs are functions of the transverse momentum of the partons in the nucleon, in addition to the longitudinal momentum fraction $x_{b j}$. While there are different models addressing the functional forms of the transverse momentum dependence of the TMD PDFs [52], the Gaussian ansatz is most widely used in the phenomenological studies of the world data of SIDIS and $e^{+} e^{-}$annihilation [4, 5, 10, 53].

The twist is widely used in categorizing the TMD PDFs and the terms in the expression of the SIDIS cross section. One of the generally used definitions is according to the powers of (mass $/ Q$ ) in a term, and the twist of a term with a factor (mass $/ Q)^{t-2}$ is defined to be $t[54]$. The lowest twist (leading twist) is $t=2$.

At the leading twist, there are eight TMD PDFs for a nucleon [55, 56]. These eight TMD PDFs are presented in Fig. 2.3 according to the nucleon and quark polarizations. In this figure, U, L and T represent no polarization, longitudinal polarization and transverse polarization, respectively. Only three leading-twist collinear PDFs do not vanish, and they can be obtained by integrating out the transverse momentum of the three TMD PDFs on the diagonal in Fig. 2.3, namely the unpolarized $\left(f_{1}\right)$, the helicity $\left(g_{1}\right)$, and the transversity $\left(h_{1}\right)$.


Figure 2.3: The eight leading twist TMD PDFs with different nucleon and quark polarizations.

Definitions of the twist-2, twist-3 and twist-4 TMD PDFs in the simple partonmodel picture, can be found in [51]. The TMD PDFs and SIDIS cross sections beyond the simple quark-parton model can be better understood with a closer look into the formalism of TMD factorization.

The Collins-Soper-Sterman (CSS) formalism of factorization was proposed by Collins et al. in the 1980s [57, 58, 59]. An updated version of the CSS formalism was later developed [60]. The CSS formalism involves the TMD PDFs and the TMD FFs with the evolution equations and has properties like the universality. In the CSS formalism the cross section is separated into a $W$ term and a $Y$ term. The pure TMD-factorization term $(W)$ dominates in the $q_{T} \ll Q$ region, and the term using the
collinear factorization $(Y)$ provides the necessary corrections in the $q_{T} \gtrsim Q$ region. Lots of theoretical efforts have been put in the details of the evolution, and the CSS formalism is still being updated to properly relate the experimental measurements to the TMD theory [61].

A full description of the TMD factorization and the CSS formalism is beyond the scope of this dissertation. In the following, key features of the TMD factorization closely related to the SIDIS phenomenology are discussed.

The SIDIS cross section in the simple parton-model picture is a hard partonic subprocess folded with the TMD PDFs and the TMD FFs. Following [62], the hadronic tensor in Eq. (2.28) can be expressed as

$$
\begin{align*}
W^{\mu \nu}= & \sum_{q}\left|\mathcal{H}_{q}(Q)^{2}\right|^{\mu \nu} \int d^{2} \mathbf{k}_{\perp} d^{2} \mathbf{p}_{\perp} \delta^{(2)}\left(\mathbf{p}_{\perp}+z_{h} \mathbf{k}_{\perp}-\mathbf{P}_{\mathbf{t}}\right) \\
& \times f_{q}\left(x_{b j}, k_{\perp}\right) D_{q}\left(z_{h}, p_{\perp}\right) \tag{2.34}
\end{align*}
$$

where $\left|\mathcal{H}_{q}(Q)^{2}\right|^{\mu \nu}$ is the hard partonic subprocess $\gamma^{*} q \rightarrow q$ for a virtual photon $\gamma^{*}$ scattering off a quark $q$. This simple form in Eq. (2.34) allows the TMD PDF $f_{q}\left(x_{b j}, k_{\perp}\right)$ and the TMD FF $D_{q}\left(z_{h}, p_{\perp}\right)$ to have simple probabilistic interpretations. The TMD PDF $f_{q}\left(x_{b j}, k_{\perp}\right)$ represents the probability density to find a quark of flavor $q$ with momentum fraction $x_{b j}$ and transverse momentum $k_{\perp}$ inside the nucleon. The TMD FF $D_{q}\left(z_{h}, p_{\perp}\right)$ is the probability density to find a hadron fragmenting from a quark. Longitudinally, the hadron carries a fraction $z_{h}$ of the virtual photon's momentum and it has a transverse momentum $p_{\perp}$ from the fragmentation process.

It is more complicated when the evolution is included in the TMD factorization. Firstly, there are divergences beyond the standard ultra-violet divergences associated with the renormalizations. The regularization of these divergences brings complications to the definitions of the TMD PDFs and the TMD FFs. In addition, there is an additional soft factor to take into account the effect from the soft gluons. With
this factor in Eq. (2.34), there can no longer be a simple parton-model picture.
The CSS formalism was applied to the SIDIS processes in a few different methods. One of the methods developed by Ji et al. [63] formulates the spin-independent SIDIS structure function as

$$
\begin{align*}
F\left(x_{b j}, z_{h}, P_{t}, Q^{2}\right)= & \sum_{q} e_{q}^{2} \int d^{2} \mathbf{k}_{\perp} d^{2} \mathbf{p}_{\perp} d^{2} \mathbf{l}_{\perp} \delta^{(2)}\left(\mathbf{p}_{\perp}+z_{h} \mathbf{k}_{\perp}+\mathbf{l}_{\perp}-\mathbf{P}_{\mathbf{t}}\right) \\
& \times f_{q}\left(x_{b j}, k_{\perp}, \mu^{2}, x_{b j} \zeta_{x}, \rho\right) D_{q}\left(z_{h}, p_{\perp}, \mu^{2}, \zeta_{z} / z_{h}, \rho\right) \\
& \times S\left(\mathbf{l}_{\perp}, \mu^{2}, \rho\right) H\left(Q^{2}, \mu^{2}, \rho\right) \tag{2.35}
\end{align*}
$$

where $\mu$ is a renormalization scale, $\rho$ a gluon rapidity cutoff parameter. $x_{b j}^{2} \zeta_{x}^{2}=$ $\zeta_{z}^{2} / z_{h}^{2}=Q^{2} \rho . H$ represents the contribution of the parton hard scattering. The explicit soft factor, $S$, is from soft-gluon radiation.

Efforts have been taken to retain both the essential elements of TMD factorization (the evolution and the soft factor) and the resemblance to Eq. (2.34) for an intuitive physical picture. A typical approach of this type can be found in [62] formulating the hadronic tensor as

$$
\begin{align*}
W^{\mu \nu}= & \sum_{q}\left|\mathcal{H}_{q}(Q)^{2}\right|^{\mu \nu} \int d^{2} \mathbf{k}_{\perp} d^{2} \mathbf{p}_{\perp} \delta^{(2)}\left(\mathbf{p}_{\perp}+z_{h} \mathbf{k}_{\perp}-\mathbf{P}_{\mathbf{t}}\right) \\
& \times f_{q}\left(x_{b j}, k_{\perp} ; \mu ; \zeta_{F}\right) D_{q}\left(z_{h}, p_{\perp} ; \mu ; \zeta_{D}\right)+Y^{\mu \nu} \tag{2.36}
\end{align*}
$$

where $\zeta_{F}=2 M^{2} x_{b j}^{2} e^{2\left(y_{N}-y_{s}\right)}$ and $\zeta_{D}=2\left(M_{h}^{2} / z_{h}^{2}\right) e^{2\left(y_{s}-y_{h}\right)} . y_{N}$ and $y_{h}$ are the rapidities of the nucleon and the produced hadron, respectively. The $y_{s}$ is an arbitrary low-rapidity cutoff parameter. The soft factor $S$ in Eq. (2.35) is absorbed in the definitions of the TMD PDF $f_{q}\left(x_{b j}, k_{\perp} ; \mu ; \zeta_{F}\right)$ and the TMD FF $D_{q}\left(z_{h}, p_{\perp} ; \mu ; \zeta_{D}\right)$. The $Y$ term accounting for the corrections in the large $\mathbf{P}_{\mathbf{t}}$ range is included as well.

### 2.4 TMD PDFs and SIDIS in the simple quark-parton model

In the simple quark-parton model, part of the evolution effect (beyond the collinear evolution), the soft factor, the cutoffs and the $Y$ correction term are neglected, and the structure functions of the SIDIS process can be obtained by the simple convolution of the TMD PDFs and the TMD FFs. This simple model is often called naive $x-z$ factorization or generalized parton-model (GPM) [62]. In general, the convolution is defined as [51]

$$
\begin{align*}
\mathcal{C}[w f D]= & \sum_{q} e_{q}^{2} x_{b j} \int d^{2} \mathbf{k}_{\perp} d^{2} \mathbf{p}_{\perp} \delta^{(2)}\left(\mathbf{p}_{\perp}+z_{h} \mathbf{k}_{\perp}-\mathbf{P}_{\mathbf{t}}\right) \\
& \times w\left(k_{\perp}, p_{\perp}\right) f_{q}\left(x_{b j}, k_{\perp}\right) D_{q}\left(z_{h}, p_{\perp}\right) . \tag{2.37}
\end{align*}
$$

In Eq. (2.37), the unpolarized TMD PDF $f_{q}\left(x_{b j}, k_{\perp}\right)$ and the unpolarized TMD FF $D_{q}\left(z_{h}, p_{\perp}\right)$ in the conventional notation, are used to represent a certain TMD PDF and a certain TMD FF in general.

The Gaussian ansatz is widely used in the parameterizations of the TMD PDFs and the TMD FFs $[3,4,5,10]$. For example, the unpolarized TMD PDF $f_{q}\left(x_{b j}, k_{\perp}\right)$ and the unpolarized TMD FF $D_{q}\left(z_{h}, p_{\perp}\right)$ are often parameterized as

$$
\begin{align*}
& f_{q}\left(x_{b j}, k_{\perp}\right)=f_{q}^{c}\left(x_{b j}\right) e^{-k_{\perp}^{2} /\left\langle k_{\perp}^{2}\right\rangle} /\left(\pi\left\langle k_{\perp}^{2}\right\rangle\right),  \tag{2.38}\\
& D_{q}\left(z_{h}, p_{\perp}\right)=D_{q}^{c}\left(z_{h}\right) e^{-p_{\perp}^{2} /\left\langle p_{\perp}^{2}\right\rangle} /\left(\pi\left\langle p_{\perp}^{2}\right\rangle\right) \tag{2.39}
\end{align*}
$$

The phenomenological parameters, $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$, are the Gaussian widths to describe the transverse momentum distributions.

The unpolarized SIDIS differential cross section is obtained by removing the
polarization-dependent terms in Eq. (2.31) and is expressed as

$$
\begin{align*}
\frac{d \sigma}{d x_{b j} d y d z_{h} d \phi_{S} d P_{t}^{2} d \phi_{h}}= & \frac{\alpha^{2}}{2 Q^{2} x_{b j} y}\left[\left(1+(1-y)^{2}\right) \cdot F_{U U}\right. \\
& +2(2-y) \sqrt{1-y} \cdot F_{U U}^{\cos \phi_{h}} \cos \phi_{h} \\
& \left.+2(1-y) \cdot F_{U U}^{\cos 2 \phi_{h}} \cos 2 \phi_{h}\right] \tag{2.40}
\end{align*}
$$

The structure function $F_{U U}$ involves a convolution of the unpolarized TMD PDF $f_{q}\left(x_{b j}, k_{\perp}\right)$ and the unpolarized TMD FF $D_{q}\left(z_{h}, p_{\perp}\right)$. The structure function $F_{U U}^{\cos \phi_{h}}$ at the lowest twist (twist-3) consists of a Cahn contribution and a Boer-Mulders contribution. The structure function $F_{U U}^{\cos 2 \phi_{h}}$ consists of a twist-2 Boer-Mulders contribution and a twist- 4 Cahn contribution. The Cahn contributions involve the convolution of the unpolarized TMD PDF $f_{q}\left(x_{b j}, k_{\perp}\right)$ and the unpolarized TMD FF $D_{q}\left(z_{h}, p_{\perp}\right)$. The Boer-Mulders contributions involve the convolution of the BoerMulders TMD PDF $\Delta f_{q \uparrow}\left(x_{b j}, k_{\perp}\right)=-h_{1}^{\perp}\left(x_{b j}, k_{\perp}\right) \cdot k_{\perp} / M_{p}$ and the Collins TMD FF $\Delta D_{q \uparrow}\left(z_{h}, p_{\perp}\right)=2 p_{\perp} \cdot H_{1}^{\perp}\left(z_{h}, p_{\perp}\right) /\left(z_{h} M_{h}\right)$. A unit vector is defined for convenience as $\mathbf{h} \equiv \mathbf{P}_{\mathbf{t}} /\left|\mathbf{P}_{\mathbf{t}}\right|$. The structure functions are given below with the momentum conservation condition $\mathbf{P}_{\mathbf{t}}=z_{h} \mathbf{k}_{\perp}+\mathbf{p}_{\perp}$.

$$
\begin{align*}
F_{U U}= & \sum_{q} e_{q}^{2} x \int d^{2} \mathbf{k}_{\perp} f_{q}\left(x_{b j}, k_{\perp}\right) D_{q}\left(z_{h}, p_{\perp}\right)  \tag{2.41}\\
\left.F_{U U}^{\cos \phi_{h}}\right|_{\mathrm{Cahn}}= & -2 \sum_{q} e_{q}^{2} x \int d^{2} \mathbf{k}_{\perp} \frac{\mathbf{k}_{\perp} \cdot \mathbf{h}}{Q} f_{q}\left(x_{b j}, k_{\perp}\right) D_{q}\left(z_{h}, p_{\perp}\right),  \tag{2.42}\\
\left.F_{U U}^{\cos \phi_{h}}\right|_{\mathrm{BM}}= & \sum_{q} e_{q}^{2} x \int d^{2} \mathbf{k}_{\perp} \frac{k_{\perp}}{Q} \frac{P_{t}-z_{h} \mathbf{k}_{\perp} \cdot \mathbf{h}}{p_{\perp}} \\
& \times \Delta f_{q \uparrow}\left(x_{b j}, k_{\perp}\right) \Delta D_{q \uparrow}\left(z_{h}, p_{\perp}\right) \tag{2.43}
\end{align*}
$$

$$
\begin{align*}
\left.F_{U U}^{\cos 2 \phi_{h}}\right|_{\mathrm{BM}}= & \sum_{q} e_{q}^{2} x \int d^{2} \mathbf{k}_{\perp} \frac{P_{t} \mathbf{k}_{\perp} \cdot \mathbf{h}+z_{h}\left[k_{\perp}^{2}-2\left(\mathbf{k}_{\perp} \cdot \mathbf{h}\right)^{2}\right]}{2 k_{\perp} p_{\perp}} \\
& \times \Delta f_{q \uparrow}\left(x_{b j}, k_{\perp}\right) \Delta D_{q \uparrow}\left(z_{h}, p_{\perp}\right),  \tag{2.44}\\
\left.F_{U U}^{\cos 2 \phi_{h}}\right|_{\text {Cahn }}= & 2 \sum_{q} e_{q}^{2} x \int d^{2} \mathbf{k}_{\perp} \frac{2\left(\mathbf{k}_{\perp} \cdot \mathbf{h}\right)^{2}-k_{\perp}^{2}}{Q^{2}} f_{q}\left(x_{b j}, k_{\perp}\right) D_{q}\left(z_{h}, p_{\perp}\right) . \tag{2.45}
\end{align*}
$$

The parameterizations of the Boer-Mulders TMD PDF and the Collins TMD FF usually involves more parameters compared with the unpolarized TMD PDF and the unpolarized TMD FF in Eqs. (2.38) and (2.39). The parameterization of the Boer-Mulders TMD PDF in [5] is expressed as

$$
\begin{equation*}
\Delta f_{q \uparrow}\left(x_{b j}, k_{\perp}\right)=\sqrt{2 e} \frac{\Delta f_{q \uparrow}\left(x_{b j}\right) k_{\perp}}{\pi\left\langle k_{\perp}^{2}\right\rangle M_{\mathrm{BM}}} e^{-k_{\perp}^{2} /\left\langle k_{\perp}^{2}\right\rangle_{\mathrm{BM}}} \tag{2.46}
\end{equation*}
$$

where the Boer-Mulders Gaussian width is defined as

$$
\begin{equation*}
\left\langle k_{\perp}^{2}\right\rangle_{\mathrm{BM}}=\left\langle k_{\perp}^{2}\right\rangle M_{\mathrm{BM}}^{2} /\left(\left\langle k_{\perp}^{2}\right\rangle+M_{\mathrm{BM}}^{2}\right), \tag{2.47}
\end{equation*}
$$

and the collinear part is parameterized as

$$
\begin{equation*}
\Delta f_{q \uparrow}\left(x_{b j}\right)=N_{q}(\alpha+\beta)^{\alpha+\beta} x_{b j}^{\alpha}\left(1-x_{b j}\right)^{\beta} f_{q}\left(x_{b j}\right) /\left(\alpha^{\alpha} \beta^{\beta}\right) \tag{2.48}
\end{equation*}
$$

The quantities $M_{\mathrm{BM}}, N_{q}, \alpha$ and $\beta$ are the phenomenological parameters.
The parameterization of the Collins TMD FF in [5] is expressed as

$$
\begin{equation*}
\Delta D_{q \uparrow}\left(z_{h}, p_{\perp}\right)=\sqrt{2 e} \frac{\Delta D_{q \uparrow}\left(z_{h}\right) p_{\perp}}{\pi\left\langle k_{\perp}^{2}\right\rangle M_{C}} e^{-p_{\perp}^{2} /\left\langle k_{\perp}^{2}\right\rangle_{C}}, \tag{2.49}
\end{equation*}
$$

where the Collins Gaussian width is defined as

$$
\begin{equation*}
\left\langle p_{\perp}^{2}\right\rangle_{C}=\left\langle p_{\perp}^{2}\right\rangle M_{C}^{2} /\left(\left\langle k_{\perp}^{2}\right\rangle+M_{C}^{2}\right) \tag{2.50}
\end{equation*}
$$

and the collinear part is parameterized as

$$
\begin{equation*}
\Delta D_{q \uparrow}\left(z_{h}\right)=N_{q}^{C}(\gamma+\delta)^{\gamma+\delta} z_{h}^{\gamma}\left(1-z_{h}\right)^{\delta} D_{q}\left(z_{h}\right) /\left(\gamma^{\gamma} \delta^{\delta}\right) \tag{2.51}
\end{equation*}
$$

The quantities $M_{C}, N_{q}^{C}, \gamma$, and $\delta$ are the phenomenological parameters.
Using the Gaussian ansatz in Eqs. (2.38) and (2.39), the convolution of TMD PDF and TMD FF can be carried out analytically. $F_{U U}$ and the Cahn parts of the structure functions $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ in Eqs. (2.41), (2.42) and (2.45) after convolution can be expressed as

$$
\begin{align*}
F_{U U} & =\sum_{q} f_{q}^{c} D_{q}^{c} \frac{e_{q}^{2} x_{b j}}{\pi\left\langle P_{t}^{2}\right\rangle} e^{-P_{t}^{2} /\left\langle P_{t}^{2}\right\rangle},  \tag{2.52}\\
F_{U U}^{\cos \phi_{h}} & =-2 \sum_{q} f_{q}^{c} D_{q}^{c} \frac{P_{t} z_{h} e_{q}^{2} x_{b j}\left\langle k_{\perp}^{2}\right\rangle}{\pi Q\left\langle P_{t}^{2}\right\rangle^{2}} e^{-P_{t}^{2} /\left\langle P_{t}^{2}\right\rangle},  \tag{2.53}\\
F_{U U}^{\cos 2 \phi_{h}} & =2 \sum_{q} f_{q}^{c} D_{q}^{c} \frac{P_{t}^{2} z_{h}^{2} e_{q}^{2} x_{b j}\left\langle k_{\perp}^{2}\right\rangle^{2}}{\pi Q^{2}\left\langle P_{t}^{2}\right\rangle^{3}} e^{-P_{t}^{2} /\left\langle P_{t}^{2}\right\rangle}, \tag{2.54}
\end{align*}
$$

where $\left\langle P_{t}^{2}\right\rangle=\left\langle p_{\perp}^{2}\right\rangle+z_{h}^{2}\left\langle k_{\perp}^{2}\right\rangle$. The Boer-Mulders parts after convolution can be found in [5].

### 2.5 Experiments and phenomenology of TMD

In this section, the phenomenological studies of the TMDs are reviewed. While it has been shown that the evolution effect is expected to be significant, most of the existing studies used the simple formalism (GPM) described in section 2.4. In this simple formalism, the structure functions of the SIDIS process are obtained by the simple convolution of the TMD PDFs and the TMD FFs as in Eq. (2.37). An example showing the power of this formalism is the agreement between the model descriptions and the experimental results of the Sivers and the Collins effects [53]. The Sivers effect emerges from the convolution of the Sivers TMD PDF and the unpolarized TMD FF. The Collins effect is from the convolution of the transversity TMD PDF and the Collins TMD FF. The Sivers and the Collins effects are related to different azimuthal modulations in the SIDIS process on a transversely polarized
nucleon [51, 53].
While factorization originates in the high energy limit $\left(Q \gg \Lambda_{\mathrm{QCD}}\right.$ or $Q \gg$ $\left.M_{\text {nucleon }}\right)[12,60]$, and at low $Q^{2}$ the description with hadronic degrees of freedom is more widely used [28], the applicability of the quark-parton model with factorization in the modest $Q^{2}$ ranges has been observed in the quark-hadron duality [64, 65].

In the polarized and the unpolarized SIDIS processes, the azimuthal modulations were found to be sizable in experiments [66, 67, 68, 69]. The intrinsic transverse momenta of the quarks are expected to play an important role in these observations [3, 70]. In the global fitting with different types of data (multiplicities and/or asymmetries) in different kinematic ranges, very different values of $\left\langle k_{\perp}^{2}\right\rangle$ were extracted. Namely $\left\langle k_{\perp}^{2}\right\rangle$ is at the level of $0.2 \mathrm{GeV}^{2}$ in [3, 10], at the level of $0.5 \mathrm{GeV}^{2}$ in [4] and less than $0.05 \mathrm{GeV}^{2}$ in [5]. While the multiplicities and asymmetries from experiments have been fitted with ratios of theoretical cross sections, the corresponding study for the absolute cross sections is rather limited.

In the following subsections, several typical existing phenomenological studies of TMDs based on the SIDIS data are reviewed.

### 2.5.1 Unpolarized SIDIS phenomenology without TMD evolution effect

In [3], the data from the European Muon Collaboration (EMC) in 1987 [66] were fitted with the unpolarized SIDIS differential cross section, integrating out all the variables except $\phi_{h}$, to probe the $\phi_{h}$ dependence of the unpolarized SIDIS cross section. The comparisons of the data and the fitting in the $\phi_{h}$ bins from [3] are presented in Fig. 2.4.

In [3], the $P_{t}$ behavior of another set of EMC data in 1991 [71] was also fitted. Instead of directly fitting the SIDIS cross section, the ratio of the SIDIS cross section and inclusive DIS cross section (multiplicity) was fitted. The comparisons of the data and the fitting in the $P_{t}$ bins from [3] are presented in Fig. 2.5. The fitting parameters


Figure 2.4: Fit on the $\phi_{h}$ dependence of the EMC data: the solid curves are up to twist-3, the dashed curves include the higher twist terms. The figure is taken from [3].
were the Gaussian widths $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$. The results combining the fitting on the $\phi_{h}$ and $P_{t}$ dependence were $\left\langle k_{\perp}^{2}\right\rangle=0.25 \mathrm{GeV}^{2}$ and $\left\langle p_{\perp}^{2}\right\rangle=0.20 \mathrm{GeV}^{2}$.

In 2014, the multiplicity data concerning the $P_{t}$ dependence of the SIDIS process from HERMES [72] and COMPASS [73] were fitted in the study [4] while the $\phi_{h}$ dependence was not included. The comparisons of the fitting and the HERMES data with $\pi^{+}$and $\pi^{-}$as produced hadrons are presented in Figs. 2.6 and 2.7. The results of fitting the HERMES data in [4] were $\left\langle k_{\perp}^{2}\right\rangle=0.57 \pm 0.08 \mathrm{GeV}^{2}$ and $\left\langle p_{\perp}^{2}\right\rangle=0.12 \pm 0.01$ $\mathrm{GeV}^{2}$. While the $\chi_{d o f}^{2}=1.69$ value from the fitting is quite satisfactory, the $\left\langle k_{\perp}^{2}\right\rangle$ value deviates from other studies significantly. The results of fitting the COMPASS data in [4] were $\left\langle k_{\perp}^{2}\right\rangle=0.61 \pm 0.20 \mathrm{GeV}^{2}$ and $\left\langle p_{\perp}^{2}\right\rangle=0.19 \pm 0.02 \mathrm{GeV}^{2}$.

In [4], instead of using $\left\langle p_{\perp}^{2}\right\rangle$ as a constant, the form $\left\langle p_{\perp}^{2}\right\rangle=N \cdot z_{h}^{a}\left(1-z_{h}\right)^{b}$ $\mathrm{GeV}^{2}$ was tested. This change improved $\chi_{d o f}^{2}=1.69$ to $\chi_{d o f}^{2}=1.63$, but gave large uncertainties to the parameters $N, a$ and $b$. The functional form, in which $\left\langle k_{\perp}^{2}\right\rangle$


Figure 2.5: Fit on $P_{t}$ dependence of EMC data: solid curves are up to twist3, dashed curves include higher twist terms. The shadowed region corresponds to varying the fitting parameters by $20 \%$. The figure is taken from [3].
depends on $x_{b j}$, was also tested. This form was from [74], as

$$
\begin{equation*}
\left\langle k_{\perp}^{2}\right\rangle=\left\langle k_{\perp}^{2}\right\rangle_{0} \frac{\left(1-x_{b j}\right)^{\alpha} x_{b j}^{\sigma}}{\left(1-x_{0}\right)^{\alpha} x_{0}^{\sigma}}, \tag{2.55}
\end{equation*}
$$

where $x_{0}=0.1$. There was no significant improvement in the fitting quality by including an $x_{b j}$ dependence of $\left\langle k_{\perp}^{2}\right\rangle$. The improvement in the fitting quality was also negligible by including a flavor dependence in $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ [4].

In [5], in addition to the multiplicity data from HERMES and COMPASS [72, 73], the data on unpolarized azimuthal modulations from HERMES and COMPASS $[75,76]$ were fitted simultaneously.

The $\cos \phi_{h}$ and $\cos 2 \phi_{h}$ modulations (also called asymmetries in [5]) are defined


Figure 2.6: Fit on $P_{t}$ dependence of HERMES $\pi^{+}$production SIDIS data. The figure is taken from [4].
as

$$
\begin{align*}
A^{\cos \phi_{h}}=2\left\langle\cos \phi_{h}\right\rangle & =2 \frac{\int d \phi_{h} d \sigma \cos \phi_{h}}{\int d \phi_{h} d \sigma}  \tag{2.56}\\
A^{\cos 2 \phi_{h}}=2\left\langle\cos 2 \phi_{h}\right\rangle & =2 \frac{\int d \phi_{h} d \sigma \cos 2 \phi_{h}}{\int d \phi_{h} d \sigma} . \tag{2.57}
\end{align*}
$$

HERMES $M_{D}^{\pi^{-}}$


Figure 2.7: Fit on $P_{t}$ dependence of HERMES $\pi^{-}$production SIDIS data. The figure is taken from [4].

In terms of structure functions, Eqs. (2.56) and (2.57) can be expressed as

$$
\begin{align*}
A^{\cos \phi_{h}} & =\frac{2(2-y) \sqrt{1-y}}{1+(1-y)^{2}} \frac{F_{U U}^{\cos \phi_{h}}}{F_{U U}}  \tag{2.58}\\
A^{\cos 2 \phi_{h}} & =\frac{2(1-y)}{1+(1-y)^{2}} \frac{F_{U U}^{\cos 2 \phi_{h}}}{F_{U U}} . \tag{2.59}
\end{align*}
$$

In [5], $\left\langle k_{\perp}^{2}\right\rangle$ was set as a free parameter, while $\left\langle p_{\perp}^{2}\right\rangle=A+B z_{h}^{2}$ where $A$ and $B$ were the other two free parameters. The structure function $F_{U U}$ and the Cahn parts of the structure functions $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ are explicitly presented in Eqs. (2.52),
(2.53) and (2.54). The parameterizations of the Boer-Mulders TMD PDF and the Collins TMD FF can be found in [5], and their contributions in the fitting were found to be marginal in [5]. The best fit curves for $\left\langle\cos \phi_{h}\right\rangle$ from the simultaneous fitting of the HERMES data on the multiplicities and the $\left\langle\cos \phi_{h}\right\rangle$ and $\left\langle\cos 2 \phi_{h}\right\rangle$ modulations is presented in Fig. 2.8. When fitting the $\left\langle\cos 2 \phi_{h}\right\rangle$ modulation, the Cahn effect was set to zero. The minimal $\chi^{2}$ fitting results for the HERMES data


Figure 2.8: Best fit curves for $\left\langle\cos \phi_{h}\right\rangle$ using the HERMES data on multiplicities, $\left\langle\cos \phi_{h}\right\rangle$ and $\left\langle\cos 2 \phi_{h}\right\rangle$. Cahn effect in $\left\langle\cos 2 \phi_{h}\right\rangle$ was set to zero. The figure is taken from [5].
were $\left\langle k_{\perp}^{2}\right\rangle=0.037 \pm 0.004 \mathrm{GeV}^{2}, A=0.126 \pm 0.004 \mathrm{GeV}^{2}$ and $B=0.506 \pm 0.045 \mathrm{GeV}^{2}$.

The $\left\langle\cos \phi_{h}\right\rangle$ data constrained $\left\langle k_{\perp}^{2}\right\rangle$ to be a very small value ( $0.037 \mathrm{GeV}^{2}$ ), compared with the much larger value $\left(0.57 \mathrm{GeV}^{2}\right)$ from [4] in which only the multiplicity data were fitted.

### 2.5.2 Polarized SIDIS phenomenology without TMD evolution effect

Besides the unpolarized SIDIS, the asymmetries in polarized SIDIS have been studied intensively in recent years. The Sivers asymmetry $\left(A_{U T}^{\sin \left(\phi_{h}-\phi_{S}\right)}\right)$, related to the convolution of the Sivers TMD PDF and the unpolarized TMD FF, can be extracted from the single-spin-asymmetry of a transversely polarized target.

An example of the phenomenological studies of the Sivers asymmetries without considering the evolution effect can be found in [6]. The comparisons of the fitting and the COMPASS data [77] taken from [6] is presented in Fig. 2.9.

The Collins asymmetry $\left(A_{U T}^{\sin \left(\phi_{h}+\phi_{S}\right)}\right)$, related to the convolution of the Transversity TMD PDF and the Collins TMD FF has also been studied. An example can be found in [7]. The comparisons between this fitting and the COMPASS data [78] is presented in Fig. 2.10.

### 2.5.3 SIDIS phenomenology with TMD evolution effect

As discussed in previously, the QCD evolution could have an important effect in the studies of TMDs. However, the phenomenological studies with the QCD evolution effects are limited so far. At the time this dissertation is being written, there is no published phenomenological study with the evolution effect of the unpolarized SIDIS process.

A recent study of the Sivers effect with the evolution can be found in [8]. The data from HERMES [79], COMPASS [80, 81] and JLab [82] were simultaneously fitted with the evolution effect taken into account by using the Sudakov factor in [8]. The comparisons of the fitting and the HERMES data in [8] is presented in


Figure 2.9: COMPASS data vs. fitting: Sivers asymmetries. The solid lines are from the best fitting. The shaded areas correspond to the statistical uncertainties of the fitting parameters. The figure is taken from [6].

Fig. 2.11. A noticeable feature of [8] is that this study had the universality feature thus the result from this SIDIS study was able to make predictions for the Drell-Yan processes. The minimal $\chi_{\text {dof }}^{2}$ in [8] was around 1.3, at the same level as the fitting for the Sivers asymmetries without the evolution effect in [6].

A recent study of the Collins effect with the CSS formalism for the evolution can be found in [9]. The SIDIS data from HERMES [79], COMPASS [83] and JLab [82], and the $e^{+} e^{-}$annihilation data from BELLE [84, 85] and BaBar [86] were simultaneously fitted. The comparisons of the fitting and the HERMES data in [9] is presented in Fig. 2.12. The minimal $\chi_{d o f}^{2}$ in [9] is around 0.88 , at the same level as the fitting for the Collins asymmetries without the evolution effect in [7].


Figure 2.10: COMPASS data vs. fitting: Collins asymmetries. The solid lines are from the best fitting. The shaded areas correspond to the statistical uncertainties of the fitting parameters. The figure is taken from [7].

### 2.5.4 Motivation of the study in this dissertation

As reviewed in this chapter, the SIDIS measurements on proton have been carried out by a number of experiments and the world data have been used for phenomenology studies $[4,5,10,53,64,65,72,73,75,76]$. However, the SIDIS data on the neutron are rather limited, since there is no stable free neutron target. Using a polarized ${ }^{3} \mathrm{He}$ target as an effective polarized neutron target for experimental studies related to the spin structure of the neutron is uniquely advantageous, due to the dominant neutron spin contribution to the ${ }^{3} \mathrm{He}$ spin [87]. The SIDIS experiment E06-010 in Hall A of JLab was carried out with a 5.9 GeV polarized electron beam and a transversely polarized ${ }^{3} \mathrm{He}$ target, between October 2008 and February 2009. The experiment covered a kinematic range $0.12<x_{b j}<0.45,1<Q^{2}<4(\mathrm{GeV} / \mathrm{c})^{2}$,


Figure 2.11: HERMES data vs. fitting: Sivers asymmetries. The figure is taken from [8].
$0.45<z_{h}<0.65$, and $0.05<P_{t}<0.55 \mathrm{GeV} / \mathrm{c}$. Studies on the data of E06-010 for single-spin asymmetries (SSAs) and double-spin asymmetries (DSAs) have been carried out [82, 88, 89, 90]. These first SIDIS asymmetry results from ${ }^{3} \mathrm{He}$ as an effective neutron target were related to TMD PDFs such as transversity, Sivers, pretzelosity, trans-helicity, and TMD FFs such as Collins.

Data on the absolute SIDIS cross sections would enhance the studies of the TMDs, as the evolution effect is much stronger on the absolute cross sections compared with the ratios [91]. In recent years, the unpolarized SIDIS processes have attracted considerable interest due to providing special insights into the TMD evolution effect [92].


Figure 2.12: HERMES data vs. fitting with evolution: Collins asymmetries. The figure is taken from [9].

In the following chapters, the configuration of experiment E06-010, the data analysis and the results of the unpolarized SIDIS differential cross section from a ${ }^{3} \mathrm{He}$ target will be presented.

## The Experiment

The SIDIS experiment E06-010 in Hall A of the Thomas Jefferson National Accelerator Facility (Jefferson Lab or JLab) was carried out with a 5.89 GeV polarized electron beam and a transversely polarized ${ }^{3} \mathrm{He}$ target, between October 2008 and February 2009. The experiment covered a kinematic range $0.12<x_{b j}<0.45$, $1<Q^{2}<4(\mathrm{GeV} / \mathrm{c})^{2}, 0.45<z_{h}<0.65$, and $0.05<P_{t}<0.55 \mathrm{GeV} / \mathrm{c}$.

In the subsequent sections, an overview of JLab is presented first, followed by the discussion about the apparatus related to experiment E06-010.

### 3.1 Overview

An overview of the Continuous Electron Beam Accelerator Facility (CEBAF) and the three experimental halls in JLab is taken from [93] and presented in Fig. 3.1.

There were three experimental halls at the time experiment E06-010 was carried out, namely Hall A, B and C. As shown in Fig. 3.1, Hall A is the largest one among the three experimental halls at JLab. A review of the recent $12-\mathrm{GeV}$ upgrade of JLab and the new facilities can be found in [94].

A 3D overview of the configuration of experiment E06-010 is taken from [95] and


Figure 3.1: An overview of JLab.
presented in Fig. 3.2.
For the production runs, the experiment used a longitudinally polarized 5.89 GeV electron beam with an average current of $12 \mu \mathrm{~A}$. The average beam polarization was $(76.8 \pm 3.5) \%$. Beam energies at 1.23 GeV and 2.4 GeV were used for elastic electronproton (ep) calibration runs.

The Hall A $40-\mathrm{cm}$ long polarized ${ }^{3} \mathrm{He}$ target was used in the experiment. The target cell contained about 10 amg of ${ }^{3} \mathrm{He}$ and a small amount of $\mathrm{N}_{2}$. There were three pairs of Helmholtz coils perpendicular to each other around the ${ }^{3} \mathrm{He}$ target providing magnetic fields with different directions according to the experiment for defining the direction of the ${ }^{3} \mathrm{He}$ polarization. Only one of the three pairs of coils is shown in Fig. 3.2.

The BigBite spectrometer $[96,97,98]$ is placed to the beam right facing the beam dump, at a central polar angle of $30^{\circ}$ as in Fig. 3.2. In the lab frame, the beam right is defined as the $-\mathbf{x}$ direction $(-\mathbf{y} \times \mathbf{z})$, where the $\mathbf{y}$ direction is vertically up and the $\mathbf{z}$ direction is along the beam direction. The BigBite spectrometer was configured to detect the scattered electrons in the experiment. The angular acceptance of the BigBite was: $(-140,140) \mathrm{mrad}$ for the in-plane angle and $(-240,240) \mathrm{mrad}$ for the out-of-plane angle. The BigBite's polar and azimuthal angular acceptance ranges in the lab frame were $23^{\circ}$ to $40^{\circ}$ and $245^{\circ}$ to $300^{\circ}$, respectively. The momentum acceptance range of the BigBite was from $0.6 \mathrm{GeV} / \mathrm{c}$ to $2.5 \mathrm{GeV} / \mathrm{c}$.

The (left) high-resolution spectrometer (HRS, or Left-HRS) [99] was placed to the beam left, at a central polar angle of $16^{\circ}$. The HRS was configured to detect the electro-produced hadrons in the experiment. The angular acceptance range of the HRS was relatively small: $(-30,30)$ mrad for the in-plane angle and $(-60,60)$ mrad for the out-of-plane angle. In the lab frame, the polar and azimuthal angular acceptance ranges of the HRS were $13.5^{\circ}$ to $18.5^{\circ}$ and $78^{\circ}$ to $102^{\circ}$, respectively. The momentum acceptance of the HRS was set in the range of $(1.0 \pm 4.5 \%) \times 2.35 \mathrm{GeV} / \mathrm{c}$.

### 3.2 The electron beam

In this section, the JLab accelerator before the recent 12 GeV upgrade and the electron beam used in experiment E06-010 are discussed. The accelerator at JLab consisted of one injector, two super-conducting linear accelerators (linacs), and two recirculation arcs. Electrons were accelerated by the linacs and circulated up to five times. Electron beams with high polarization were delivered to Hall A and C at high current and delivered to Hall B at low current. The accelerator and the three experimental Halls are shown in Fig. 3.1. Details of the accelerator can be found in [93, 95, 100].


Figure 3.2: A 3D overview of the configuration of E06-010. Only part of the target system is shown, namely the target ladder, collimators and one of the three field coils. The acceptance of the Left-HRS and BigBite spectrometer is visualized as the grey and yellow blocks, respectively.

- The electron beam energy during experiment E06-010 was monitored by the
"Tiefenbach" value and cross checked by the Arc measurement. The "Tiefenbach" method yielded the following beam energy:

$$
\begin{equation*}
E_{\text {beam }}=5891.3 \pm 2.5 \mathrm{MeV} \tag{3.1}
\end{equation*}
$$

The Arc measurement yielded the following beam energy:

$$
\begin{equation*}
E_{\text {beam }, \text { Arc }}=5889.4 \pm 0.5(\text { Stat. }) \pm 1.0 \text { (Sys.) MeV } \tag{3.2}
\end{equation*}
$$

where Stat. and Sys. represent the statistical and systematic uncertainties of the Arc measurement, respectively.

- The beam current was monitored by the beam current monitors (BCMs) in Hall A [99]. The averaged beam current was $12 \mu \mathrm{~A}$ during experiment E06-010. The beam current was calculated using the signal from the two RF cavities of the BCMs in this experiment. The RF cavities were calibrated by the "OLO2" cavity which measured the beam current at the injector [93]. The beam charge was from the integration of the beam current and had a precision at the level of $1 \%$.
- The beam helicity was flipped at 30 Hz during the experiment. The charge asymmetry of the beam with opposite helicities was less than 10 ppm for the entire experiment [95].
- The beam position in the experiment was monitored by two Beam Position Monitors (BPMA and BPMB), and recorded in the data. Due to the need of preserving the ${ }^{3} \mathrm{He}$ target cell window ( $\sim 0.1 \mathrm{~mm}$ ) during the experiment, the electron beam was rastered into a $3 \times 3 \mathrm{~mm}^{2}$ square, by using two sets of steering magnets located 23 meters upstream from the target.


### 3.3 Polarized ${ }^{3} \mathrm{He}$ target

In this section, the important aspects related to the polarized ${ }^{3} \mathrm{He}$ target will be discussed.

### 3.3.1 Polarized ${ }^{3}$ He target as effective polarized neutron target

Neutron has a half-life of $880.2 \pm 1.0 \mathrm{~s}$ [12], thus there is no stable neutron target. The lack of a stable neutron target leads to the usage of effective neutron targets such as deuteron and ${ }^{3} \mathrm{He}$, for studying the structure of the neutron.

Using a polarized ${ }^{3} \mathrm{He}$ target as an effective polarized neutron target for experimental studies related to the spin structure of the neutron is uniquely advantageous,
due to the dominant neutron spin contribution to the ${ }^{3} \mathrm{He}$ nuclear spin [87]. This can be understood by looking at the ${ }^{3} \mathrm{He}$ nuclear wavefunction. The ground state of polarized ${ }^{3} \mathrm{He}$ is dominated by the $S$ state (with $\sim 90 \%$ probability) in which the spins of the two protons cancel each other and the neutron spin comprises the entire ${ }^{3} \mathrm{He}$ spin. There are also a $P$ state with negligible probability, an $S^{\prime}$ state with $\sim 1.5 \%$ probability and a $D$ state with $\sim 8 \%$ probability [101]. An illustration of the ground state of ${ }^{3} \mathrm{He}$ is taken from [95] and presented in Fig. 3.3.


Figure 3.3: Illustration of ${ }^{3} \mathrm{He}$ nucleus ground state.

The effective polarizations of the neutron $\left(P_{n}\right)$ and the proton $\left(P_{p}\right)$ in a fully polarized ${ }^{3} \mathrm{He}$ are directly related to the probabilities of the $\mathrm{S}, \mathrm{S}$ ' and D wave in the ground state of the ${ }^{3} \mathrm{He}$ nucleus [87], and can be expressed as:

$$
\begin{align*}
P_{n} & =1-\frac{2}{3}\left[P\left(S^{\prime}\right)+2 P(D)\right]  \tag{3.3}\\
P_{p} & =-\frac{1}{3}\left[P(D)-P\left(S^{\prime}\right)\right] \tag{3.4}
\end{align*}
$$

where $P\left(S^{\prime}\right)$ and $P(D)$ represent the probabilities of the $S^{\prime}$ and the $D$ state, respectively. The normalization $P(S)+P\left(S^{\prime}\right)+P(D)=1$ is assumed.

We follow [87] to illustrate Eqs. (3.3) and (3.4). The spin-dependent projection
operators for the nucleons from [87] are expressed as:

$$
\begin{align*}
& \hat{P}_{n}^{ \pm}=\sum_{i=1}^{3} \frac{1-\tau_{3}(i)}{2} \frac{1 \pm \sigma_{3}(i)}{2}  \tag{3.5}\\
& \hat{P}_{p}^{ \pm}=\sum_{i=1}^{3} \frac{1+\tau_{3}(i)}{2} \frac{1 \pm \sigma_{3}(i)}{2}, \tag{3.6}
\end{align*}
$$

where $\hat{P}_{n}\left(\hat{P}_{p}\right)$ counts the number of neutrons (protons) which are aligned $(+)$ or antialigned (-) with the ${ }^{3} \mathrm{He}$ spin and $\tau_{3}(i)\left(\sigma_{3}(i)\right)$ is the third $\mathrm{SU}(2)$ isospin (spin) operator for the $i$ th nucleon. The sum over the three nucleons is expressed by $\sum_{i=1}^{3}$. The following relations are obtained based on the well-studied ground state wavefunction $|\psi\rangle$ of the ${ }^{3} \mathrm{He}$ nucleus:

$$
\begin{align*}
\sum_{i=1}^{3}\langle\psi| \tau_{3}(i)|\psi\rangle & =1  \tag{3.7}\\
\sum_{i=1}^{3}\langle\psi| \sigma_{3}(i)|\psi\rangle & =P(S)+P\left(S^{\prime}\right)-P(D),  \tag{3.8}\\
\sum_{i=1}^{3}\langle\psi| \tau_{3}(i) \sigma_{3}(i)|\psi\rangle & =-\left[P(S)-\frac{1}{3} P\left(S^{\prime}\right)+\frac{1}{3} P(D)\right] . \tag{3.9}
\end{align*}
$$

The effective polarizations can be expresses as:

$$
\begin{align*}
P_{n} & =\langle\psi| \frac{\hat{P}_{n}^{+}-\hat{P}_{n}^{-}}{\hat{P}_{n}^{+}+\hat{P}_{n}^{-}}|\psi\rangle,  \tag{3.10}\\
P_{p} & =\langle\psi| \frac{\hat{P}_{p}^{+}-\hat{P}_{p}^{-}}{\hat{P}_{p}^{+}+\hat{P}_{p}^{-}}|\psi\rangle . \tag{3.11}
\end{align*}
$$

Substituting Eqs. (3.7), (3.8) and (3.9) into Eqs. (3.10) and (3.11), the results in Eqs. (3.3) and (3.4) can be obtained. Based on different calculations of $|\psi\rangle$ the
averaged $P_{n}$ and $P_{p}$ are [87, 102]:

$$
\begin{align*}
P_{n} & =0.86 \pm 0.002  \tag{3.12}\\
P_{p} & =-0.028 \pm 0.004 \tag{3.13}
\end{align*}
$$

### 3.3.2 Polarized ${ }^{3}$ He target in experiment E06-010

The ${ }^{3} \mathrm{He}$ target cells used in experiment E06-010 were cells made of GE180 glass. A schematic figure for the ${ }^{3} \mathrm{He}$ target cell is presented in Fig. 3.4.


Figure 3.4: Schematic figure for a ${ }^{3} \mathrm{He}$ target cell.

Each of the cells consisted of three parts: the pumping chamber, the transfer tube and the target chamber. The target chamber was put in the electron beam providing the target ${ }^{3} \mathrm{He}$ nuclei for the scattering experiment. The pumping chamber was under the spin-exchange optical pumping (SEOP) process [103] , and enabled the ${ }^{3} \mathrm{He}$ nuclei inside the cell to be polarized. The transfer tube allowed the polarized ${ }^{3} \mathrm{He}$ gas in the pumping chamber to diffuse to the target chamber. About 8 amg of ${ }^{3} \mathrm{He}$ gas was filled in the glass cell. A small amount of $\mathrm{N}_{2}$ and a mixture of Rb and K metal were filled in the cell as they are necessary when polarizing ${ }^{3} \mathrm{He}$ using SEOP. A graphical
illustration of SEOP is put in Fig. 3.5.

## Spin Exchange Optical Pumping (SEOP) of ${ }^{3} \mathrm{He}$ cell



Figure 3.5: The pumping chamber of the cell is heated to more than $200^{\circ} \mathrm{C}$, to produce sufficient densities of Rb and K vapors. A circularly polarized laser beam with 794.8 nm wavelength is applied to the pumping chamber. The Rb atoms go through the process in the left top plot and become polarized. The polarized Rb atoms exchange spin with the K atoms and the ${ }^{3} \mathrm{He}$ nuclei. The polarized K atoms also exchange spin with the ${ }^{3} \mathrm{He}$ nuclei.

In SEOP, the vapor of the alkali metal Rb is polarized by the circularly polarized laser at wavelength of 794.8 nm . The outermost electron in a Rb atom at the ground state $\left(5 S_{1 / 2}\right)$ is excited to the $5 P_{1 / 2}$ state. Both of these states split through Zeeman splitting in an external magnetic field, to sub-levels $m_{J}= \pm 1 / 2$. Using a laser beam with spin parallel to the magnetic field, the excitation is from $5 S_{1 / 2}\left(m_{J}=-1 / 2\right)$ to $5 P_{1 / 2}\left(m_{J}=1 / 2\right)$. The electrons at the excited states decay to both the $m_{J}= \pm 1 / 2$
sub-levels of the state $5 S_{1 / 2}$. By pumping with a polarized laser, Rb atoms with one of the $m_{J}= \pm 1 / 2$ sub-levels quickly becomes dominant, thus polarized.

When the excited Rb atoms decay, they emit photons with the same wavelength as the pumping laser, which can depolarize the Rb atoms and jeopardize the pumping. By introducing a small amount of $\mathrm{N}_{2}$ molecules, the decay of the excited Rb atoms becomes dominated by the radiation-less process ( $\sim 95 \%$ ), thus the efficiency of the pumping is much enhanced.

The polarized Rb atoms exchange spins with K atoms at a large rate, making the K atoms also highly polarized. Both the polarized Rb and K atoms exchange spins with the ${ }^{3} \mathrm{He}$ nuclei, making the ${ }^{3} \mathrm{He}$ gas polarized. While the spin exchange between both types of the alkali atoms ( Rb and K ) and ${ }^{3} \mathrm{He}$ nuclei is dominated by hyperfine interactions during binary collisions [104, 105], K atoms have an order of magnitude higher efficiency in spin exchange than Rb atoms, thus SEOP is accelerated by using a mixture of Rb and K , instead of using only Rb . The efficiency, with a dependence on the $[\mathrm{Rb}] /[\mathrm{K}]$ density ratio, was studied in $[106]$. The $[\mathrm{Rb}] /[\mathrm{K}]$ density ratio in experiment E06-010 was around 5 [95].

Another part of the system of a polarized ${ }^{3} \mathrm{He}$ target is a magnetic holding field, defining the direction of the ${ }^{3} \mathrm{He}$ polarization. Three pairs of Helmholtz coils were used in experiment E06-010, which enabled one to set the magnetic field to any direction by tuning the currents in the coils. In the experiment, the magnetic field had two configurations for the transverse polarization of ${ }^{3} \mathrm{He}$, namely the vertical direction (up-down) and the transverse direction (left-right), both of which were perpendicular to the electron beam. A figure for the Helmholtz coil system taken from [95] is presented in Fig. 3.6.


Figure 3.6: The Helmholtz coil system of experiment E06-010.

### 3.3.3 Target polarization measurement

In the experiment, the percentage of polarization of the ${ }^{3} \mathrm{He}$ gas in the target cell was measured by using the nuclear magnetic resonance (NMR) [107] and electron paramagnetic resonance (EPR) techniques [108].

In classical electrodynamics, a particle with a magnetic moment $\vec{M}$ and a gyromagnetic ratio $\gamma$ follows an equation of rotation as below, when it is placed in a magnetic field $\vec{B}_{0}$.

$$
\begin{equation*}
\frac{d \vec{M}}{d t}=\gamma \vec{M} \times \vec{B}_{0} \tag{3.14}
\end{equation*}
$$

Another magnetic field perpendicular to $\vec{B}_{0}$, rotating with an angular frequency $-\vec{\omega}_{r f}$ is then added to the system. In a frame rotating with an angular frequency $-\overrightarrow{\omega_{r f}}$,

Eq. (3.14) becomes

$$
\begin{equation*}
\frac{d \vec{M}}{d t}=\gamma \vec{M} \times \vec{B}_{e f f} \tag{3.15}
\end{equation*}
$$

where $\vec{B}_{\text {eff }}$ is the effective magnetic field in the rotating frame and is expressed in Eq. (3.16) below. To be concise, one can define $\vec{B}_{0}$ to be in the $\hat{z}$ direction. The perpendicular magnetic field $\vec{B}_{r f}$ is in the $\hat{y}$ direction.

$$
\begin{equation*}
\vec{B}_{e f f}=\left(B_{0}-\omega_{r f} / \gamma\right) \hat{z}+B_{r f} \hat{y} \tag{3.16}
\end{equation*}
$$

When $\vec{B}_{\text {eff }}$ is changing in an adiabatically slow manner, the direction of $\vec{M}$ always follows the direction of $\vec{B}_{\text {eff }}$. In practice, when using NMR to measure the polarization, the rotating magnetic field is provided by a radio-frequency (RF) electromagnetic field with a constant frequency $\omega_{r f}$. The magnetic field $\vec{B}_{0}$ is changing adiabatically in time. The adiabatic condition is fulfilled by requiring

$$
\begin{equation*}
\left|\frac{d B_{0}(t)}{d t}\right| \ll \gamma B_{r f}^{2} \tag{3.17}
\end{equation*}
$$

The angle between the $\hat{z}$ axis and the direction of $\vec{M}$ (same as $\vec{B}_{\text {eff }}$ ) as $\theta(t)$ can be expressed as

$$
\begin{equation*}
\theta(t)=\arcsin \frac{B_{r f}}{\sqrt{\left(B_{0}(t)-\omega_{r f} / \gamma\right)^{2}+B_{r f}^{2}}} \tag{3.18}
\end{equation*}
$$

A pair of pick-up coils is placed in the $\hat{x}$ direction to pick up the signal induced by the rotating $\vec{M}$. The signal the coils pick up is proportional to $\sin \theta(t)$. The resonance magnetic field is defined as $B_{0}(t)=\omega_{r f} / \gamma$. At the resonance, the angle $\theta(t)$ is at the largest value, maximizing the NMR signal recorded by the pick-up coil.

Besides the adiabatic condition in Eq. 3.17, this type of NMR measurement also requires the field $B_{0}(t)$ to change fast enough, so the relaxation of polarization is
negligible during the sweep passing through the resonance. This is expressed as

$$
\begin{equation*}
\frac{1}{B_{r f}}\left|\frac{d B_{0}(t)}{d t}\right| \gg \frac{1}{T_{1}}, \tag{3.19}
\end{equation*}
$$

where $T_{1}$ is the longitudinal relaxation time of the polarization.
Due to the requirements of "adiabatic" in Eq. (3.17) and "fast" in Eq. (3.19), this way of doing NMR measurement is named adiabatic-fast-passage (AFP) NMR.

The AFP-NMR needs to be calibrated with the absolute value of polarization. Such calibration can be achieved by the electron paramagnetic resonance (EPR).

The EPR polarimetry utilizes the Zeeman splitting of the Rb states [108], which is proportional to the ambient magnetic field. The frequency corresponding to a specific Zeeman splitting can be locked by the frequency-modulation (FM) sweep method. Flipping the spin of the polarized ${ }^{3} \mathrm{He}$ changes the magnetic field due to the magnetic moments of the ${ }^{3} \mathrm{He}$ nuclei. This change of magnetic field is reflected in the change of the Zeeman frequency. With the knowledge of the ${ }^{3} \mathrm{He}$ density, amplitude of magnetic holding field and a few constants, the absolute ${ }^{3} \mathrm{He}$ polarization can be determined from the change of the Zeeman frequency.

During the EPR measurement, a different type of AFP is used to flip the spin of polarized ${ }^{3} \mathrm{He}$ : the magnetic holding field is kept at a constant magnitude while sweeping the RF frequency in an adiabatic fast manner. In this way, the Zeeman frequency change before and after the spin flip is totally due to the magnetic moment of the polarized ${ }^{3} \mathrm{He}$ gas.

Experimentally carrying out the NMR-AFP and EPR measurements has been well established for many years at JLab and by the Medium Energy Physics group at Duke University $[95,109,110,111]$.

### 3.3.4 Polarization dynamics

In this subsection, the polarization dynamics of the ${ }^{3} \mathrm{He}$ gas in a one-chamber target is presented first to explain the physics in a simple and intuitive manner. Then the polarization dynamics for a two-chamber cell as used in experiment E06-010 is presented as a generalization of the one-chamber case.

In a single chamber, the number of ${ }^{3} \mathrm{He}$ nuclei with positive and negative spindirections are denoted as $N_{+}$and $N_{-}$, respectively. The total number of ${ }^{3} \mathrm{He}$ nuclei is a constant $N_{\text {tot }}=N_{+}+N_{-}$. The number of alkali metal atoms with positive and negative spin-directions are denoted as $n_{+}$and $n_{-}$, respectively. The total number of alkali metal atoms is a constant $n_{t o t}=n_{+}+n_{-}$. The polarization of the ${ }^{3} \mathrm{He}$ gas $(P)$ and the polarization of the alkali metal vapor $\left(P_{A}\right)$ are defined as the following:

$$
\begin{align*}
P & =\frac{N_{+}-N_{-}}{N_{+}+N_{-}}  \tag{3.20}\\
P_{A} & =\frac{n_{+}-n_{-}}{n_{+}+n_{-}} \tag{3.21}
\end{align*}
$$

The spin exchange dynamics can be expressed as:

$$
\begin{align*}
\frac{d N_{+}}{d t} & =N_{-} \frac{n_{+}}{n_{t o t}} \gamma_{S E}-N_{+} \frac{n_{-}}{n_{t o t}} \gamma_{S E}-\frac{1}{2}\left(N_{+}-N_{-}\right) \Gamma,  \tag{3.22}\\
\frac{d N_{-}}{d t} & =N_{+} \frac{n_{-}}{n_{t o t}} \gamma_{S E}-N_{-} \frac{n_{+}}{n_{t o t}} \gamma_{S E}-\frac{1}{2}\left(N_{-}-N_{+}\right) \Gamma, \tag{3.23}
\end{align*}
$$

where $\gamma_{S E}$ is the spin-exchange rate between the alkali atoms and the ${ }^{3} \mathrm{He}$ nuclei and $\Gamma$ is the relaxation rate of the ${ }^{3} \mathrm{He}$ nuclei.

The differential equation for $P$ can be derived by subtracting Eq. (3.23) from Eq. (3.22) and use Eqs. (3.20) and (3.21):

$$
\begin{equation*}
\frac{d P}{d t}=\gamma_{S E} P_{A}-\gamma_{S E} P-\Gamma P \tag{3.24}
\end{equation*}
$$

Assuming $P(t=0)=0$, Eq. (3.24) is solved as

$$
\begin{equation*}
P(t)=P_{A} \frac{\gamma_{S E}}{\gamma_{S E}+\Gamma}\left[1-e^{-\left(\gamma_{S E}+\Gamma\right) t}\right] \tag{3.25}
\end{equation*}
$$

The maximum of the ${ }^{3} \mathrm{He}$ polarization in the cell is

$$
\begin{equation*}
P_{\max }=P(t=\infty)=P_{A} \frac{\gamma_{S E}}{\gamma_{S E}+\Gamma} \tag{3.26}
\end{equation*}
$$

The ${ }^{3} \mathrm{He}$ polarization is maximized by maximizing the polarization of the alkali vapor $P_{A}$, maximizing the spin-exchange rate $\gamma_{S E}$ and minimizing the relaxation rate $\Gamma$.

The alkali-vapor polarization $P_{A}$ can be increased by increasing the power of the pumping laser and tuning the spectrum of the laser for better laser-power absorption by the alkali vapor. In experiments using polarized ${ }^{3} \mathrm{He}$ cell with SEOP, $P_{A} \approx 100 \%$. The spin-exchange rate $\gamma_{S E}$ can be tuned by changing the proportions of Rb and K in the cell.

The relaxation mechanism includes the nuclear dipolar interactions between two nearby ${ }^{3} \mathrm{He}$ nuclei, collisions between ${ }^{3} \mathrm{He}$ nuclei and the cell wall, and the non-uniform-magnetic-field effect. Babcock et al. proposed there could be an additional relaxation effect and added an "X-factor" in Eq. (3.26) [112]. The resulting maximal ${ }^{3}$ He polarization is expressed as

$$
\begin{equation*}
P_{\max }=P_{A} \frac{\gamma_{S E}}{(1+X) \cdot \gamma_{S E}+\Gamma} \tag{3.27}
\end{equation*}
$$

The origin of the "X-factor" is not understood yet.
In experiment E06-010, the ${ }^{3} \mathrm{He}$ target cell consisted of two chambers, the pumping chamber and the target chamber. The polarization dynamics for one-chamber cell can be generalized to describe a two-chamber cell by including the diffusion effect between the two chambers $[95,113]$. The differential equations for the polarization of the pumping chamber $P_{p c}$ and the polarization of the target chamber $P_{t c}$ are
expressed as

$$
\begin{align*}
\frac{d P_{p c}}{d t} & =\gamma_{S E}\left(P_{A}-P_{p c}\right)-\Gamma_{p c} P_{p c}-d_{p c}\left(P_{p c}-P_{t c}\right)  \tag{3.28}\\
\frac{d P_{t c}}{d t} & =d_{t c}\left(P_{p c}-P_{t c}\right)-\Gamma_{t c} P_{t c} \tag{3.29}
\end{align*}
$$

The diffusion rate $d_{p c}\left(d_{t c}\right)$ denotes the rate of diffusion from the target (pumping) chamber to the pumping (target) chamber. These differential equations can be solved as

$$
\begin{align*}
P_{p c}(t) & =P_{p c, \max }+C_{p c}^{s} \exp \left(-\Gamma_{s} t\right)+C_{p c}^{f} \exp \left(-\Gamma_{f} t\right),  \tag{3.30}\\
P_{t c}(t) & =P_{t c, \max }+C_{t c}^{s} \exp \left(-\Gamma_{s} t\right)+C_{t c}^{f} \exp \left(-\Gamma_{f} t\right) . \tag{3.31}
\end{align*}
$$

The maximal polarizations $P_{p c, \max }$ and $P_{t c, \max }$ are expressed as

$$
\begin{align*}
P_{p c, \text { max }} & =P_{A} \frac{\gamma_{S E} f_{p c}}{\gamma_{S E} f_{p c}+\Gamma_{p c} f_{p c}+\Gamma_{t c} f_{t c} d_{t c} /\left(d_{t c}+\Gamma_{t c}\right)},  \tag{3.32}\\
P_{t c, \text { max }} & =P_{p c, \text { max }} \frac{d_{t c}}{d_{t c}+\Gamma_{t c}}, \tag{3.33}
\end{align*}
$$

where $f_{p c}\left(f_{t c}\right)$ is the fraction of the ${ }^{3} \mathrm{He}$ nuclei in the pumping (target) chamber. The explicit expressions for $C_{p c}^{f}, C_{t c}^{f}, \Gamma_{f}$ and $\Gamma_{s}$ can be found in [113].

The rates $d_{t c}$ and $\Gamma_{t c}$ were determined on a cell-by-cell basis in experiment E06-010 to determine the polarization ratio $P_{t c, \max } / P_{p c, \max }$ using Eq. (3.33). The polarization in the pumping chamber $P_{p c}$ was measured and recorded throughout the experiment and the polarization in the target chamber was derived using the polarization ratio. The uncertainty of the target-chamber polarization was estimated to be $3.4 \%$ [95].

### 3.4 BigBite spectrometer

In experiment E06-010, the scattered electrons were detected by the BigBite spectrometer [96, 97, 98]. A schematic of the BigBite spectrometer from [93] is presented in Fig. 3.7.


Figure 3.7: A schematic of the BigBite spectrometer.

The BigBite spectrometer was placed to the beam right facing the beam dump. The central polar angle of the BigBite was set at $30^{\circ}$. The solid angle acceptance of the BigBite was about 64 msr . The BigBite's polar and azimuthal angular acceptance ranges in the lab frame were $23^{\circ}$ to $40^{\circ}$ and $245^{\circ}$ to $300^{\circ}$, respectively. The momentum acceptance range of the BigBite was from $0.6 \mathrm{GeV} / \mathrm{c}$ to $2.5 \mathrm{GeV} / \mathrm{c}$.

The BigBite spectrometer consisted of a single dipole magnet, eighteen planes of multiwire drift chambers in three groups and a scintillator plane between the lead-glass preshower and shower calorimeters. The knowledge of the magnetic field and the information from the drift chambers, were used to reconstruct the tracks of charged particles. The trigger was formed by summing the signals from the preshower and shower calorimeters. The preshower and shower energy depositions with the
reconstructed momentum were utilized for the particle identification (PID) in the BigBite [93, 96, 97, 98].

The BigBite magnet provided about 1.2 T magnetic field from about 710 A current. In this magnetic field, charged particles were bent up or down based on the signs of their charges. This feature enabled a high-precision discrimination of the particles with opposite signs of charges. The acceptance for the bent-up and bentdown particles was different. There were a series of reference runs with reversed BigBite magnetic field in which the positron events had the same acceptance as the electron events in the production runs. These reference runs were for studying the background from electron-positron pair-production.

The Multi-Wire-Drift-Chamber (MWDC) for the reconstruction of tracks consisted of 18 wire planes. Each of the three chambers of the MWDC had six wire planes which were grouped into three types of directions ( x at $0^{\circ}$, u at $30^{\circ}$ and v at $-30^{\circ}$ ). Two adjacent wires were 1 cm way from each other. A figure of the MWDC is taken from [93], and is presented in Fig. 3.8.


Figure 3.8: The MWDC of the BigBite.

The lead-glass preshower and shower calorimeters provided the trigger and the PID for electrons in the BigBite. There were 54 preshower blocks in two columns. Each of the preshower block was made of TF-5 lead-glass and had the size of 8.5 cm $\times 34 \mathrm{~cm} \times 8.5 \mathrm{~cm}$. The shower blocks were made of TF-2 lead-glass with the same size as the preshower blocks. The 189 shower blocks were arranged in seven columns each of which had 27 rows. A schematic of the preshower, shower and the scintillator between them is shown in Fig. 3.9.


Figure 3.9: A schematic of the pre-shower, shower and the scintillator of the BigBite.

Electrons and hadrons generate electromagnetic showers and deposit certain amounts of energy in the preshower and shower calorimeters, recorded in the data on an event-by-event basis. The energy depositions in the preshower/shower system from
electrons were generally larger than those from hadrons. The PID in the BigBite was based on the ability to discriminate electrons and hadrons by combining the information of the preshower/shower energy deposition and the reconstructed momentum.

### 3.5 High-Resolution Spectrometer (HRS)

The High-Resolution Spectrometer (HRS) consisted of three super-conducting quadruples, a 6.6 m dipole magnet and the detector hut [99]. A schematic figure for the Left HRS is shown in Fig. 3.10.


Figure 3.10: A schematic of the Left HRS.

The HRS was placed to the beam left. The central polar angle of the HRS was set at $16^{\circ}$. The solid angle acceptance of the HRS was about 6 msr . The polar and azimuthal angular acceptance ranges of the HRS were $13.5^{\circ}$ to $18.5^{\circ}$ and $78^{\circ}$ to $102^{\circ}$, respectively. The momentum acceptance range of the HRS was set in the range of $(1.0 \pm 4.5 \%) \times 2.35 \mathrm{GeV} / \mathrm{c}$.

The tracking information from the HRS was from a pair of Vertical Drift Chambers (VDCs). The trigger was provided by two scintillator planes. Four detectors in the HRS were used for PID: a $\mathrm{CO}_{2}$ gas Čerenkov detector for electron identification, an aerogel Čerenkov detector for pion identification, a ring imaging Čerenkov (RICH) detector for $\pi^{ \pm}, K^{ \pm}$, and proton identification, and two layers of lead-glass calorimeter for electron-hadron separation.

### 3.6 Trigger and Data Acquisition

The CEBAF Online Data Acquisition (CODA) system was used for the data acquisition (DAQ) of experiment E06-010. The Experimental Physics and Industrial Control System (EPICS) was used in addition, providing the assisting information at a lower rate (every few seconds), such as the beam position, beam current, beam energy and magnet status.

The triggers in the experiment included the singles triggers on the BigBite and the HRS, and the coincidence trigger by combining the single triggers. The eight types of triggers in the experiment are listed in Table. 3.1.

Table 3.1: Triggers in experiment E06-010.

| Trigger | Description |
| :---: | :---: |
| T1 | Low threshold on BigBite lead-glass |
| T2 | BigBite gas Cerenkov singles |
| T3 | Left HRS singles |
| T4 | Left HRS efficiency |
| T5 | Coincidence between BigBite and Left HRS (T1.AND.T3) |
| T6 | High threshold on BigBite lead-glass |
| T7 | BigBite Cerenkov and lead-glass overlap |
| T8 | 1024 Hz clock |

The single BigBite triggers T1 and T6 were formed by requiring a certain amount of total energy deposited in the preshower and shower calorimeters at low and high
threshold, respectively. The single HRS trigger T3 was formed by requiring both of the scintillator planes to have a paddle with a hit on both sides of four PMTs. The coincidence trigger T 5 was constructed by putting logic AND between T 1 and T 3 .

The accumulated information (clock, raw counts for triggers and beam charge) was recorded in the scalers for normalizing the experimental data and calculating the DAQ livetime.

Depending on the electronic modules being used and the event rate, the DAQ system could lose some events due to its busy state. The livetime $f_{\text {live }}$ is defined in the following equation to take this effect into account:

$$
\begin{equation*}
f_{\text {live }}=\frac{T_{\text {recorded }}}{T_{\text {raw }}} \tag{3.34}
\end{equation*}
$$

where $T_{\text {recorded }}$ is the counts of the recorded triggers and $T_{\text {raw }}$ is the counts of the raw triggers.

### 3.7 Calibrations of the experiment

### 3.7.1 Calibrations of the BigBite spectrometer

There were several types of runs for calibrating the track reconstruction of the BigBite, as listed below.

- No-field runs: finding the locations of the MWDC.
- Multi-Carbon-Foil runs: calibrating the interaction vertices.
- Sieve runs: calibrating the angles at two incident beam energies with the multifoil carbon and hydrogen gas targets.
- Hydrogen runs at elastic kinematics: calibrating the momentum with the elastic scattering data at two beam energies.

The reconstruction of the angular information relied on the runs using the target foils and the sieve. The positions of each foil and the sieve holes were precisely known. The positions of the carbon foils and the reconstructed data are presented in Fig 3.11. The resolution of the vertex $z$ is about 1 cm .


Figure 3.11: The positions of the seven carbon foils (red lines) and the reconstructed data (black line).

In the sieve run with the carbon foils, the vertex positions (the foils), and the sieve positions were precisely measured, thus the angular information of the events from a certain foil and passed through a certain sieve hole is known. A 2D plot of the sieve and the angular constraint provided by one of the holes are presented in Fig 3.12.

The relations between $\left(d x_{t a r}, d y_{t a r}\right)$ and $\left(\theta_{p h y}, \phi_{p h y}\right)$ can be expressed as [114]

$$
\begin{align*}
\theta_{p h y} & =\arccos \left[\left(\cos \theta_{0}-d y_{t a r} \sin \theta_{0} \sin \phi_{0}\right) / \sqrt{1+d x_{t a r}^{2}+d y_{t a r}^{2}}\right]  \tag{3.35}\\
\phi_{p h y} & =\arctan \left[\left(d y_{t a r} \cos \theta_{0}+\sin \theta_{0} \sin \phi_{0}\right) /\left(\sin \theta_{0} \cos \phi_{0}+d x_{t a r}\right)\right] \tag{3.36}
\end{align*}
$$



Figure 3.12: A 2D plot of all the events from the sieve run (black dots) is presented in the left big panel. The events from the middle foil in the vertex z range of $-3<v_{z}<3 \mathrm{~cm}$ (red dots) are drawn on top of the black dots. The differences ( $\delta \mathrm{s}$ ) between the reconstructed angles and the central angles using the red-dot events are presented in the four panels on the right side. The quantities $d x_{t a r}, d y_{t a r}, \theta_{p h y}$ and $\phi_{p h y}$ represent the out-of-plane angle, the in-plane angle, the physical polar angle and the physical azimuthal angle, respectively.
where $\theta_{0}$ and $\phi_{0}$ are the central polar and azimuthal angles of a detector. The central angles of the BigBite and the HRS are presented in Table 3.2.

| Type of angle | Value |
| :--- | :---: |
| BigBite central polar angle $\theta_{0}^{b b}$ | $30^{\circ}$ |
| BigBite central azimuthal angle $\phi_{0}^{b b}$ | $270^{\circ}$ |
| HRS central polar angle $\theta_{0}^{\text {hrs }}$ | $16^{\circ}$ |
| HRS central azimuthal angle $\phi_{0}^{\text {hrs }}$ | $90^{\circ}$ |

Table 3.2: The central polar and azimuthal angles of the BigBite and the HRS.

The relation between the angles in Eqs. (3.35) and (3.36) can be understood by looking at the coordinate systems presented in Fig 3.13. In the detector coordinate


Figure 3.13: A top view of the physical (subscript phy) and the detector (subscript $b b$ and $h r s$ ) coordinate systems.
system (DCS), the $z_{0}$ axis points from the interaction vertex to the detector center, the $x_{0}$ axis points vertically down (into the paper), and the $y_{0}$ axis is in the direction $\hat{y}_{0}=-\hat{x}_{0} \times \hat{z}_{0}$. The vector for the direction of an event in the DCS can be expressed as

$$
\begin{equation*}
\vec{v}=\left(d x_{t a r}, d y_{t a r}, 1\right), \tag{3.37}
\end{equation*}
$$

where the out-of-plane (out of the $y_{0}-z_{0}$ plane) angle $d x_{t a r}$ is defined as the ratio $\left(x_{t a r} / L\right)$ of the length $x_{t a r}=d x_{t a r} \times L$ and a fixed distance $L$, and the in-plane (in the $y_{0}-z_{0}$ plane) angle $d y_{t a r}$ is defined similarly. These definitions are especially useful for using the sieve plate with a fixed $L$ to calibrate the detector angles. The same vector in the physical coordinate system (PCS) can be easily derived as

$$
\begin{equation*}
\overrightarrow{v^{\prime}}=\left(\sin \theta_{0} \cos \phi_{0}+d x_{t a r}, d y_{t a r} \cos \theta_{0}+\sin \theta_{0} \sin \phi_{0}, \cos \theta_{0}-d y_{t a r} \sin \theta_{0} \sin \phi_{0}\right)(. \tag{3.38}
\end{equation*}
$$

The polar and azimuthal angles of the vector $\overrightarrow{v^{\prime}}$ are found as in Eqs. (3.35) and (3.36). The $\theta_{p h y}^{b b}$ and $\phi_{p h y}^{b b}$ resolutions were about $0.25^{\circ}$ and $0.6^{\circ}$, respectively.

After the angle-reconstruction calibrations, the elastic electron-proton scattering runs at 1.23 and 2.4 GeV beam energy were used to calibrate the momentum reconstruction. In the elastic scattering process, the momentum of the scattered electron can be calculated based on energy-momentum conservation, with a known polar angle. The momentum resolution was about $1 \%$.

The BigBite calorimeter was used for both the threshold of the BigBite trigger and the PID. The two main aspects of the calibration are listed below.

- Cosmic ray calibration: match the gains for all PMTs by changing the highvoltage (HV) settings.
- Elastic scattering calibration: used the reconstructed momentum in the elastic ep runs to tune the ADC gains in software.

The average energy resolution was about $\sigma_{E / P} \approx 8 \%$. Using the preshower and shower energy for PID will be discussed in section 4.2.2. A description of the total shower energy deposition was built into the Monte Carlo simulation for the experiment, which will be discussed in section 4.3.2.

### 3.7.2 Calibrations of the left HRS

The calibration procedure for reconstructing the vertex, angles and momentum are listed below.

- Vertex calibration of the HRS used the multi-carbon-foil target. The reconstructed vertex matched actual positions of the foils, as shown in Fig 3.14. The mean resolution of the vertex reconstruction was about 6 mm [95].
- The angular calibration of the HRS was based on the sieve runs using the multi-carbon-foil target. The reconstructed events matched the actual sieve pattern, as presented in Fig 3.15. The $\theta_{p h y}$ and $\phi_{p h y}$ resolutions of the HRS were about $0.07^{\circ}$ and $0.6^{\circ}$, respectively.
- The momentum calibration of the HRS was based on the elastic electron-carbon scattering using the multi-carbon-foil target. The real momentum was calulated using the scattering angle and the corrections due to the collision energy losses. A resolution better than $5 \times 10^{-4}$ was achieved [95].


Figure 3.14: The positions of the seven carbon foils (red lines) and the reconstructed data (black line), in the HRS calibration runs.

The calibration of the detectors for PID in the HRS are listed below.

- The gas Čerenkov detector was calibrated for the discrimination between leptons (electrons and positrons) and hadrons. The lepton events were mainly in high ADC channels, while the hadron events were mostly below the threshold.
- The aerogel Čerenkov detector was calibrated for distinguishing pions from kaons and protons. The pion events occupied in high ADC channels, while the kaons and proton were mostly in the low ADC channels.


Figure 3.15: The reconstructed sieve-slit pattern. Each plot is one of the carbon foils, and each crossing of the magenta lines represents the physical location of a sieve hole.

- The lead-glass calorimeter was calibrated for distinguishing pions from leptons. Leptons deposit more energy than hadrons due to the EM shower, and their spectra could be clearly discriminated.

The procedures of using these detector for the HRS PID will be discussed in detail in section 4.2.1.

### 3.7.3 Calibrations of the timing

In experiment E06-010, a coincidence timing (CT, also called coincidence time-ofFlight, cTOF) was defined as the time difference between the production of two particles detected by the BigBite and the HRS. The CT spectra between different particles were different due to different flight times. The CT was used to suppress the random coincidence background, and also in the PID for different hadrons detected by the HRS. The calibrations of CT aimed at setting the offsets of $\left(e, e^{\prime} \pi^{ \pm}\right)$peaks at

0 , and optimizing the resolution.
The CT spectrum of ${ }^{3} \mathrm{He}\left(\mathrm{e}, \mathrm{e}^{\prime} \mathrm{h}^{+}\right) \mathrm{X}$ is presented in Fig 3.16. In this spectrum, the electrons detected by the BigBite were selected by the PID techniques introduced in the next chapter, no PID cut was applied to the HRS, and all the production runs in which the HRS detected positively charged particles were combined.


Figure 3.16: The CT spectrum of $\left(e, e^{\prime} h\right)$. The $\pi^{+}$peak centers at CT $=0 \mathrm{~ns}$, the $K^{+}$peak centers at -1.7 ns , and the proton peak centers at -6.7 ns .

## 4

## Data analysis

The goal of the data analysis is to extract the unpolarized SIDIS differential cross sections from the data. The detector calibration, data reconstruction and data quality check have been carried out in the asymmetry studies of experiment E06-010 [93, 95]. The data analysis for the unpolarized SIDIS differential cross section is more complicated than that for the asymmetry studies due to the need of a thorough understanding and description of the experimental acceptance as well as a good control of the systematic uncertainties, as some of which were less important due to the cancellation in the asymmetry studies. Dedicated developments and updates of the detector models in the simulation enabled a good description of the experimental acceptance, and have been successfully used in single electron channels as well as coincidence SIDIS channels. Detailed studies of the systematic uncertainties have been carried out thoroughly for the cross section extraction and the overall systematic uncertainty is mostly under $10 \%$. In addition, radiative corrections, subtraction of the radiative tails from the exclusive scattering channels, and bin-centering corrections have been applied. In this chapter, the general procedures of the data analysis will be presented first. Then each element in the entire analysis will be discussed.

### 4.1 General procedures of data analysis

The general procedure of the data analysis is listed below.

- The data in each experimental run were combined, and the data events were put in certain kinematic bin sets.
- The events in each bin contained both the "good" and the "bad" events. The bad events are those consisted of various contaminations/backgrounds, and were subtracted or corrected for. The efficiencies, related to the fact that some of the good events were not recorded in the data or were cut away in the analysis process, were corrected for.
- The raw differential cross section in each bin was obtained by applying the acceptance correction to the number of data events, with a Monte Carlo simulation that was able to describe the experimental acceptance.
- The raw differential cross sections from the data included not only the physics of interest, but also the radiative effects (REs). Radiative corrections (RCs) were carried out to extract the Born differential cross sections from the data, by using the simulations with and without the REs.
- The bin-centering corrections were carried out, which allowed the comparison between the experimental results and various models/theories.

An important quantity for normalizing the combined data runs is the luminosity, which was obtained from the information recorded in each run of the experiment. In each run, the beam charge, the data-acquisition (DAQ) livetime and the target temperature were recorded in the data together with the information on the detected particles from the detectors. The number of beam-electrons was calculated from the recorded beam charge for each run. The target number density was calculated in
each run based on the filling density of ${ }^{3} \mathrm{He}$ gas in the target, the target geometry and the target temperature values at different parts of the target cell during the experiment. Conventionally the luminosity is defined as the product of the number of beam-electrons $N_{e}$, the target number density $\rho_{t a r}$ and the target length $l_{t a r}$. In each run, due to the DAQ livetime, the luminosity corresponding to the recorded data (effective luminosity) is the product of the conventional luminosity and the DAQ livetime $f_{\text {live }}$. In order to include the livetime correction in the data normalization procedure, we defined the effective luminosity $L_{d}$ as in the following expression:

$$
\begin{equation*}
L_{d}=N_{e} \cdot f_{\text {live }} \cdot \rho_{t a r} \cdot l_{t a r} . \tag{4.1}
\end{equation*}
$$

The individual runs were combined together in the analysis and the effective luminosity related to each of the combined runs were summed to normalize the data when extracting the differential cross sections. Overall detection efficiencies of the detectors were also included in the normalization of the data. Using the elastic electron-proton (ep) calibration runs and the inclusive DIS channel to determine these overall efficiencies is discussed in the following sections.

A number of corrections needed to be applied to the data in order to extract the differential cross sections, namely for the efficiency, the contamination/background subtraction and the acceptance. In order to compare the experimental results with theoretical/phenomenological models, the radiative corrections and the bin-centering corrections need to be applied in addition. The differential cross section from the data in a specific bin before the radiative corrections and the bin-centering corrections is denoted as $\left\langle\frac{d \sigma}{d P H S}\right\rangle_{\text {data }}$, and can be expressed as the following:

$$
\begin{equation*}
\left\langle\frac{d \sigma}{d P H S}\right\rangle_{\text {data }}=N_{\text {data }} \cdot f_{c o r r} \cdot \frac{1}{L_{d}} \cdot \frac{L_{s}}{N_{p h s}}, \tag{4.2}
\end{equation*}
$$

where $N_{\text {data }}$ is the number of events from the data in this bin, $f_{\text {corr }}$ is the factor for the data correction, $L_{d}$ is the effective luminosity, $L_{s}$ is the phase-space simulation
luminosity and $N_{p h s}$ the number of events from the phase-space simulation in this bin.

The data-correction factor $f_{\text {corr }}$ included the correction for the efficiency and the contamination/background, and can be expressed as

$$
\begin{equation*}
f_{\text {corr }}=\left[1-\sum_{i} f_{\text {contam }}(i)\right] \prod_{j} \frac{1}{f_{e f f}(j)}, \tag{4.3}
\end{equation*}
$$

where $f_{\text {contam }}(i)$ is the contamination fraction of the $i$ th type and $f_{\text {eff }}(j)$ is the efficiency of the $j$ th type. The fraction $f_{\text {contam }}(i)$ was defined as the ratio of the number of events from $i$ th type of contamination/background over the total number of events. The types of contamination/background included the events from the radiative tails of the exclusive channels, the $\mathrm{N}_{2}$ background, the random-coincidence background and the remaining contaminations after the PID cuts. The types of efficiencies included the PID-cut efficiencies and the overall detection efficiencies of the BigBite and the HRS. The correction for the $\pi^{ \pm}$decay was included in Eq. (4.3) as $f_{e f f}(d e c a y)$.

The angular and momentum cuts of the BigBite and the HRS, the total-showerenergy $\left(E_{T S}\right)$ cut of the BigBite, the vertex $z$ cut (common for both detectors), the DIS-selecting kinematic cuts, the cut on the mass of the undetected final-state particles $W^{\prime}$ (for SIDIS events), and the coincidence-time (CT) cut (for SIDIS events) used in this study are listed in Table 4.1.

### 4.2 Efficiency and contamination corrections

### 4.2.1 Left HRS PID

The $\mathrm{CO}_{2}$ gas Čerenkov detector, the aerogel Čerenkov detector, the lead-glass calorimeter, and the reconstructed momentum of each event were used to do the PID in the (left) HRS.

| Category | Cut |
| :--- | :---: |
| BigBite angular cut | $25^{\circ}<\theta_{b b}<37^{\circ}$ and $250^{\circ}<\phi_{b b}<290^{\circ}$ |
| BigBite momentum cut | $1.6<P_{b b}<2.5 \mathrm{GeV}$ |
| BigBite $E_{T S}$ cut | $E_{T S}>0.9 \mathrm{GeV}$ |
| HRS angular cut | $14^{\circ}<\theta_{\mathrm{hrs}}<18^{\circ}$ and $81^{\circ}<\phi_{\mathrm{hrs}}<99^{\circ}$ |
| HRS momentum cut | $2.26<P_{\mathrm{hrs}}<2.41 \mathrm{GeV}$ |
| Vertex $z$ cut | $-0.12<v_{z}<0.12 \mathrm{~m}$ |
| DIS-selecting invariant mass cut | $W>2.3 \mathrm{GeV}$ |
| DIS-selecting $Q^{2}$ cut | $Q^{2}>1 \mathrm{GeV}^{2}$ |
| SIDIS-selecting $W^{\prime}$ cut | $W^{\prime}>1.6 \mathrm{GeV}$ |
| Coincidence-time $(\mathrm{CT})$ cut | $-3<\mathrm{CT}<3 \mathrm{~ns}$ |

Table 4.1: List of cuts on the kinematics in this study.

Each PID cut removes some good events thus has a certain efficiency, and some contamination after applying all the PID cuts exists. To study the effect of a PID cut using a certain apparatus on one type of particles, one needs to select a rather clean sample first, which can be achieved by a tight PID cut using another apparatus. A tight PID cut, while being able to produce a clean sample, has a low efficiency in general.

In the runs with negative HRS-polarity, only negatively charged particles were detected, and the $\pi^{-}$and electron events were dominant. The $\pi^{-}$and electron events selected by tight PID cuts in the gas Cerenkov detector are shown in the ratio $E_{t o t} / P_{h r s}$ as in Fig. 4.1, where $E_{t o t}$ is the total energy deposition in the lead-glass calorimeter, and $P_{h r s}$ is the reconstructed momentum. The $\pi^{-}$events are mostly in the range of $E_{t o t} / P_{h r s}<0.6$, and has a peak around 0.1. The electron events are mostly in the range of $E_{t o t} / P_{h r s}>0.6$, and has a peak around 0.85 .

The $\pi^{-}$and electron events selected by the lead-glass calorimeter (the ratio $\left.E_{t o t} / P_{h r s}\right)$, observed in the gas Čerenkov detector are presented in Fig. 4.2. The $\pi^{-}$events are mostly in the range of channel ${ }_{\mathrm{GC}}<250$, where channel ${ }_{\mathrm{GC}}$ is the chan-
nel number ${ }^{1}$ of the gas Čerenkov detector. The $\pi^{-}$spectrum has a very large peak at 0 . The electron events are mostly in the range of channel ${ }_{\mathrm{GC}}>250$, and has a peak around 1300 .

The tight PID cuts in the gas Cerenkov detector and the ratio $E_{t o t} / P_{h r s}$, are listed in Table 4.2.

| Tight PID cut | Type of selection |
| :---: | :---: |
| channel $_{\text {GC }}>700$ | Electron events |
| channel $_{\text {GC }}<10$ | $\pi^{-}$events |
| $E_{t o t} / P_{\text {hrs }}>0.8$ | Electron events |
| $0.08<E_{\text {tot }} / P_{\text {hrs }}<0.11$ | $\pi^{-}$events |

Table 4.2: Tight PID cuts in the gas Čerenkov detector and the ratio $E_{t o t} / P_{h r s}$, for clean sample selection.

The scaling factors of the $\pi^{-}$and electron spectra in the lead-glass calorimeter $\left(E_{\text {tot }} / P_{\text {hrs }}\right)$ and the gas Čerenkov channel are based on a ratio $N_{\text {full }} / N_{\text {sample }}$, where $N_{\text {full }}$ is the number of events without PID cuts, and $N_{\text {sample }}$ is the number of events in the clean sample. The integration ranges for $N_{\text {full }} / N_{\text {sample }}$ used in Figs. 4.1 and 4.2 are listed in Table 4.3. It is observed that the $\pi^{-}$and electron spectra, with proper scaling, add up to the spectrum without PID cuts, and one is able to study the effects of a cut in $E_{t o t} / P_{h r s}$ on different particles. This method allows one to study the PID-cut efficiency and the fraction of contamination after applying a PID cut.

In the runs with positive HRS-polarity, only the positively charged particles were detected, and the $\pi^{+}$events were dominant. There were also a certain amount of proton events, which could not be distinguished from the $\pi^{+}$events by using the $E_{t o t} / P_{h r s}$ and channel ${ }_{\mathrm{GC}}$ spectra. The $E_{t o t} / P_{h r s}$ and channel ${ }_{\mathrm{GC}}$ spectra of positively charged particles without applying PID cuts are presented in Fig. 4.3. As the number

[^0]| Integration range | Type of spectrum |
| :--- | :---: |
| $0.8<E_{\text {tot }} / P_{\text {hrs }}<0.9$ | Electron spectrum |
| $0.08<E_{\text {tot }} / P_{\text {hrs }}<0.11$ | $\pi^{-}$spectrum |
| $2000<$ channel $_{\mathrm{GC}}<2500$ | Electron spectrum |
| $0<$ channel $_{\mathrm{GC}}<10$ | $\pi^{-}$spectrum |

Table 4.3: Integration ranges for electron and $\pi^{-}$spectra in the ratio $E_{t o t} / P_{h r s}$ and the gas Čerenkov channel.
of positron events was very small, these spectra with no PID cuts are very similar to the $\pi^{-}$spectra in Figs. 4.1 and 4.2. By applying the tight electron-selecting PID cuts in channel ${ }_{\mathrm{GC}}$ and $E_{t o t} / P_{h r s}$, one can observe the positron events in the $E_{t o t} / P_{h r s}$ and channel ${ }_{\mathrm{GC}}$ spectra, as presented in Fig. 4.4. The shape of the positron spectra are very similar to the electron spectra in Figs. 4.1 and 4.2. A small number (comparable to the number of positron events) of positively charged hadron events remained after the tight lepton PID cut, as in Fig. 4.4.


Figure 4.1: The $\pi^{-}$and electron events selected by the gas Cerenkov detector, observed in the ratio $E_{t o t} / P_{h r s}$. The green histogram contains all the events, without PID cuts. The black (red) line represents the electron ( $\pi^{-}$) events selected by a tight PID cut in the gas Cerenkov detector. The scaling of the electron and $\pi^{-}$spectra is described in the text. The right-panel plot is a zoom-in view of the left-panel plot, focusing on the junction region.


Figure 4.2: The $\pi^{-}$and electron events selected by the lead-glass calorimeter (the ratio $E_{t o t} / P_{h r s}$ ), observed in the gas Čerenkov detector. The green histogram contains all the events, without PID cuts. The black (red) line represents the electron ( $\pi^{-}$) events selected by a tight PID cut in the lead-glass calorimeter. The scaling of the electron and $\pi^{-}$spectra is described in the text. The right-panel plot is a zoom-in view of the left-panel plot, focusing on the junction region.

The proton and $\pi^{+}$events were separated by using the aerogel Čerenkov detector and the CT information. The spectra of hadrons with positive (negative) charge in the aerogel Čerenkov channel channel ${ }_{\mathrm{AC}}$, after the cut $0.08<E_{\text {tot }} / P_{h r s}<0.11$ and several different CT cuts are presented in the left (right) panel of Fig. 4.5. The spectra in each panel has been scaled to the spectrum without any CT cut, according to the number of events in the range $1000<$ channel $_{\mathrm{AC}}<1500$, without changing the shape. The spectra of negatively charged hadrons (mostly $\pi^{-}$, with a very small number of $K^{-}$) have almost the same shape under different CT cuts, with a peak at channel ${ }_{\mathrm{AC}} \approx 550$. The spectra of positively charged hadrons have two peaks from the $\pi^{+}$and proton spectra. The $\pi^{+}$spectrum has a very similar shape as the $\pi^{-}$spectrum. The proton spectra has a peak at channel ${ }_{\mathrm{AC}} \approx 100$. The CT cut can change the size of the proton spectrum relative to the $\pi^{+}$spectrum, due to the different CT behavior of $\pi^{+}$and proton.

The CT spectra of positively and negatively charged hadrons, under the cut


Figure 4.3: The $E_{t o t} / P_{h r s}$ and the gas Čerenkov spectra of positively charged particles in the HRS.
$0.08<E_{t o t} / P_{h r s}<0.11$, are presented in the left and right panel of Fig. 4.6. The $\pi^{ \pm}$ events are mostly in the range $-3<\mathrm{CT}<3 \mathrm{~ns}$, and the proton events are mostly in the range $-9<\mathrm{CT}<-4.5 \mathrm{~ns}$.

The spectrum of positively charged hadrons in the aerogel Čerenkov detector can be decomposed into the proton and pion spectra, with proper scaling factors. In this way, the efficiency of the PID cut in the aerogel Cerenkov detector and the


Figure 4.4: The $E_{t o t} / P_{h r s}$ (gas Čerenkov) spectrum of positively charged particles, with the tight electron-selecting PID cuts in the gas Čerenkov $\left(E_{t o t} / P_{h r s}\right)$.


Figure 4.5: The spectra of positively charged hadrons (left panel), and negatively charged hadrons (right panel), with different CT cuts.


Figure 4.6: The CT spectra of positively charged hadrons (left panel), and negatively charged hadrons (right panel).
contamination after the PID cut can be studied. The proton spectrum was obtained by using the cuts $0.08<E_{t o t} / P_{h r s}<0.11$ and $-8<\mathrm{CT}<-5.5 \mathrm{~ns}$. The $\pi^{-}$ spectrum is very similar to the $\pi^{+}$with the cut $0<\mathrm{CT}<0.5 \mathrm{~ns}$, and could represent the spectra of both $\pi^{+}$and $\pi^{-}$. The decomposition of the spectrum of positively charged hadrons without CT cuts is presented in Fig. 4.7. In this figure, factors $f s c_{p}$ and $f s c_{\pi}$ were used to scale the proton and pion spectra, respectively. These


Figure 4.7: The decomposition of the spectrum of positively charged hadrons without CT cuts. The black line is the spectrum without CT cuts. The blue line is the pion spectrum. The red line is the proton spectrum. The green line is the sum of blue and red line.
scaling factors are expressed as

$$
\begin{align*}
f s c_{p} & =\frac{n_{2} n_{\pi, 1}-n_{1} n_{\pi, 2}}{n_{p, 2} n_{\pi, 1}-n_{p, 1} n_{\pi, 2}}  \tag{4.4}\\
f s c_{\pi} & =\frac{-n_{2} n_{p, 1}+n_{1} n_{p, 2}}{n_{p, 2} n_{\pi, 1}-n_{p, 1} n_{\pi, 2}} \tag{4.5}
\end{align*}
$$

where $n_{i}, n_{\pi, i}$ and $n_{p, i}$ represent the numbers of events in range $i(i=1$ or 2$)$ from the total, the pion and the proton spectrum. Range $i=1$ is for the proton peak $80<$ channel $_{\mathrm{AC}}<120$, and $i=2$ is for the pion-dominating range $1000<$ channel $_{\mathrm{AC}}<1500$. It is straightforward to verify the following relations from Eqs. (4.4) and (4.5):

$$
\begin{align*}
& n_{1}=n_{\pi, 1} f s c_{\pi}+n_{p, 1} f s c_{p},  \tag{4.6}\\
& n_{2}=n_{\pi, 2} f s c_{\pi}+n_{p, 2} f s c_{p} . \tag{4.7}
\end{align*}
$$

In Fig. 4.7, it is observed that the sum (green line) of the proton and pion spectra
is almost the same as the total spectrum (black line) in all the ranges.
The PID cuts used in the HRS for selecting the $\pi^{ \pm}$events from the data of the production runs are listed in Table 4.4. The efficiencies and contaminations related

| Detector name | PID cut |
| :--- | :---: |
| Aerogel Cerenkov | channel $_{\mathrm{AC}}>150$ |
| Gas Cerenkov | channel $_{\mathrm{GC}}<250$ |
| Lead-glass calorimeter | $E_{\text {tot }} / P_{\text {hrs }}<0.6$ |

Table 4.4: PID cuts used for $\pi^{ \pm}$event selection from the production runs.
to the PID cuts above are listed in Table 4.5.

| Item | Value |
| :--- | :---: |
| Aerogel Cerenkov cut efficiency | $96.5 \%$ |
| Gas C̈erenkov cut efficiency | $99.5 \%$ |
| Lead-glass calorimeter cut efficiency | $99.3 \%$ |
| Positively charged non-pion hadron contamination to $\pi^{+}$ | $0.7 \%$ |
| Positron contamination to $\pi^{+}$ | $<10^{-5}$ |
| Negatively charged non-pion hadron contamination to $\pi^{-}$ | $<0.1 \%$ |
| Electron contamination to $\pi^{-}$ | $<10^{-4}$ |

Table 4.5: Efficiencies and contaminations related to the PID cuts in the HRS.

### 4.2.2 BigBite PID

The negatively and positively charged particles detected by the BigBite spectrometer were bent by the magnetic field in opposite directions, and could be clearly separated. A two-dimensional (2D) plot showing the clear separation of differently charged particles is presented in Fig. 4.8. The elastic electron-proton (ep) scattering data were used to check the separation of particles with different charges. The 2D plots showing the separation of particles with different charges, from the elastic ep data at 1.23 GeV beam energy is presented in Fig. 4.9. The cut(s) applied to the plots are shown


Figure 4.8: Particles detected by the BigBite spectrometer. The positively charged particles (blue dots), the negatively charged particles (red dots) and the neutral particles (black dots) are shown together in the top left panel. The neutral particles are shown in the top right panel. The positively and negatively charged particles are shown in the bottom left and right panels, respectively. The densities of the particles in the top right, and bottom panels are shown in color scales.
in each panel of Fig. 4.9, namely the combinations of the preshower cut and the $W$ cut. The preshower cut was used to reject the hadrons (mainly $\pi^{ \pm}$), as will be discussed in the following part of this subsection. The invariant mass $W$ cut was used to select the elastic ep kinematics $(0.9<W<1.0 \mathrm{GeV})$, which could help to obtain a cleaner electron sample. It can be observed that, the positively charged, negatively charged and neutral particles are well separated. When a clean electron sample is selected, more than $99.9 \%$ of the events were identified as negatively charged, which


Figure 4.9: Particles detected by the BigBite spectrometer in the elastic ep runs at 1.23 GeV beam energy. The positively charged particles (blue dots), the negatively charged particles (red dots) and the neutral particles (black dots) are shown together in each panel. In each panel, the cut(s) applied to the data, and the fraction of the negatively (positively) charged particles denoted by $R_{N}\left(R_{P}\right)$ are presented.
supports the validity of the charge separation/identification.
The PID in the BigBite spectrometer was based on the ratio of the total shower energy $\left(E_{T S}\right)$ over the reconstructed momentum $\left(P_{b b}\right)$, and the information of the preshower channel (channel ${ }_{P S}$ ). The behavior of leptons and hadrons (mostly pions) in $E_{T S} / P_{b b}$ and channel ${ }_{\text {PS }}$ can be distinguished from each other. A 2D plot of channel ${ }_{\text {PS }}$ vs. $E_{T S} / P_{b b}$ is presented in Fig. 4.10. The pion events dominate in the low preshower channels (channel ${ }_{\mathrm{PS}}<300$ ), and cover a large range of $E_{T S} / P_{b b}$. The lepton events are mostly in the range around $E_{T S} / P_{b b}=1$, and cover an extended


Figure 4.10: The 2D plot of preshower channel (channel ${ }_{\mathrm{PS}}$ ) vs. $E_{T S} / P_{b b}$.
range of the preshower channels. The PID cut for leptons in $E_{T S} / P_{b b}$ was not a constant and had a $P_{b b}$ dependence. This cut was described as

$$
\begin{equation*}
\left|\frac{E_{T S}}{P_{b b}}-\mu\left(P_{b b}\right)\right|<W_{c u t}\left(P_{b b}\right) \tag{4.8}
\end{equation*}
$$

where $\mu\left(P_{b b}\right)$ is the central value of $E_{T S} / P_{b b}$, and $W_{c u t}\left(P_{b b}\right)$ is the width of the cut. Both $\mu\left(P_{b b}\right)$ and $W_{c u t}\left(P_{b b}\right)$ were determined from the data, by Gaussian fits of $E_{T S} / P_{b b}$ in different ranges of $P_{b b}: \mu$ is equal to the Gaussian mean value, and $W_{c u t}$ is the Gaussian width $\sigma$ multiplied by a constant $N_{s}$ chosen empirically. The 2D plot of $P_{b b}$ vs. $E_{T S} / P_{b b}$, and an example Gaussian fit in the range $1.3<P_{b b}<1.5$ GeV are presented in Fig. 4.11. In this plot, a cut of channel $\mathrm{PS}_{\mathrm{PS}}>300$ was applied, to suppress the large number of pions in the low preshower channels.

The 2D plot of $P_{b b}$ vs. $E_{T S} / P_{b b}$ and the cut with $W_{c u t}=2.5 \sigma$ is presented in Fig. 4.12. The momentum-dependent quantities $\mu\left(P_{b b}\right)$ and $\sigma\left(P_{b b}\right)$ were parameterized as


Figure 4.11: A 2D plot of $P_{b b}$ vs. $E_{T S} / P_{b b}$ in the left panel. The $E_{T S} / P_{b b}$ distribution in the range $1.3<P_{b b}<1.5 \mathrm{GeV}$ (between the two black lines), and the corresponding Gaussian fit are shown in the right panel.
as

$$
\begin{align*}
\mu\left(P_{b b}\right) & =a_{0}+a_{1} P_{b b}+a_{2} P_{b b}^{2}+a_{3} P_{b b}^{3}  \tag{4.9}\\
\sigma\left(P_{b b}\right) & =b_{0}+b_{1} P_{b b}+b_{2} P_{b b}^{2}+b_{3} P_{b b}^{3} \tag{4.10}
\end{align*}
$$

where $a_{0}, a_{1}, a_{2}, a_{3}, b_{0}, b_{1}, b_{2}$ and $b_{3}$ are parameters determined by a series of fits of $E_{T S} / P_{b b}$ in different $P_{b b}$ ranges. Two sets of parameters were used in the ranges $P_{b b} \leq 0.9 \mathrm{GeV}$ or $P_{b b}>0.9 \mathrm{GeV}$, and were able to describe the $P_{b b}$ dependence of the $E_{T S} / P_{b b}$ distribution in the data. These sets of parameters are presented in Tables 4.6 and 4.7.

| Kinematic ranges | $a_{0}$ | $a_{1}$ | $a_{2}$ | $a_{3}$ |
| :--- | :---: | :---: | :---: | :---: |
| $P_{b b} \leq 0.9$ | 1.5664 | 1.1385 | 5.6513 | 0.0 |
| $P_{b b}>0.9$ | 0.7931 | 0.4426 | -0.2740 | 0.0457 |

Table 4.6: Parameters for $\mu\left(P_{b b}\right)$.

To study the PID cut in the preshower channels, the channel ${ }_{P S}$ distribution in different $P_{b b}$ ranges were studied. A 2D plot of $P_{b b}$ vs. channel ${ }_{P S}$, and the channel ${ }_{P S}$


Figure 4.12: Events and the lepton PID cut in a 2D plot of $P_{b b}$ vs. $E_{T S} / P_{b b}$. The PID cut is represented by the black lines.

| Kinematic ranges | $b_{0}$ | $b_{1}$ | $b_{2}$ | $b_{3}$ |
| :--- | :---: | :---: | :---: | :---: |
| $P_{b b} \leq 0.9$ | 0.0849 | 0.1218 | -0.1041 | 0.0 |
| $P_{b b}>0.9$ | 0.2939 | -0.3408 | 0.1763 | -0.0293 |

Table 4.7: Parameters for $\sigma\left(P_{b b}\right)$.
distribution in the range $0.9<P_{b b}<1.2$ are presented in Fig. 4.13. In this plot, the $E_{T S} / P_{b b}$ cut shown in Fig. 4.12 was applied. The hadron (pion) part was fitted with a Gaussian-convoluted Landau function, and the lepton (electron) part was fitted with a Gaussian distribution. The data spectrum was well described by the combined fit. While the data spectrum can be well described by the combined fit, it is important to check the hadron and lepton spectra individually. Due to the lack of other PID detectors, the method for studying the HRS PID in Section 4.2.1 could not be used for the BigBite. Relatively clean pion spectrum in the preshower


Figure 4.13: 2D plot of $P_{b b}$ vs. channel $l_{\text {PS }}$, and the preshower spectrum of events selected by the $E_{T S} / P_{b b}$ cut for leptons.
channels can be obtained by requiring $E_{T S} / P_{b b}<0.45$ or $E_{T S} / P_{b b}>1.55$, instead of the cut for leptons shown in Fig. 4.12. A 2D plot of $P_{b b}$ vs. channel ${ }_{P S}$, and the channel $_{\text {PS }}$ distribution in the range $0.9<P_{b b}<1.2$ are presented in Fig. 4.14. It


Figure 4.14: 2D plot of $P_{b b}$ vs. channel ${ }_{\text {PS }}$, and the preshower spectrum of pion events, selected by $E_{T S} / P_{b b}<0.45$ or $E_{T S} / P_{b b}>1.55$.
can be observed that the pion spectrum in the preshower channels can be reasonably described by the Gaussian-convoluted Landau function.

The pion contamination and PID-cut efficiency have kinematic dependence and also depend on the interaction channel under the study. In the semi-inclusive channel, the size of the pion spectrum relative to the electron spectrum is much smaller than that in the inclusive channel, as presented in Fig. 4.15. The semi-inclusive channel in this plot requires that a pion is detected by the HRS in coincidence with the negatively charge particles detected by the BigBite.


Figure 4.15: 2D plot of $P_{b b}$ vs. channel ${ }_{\text {PS }}$, and the preshower spectrum of events selected by the $E_{T S} / P_{b b}$ cut for leptons, requiring a pion is detected by the HRS in coincidence.

The fraction of the $\pi^{-}$contamination and the PID cut efficiency are defined as the following:

$$
\begin{align*}
f_{\text {contam }}(\pi) & =\frac{N_{\pi, h}}{N_{h}}  \tag{4.11}\\
f_{\text {eff }}(P S) & =1-\frac{N_{e, l}}{N_{h}-N_{\pi, h}+N_{e, l}}, \tag{4.12}
\end{align*}
$$

where $f_{\text {contam }}(\pi)$ represents the $\pi^{-}$contamination, and $f_{\text {eff }}(P S)$ represents the preshower cut efficiency. The numbers $N_{i}, N_{\pi, i}$ and $N_{e, i}$ represent the number from the data (the histogram of preshower spectrum), the number from the pion fit and the number from the electron fit, where $i=l(h)$ represents the integrated number
in the range lower (higher) than the preshower cut. The events with $i=l$ and $i=h$ are the ones being rejected and kept intact by the preshower cut, respectively.

The total number of good events (electrons) can be obtained by substituting Eqs. (4.11) and (4.12) into Eq. (4.3):

$$
\begin{equation*}
N_{e}=N_{h} \times\left(1-f_{\text {contam }}(\pi)\right) / f_{\text {eff }}(P S)=N_{h}-N_{\pi, h}+N_{e, l} \tag{4.13}
\end{equation*}
$$

where $N_{e}$ is the total number of electrons.
In the SIDIS channel, under different preshower cuts, the PID-cut efficiency and the $\pi^{-}$contamination are different. These numbers in the $\pi^{+}$and $\pi^{-}$production SIDIS channel are presented in Tables 4.8 and 4.9.

| $P_{b b}$ range $(\mathrm{GeV})$ | $E_{1}(\%)$ | $C_{1}(\%)$ | $E_{2}(\%)$ | $C_{2}(\%)$ | $E_{3}(\%)$ | $C_{3}(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0.6-0.7$ | 9.75 | 7.91 | 18.7 | 5.04 | 31.2 | 4.06 |
| $0.7-0.8$ | 8.68 | 4.76 | 16.7 | 2.96 | 28.2 | 2.31 |
| $0.8-0.9$ | 7.6 | 3.62 | 14.8 | 2.26 | 25.3 | 1.74 |
| $0.9-1.0$ | 5.43 | 3.17 | 11.0 | 1.96 | 19.5 | 1.47 |
| $1.0-1.1$ | 4.36 | 2.82 | 8.87 | 1.71 | 16.0 | 1.26 |
| $1.1-1.2$ | 3.56 | 2.6 | 7.27 | 1.57 | 13.3 | 1.15 |
| $1.2-1.3$ | 3.2 | 2.0 | 6.44 | 1.2 | 11.6 | 0.862 |
| $1.3-1.4$ | 2.88 | 1.57 | 5.74 | 0.936 | 10.3 | 0.667 |
| $1.4-1.5$ | 2.4 | 1.17 | 4.84 | 0.692 | 8.82 | 0.492 |
| $1.5-1.6$ | 2.11 | 0.903 | 4.23 | 0.531 | 7.72 | 0.374 |
| $1.6-1.7$ | 1.97 | 0.621 | 3.93 | 0.364 | 7.11 | 0.255 |
| $1.7-1.8$ | 1.65 | 0.582 | 3.33 | 0.342 | 6.1 | 0.239 |
| $1.8-1.9$ | 1.62 | 0.358 | 3.21 | 0.209 | 5.8 | 0.146 |
| $1.9-2.0$ | 1.35 | 0.375 | 2.72 | 0.222 | 5.01 | 0.155 |
| $2.0-2.1$ | 1.27 | 0.192 | 2.53 | 0.109 | 4.63 | 0.0755 |

Table 4.8: The PID-cut inefficiency $\left[E_{i}=1.0-f_{\text {eff }}\left(P S_{i}\right)\right]$ and the $\pi^{-}$contamination $\left(C_{i}\right)$ in the $\pi^{+}$production SIDIS channel under three different preshower cuts $i=1$, 2 and 3 , which are channel $l_{P S}>300$, 400 and 500 , respectively.

It is observed that, with a tighter PID cut, the fraction of the contamination is smaller, but the efficiency also becomes lower.

| $P_{b b}$ range (GeV) | $E_{1}(\%)$ | $C_{1}(\%)$ | $E_{2}(\%)$ | $C_{2}(\%)$ | $E_{3}(\%)$ | $C_{3}(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0.6-0.7$ | 9.75 | 3.0 | 18.7 | 1.87 | 31.2 | 1.49 |
| $0.7-0.8$ | 8.68 | 1.88 | 16.7 | 1.16 | 28.2 | 0.906 |
| $0.8-0.9$ | 7.6 | 1.45 | 14.8 | 0.897 | 25.3 | 0.688 |
| $0.9-1.0$ | 5.43 | 1.07 | 11.0 | 0.656 | 19.5 | 0.491 |
| $1.0-1.1$ | 4.36 | 0.825 | 8.87 | 0.499 | 16.0 | 0.368 |
| $1.1-1.2$ | 3.56 | 0.747 | 7.27 | 0.448 | 13.3 | 0.326 |
| $1.2-1.3$ | 3.2 | 0.547 | 6.44 | 0.326 | 11.6 | 0.234 |
| $1.3-1.4$ | 2.88 | 0.408 | 5.74 | 0.242 | 10.3 | 0.173 |
| $1.4-1.5$ | 2.4 | 0.297 | 4.84 | 0.175 | 8.82 | 0.124 |
| $1.5-1.6$ | 2.11 | 0.202 | 4.23 | 0.119 | 7.72 | 0.0836 |
| $1.6-1.7$ | 1.97 | 0.142 | 3.93 | 0.0831 | 7.11 | 0.0583 |
| $1.7-1.8$ | 1.65 | 0.133 | 3.33 | 0.0781 | 6.1 | 0.0545 |
| $1.8-1.9$ | 1.62 | 0.088 | 3.21 | 0.0513 | 5.8 | 0.0357 |
| $1.9-2.0$ | 1.35 | 0.102 | 2.72 | 0.0601 | 5.01 | 0.0419 |
| $2.0-2.1$ | 1.27 | 0.0347 | 2.53 | 0.0199 | 4.63 | 0.0137 |

Table 4.9: The PID-cut efficiency $\left[E_{i}=1.0-f_{e f f}\left(P S_{i}\right)\right]$ and the $\pi^{-}$contamination $\left(C_{i}\right)$ in the $\pi^{-}$production SIDIS channel under three different preshower cuts $i=1$, 2 and 3 , which are channel $l_{P S}>300$, 400 and 500 , respectively.

### 4.2.3 Photon-induced contamination

A more significant contamination in the electron events was from the photon-induced electrons from the pair-production. The $\pi^{0}$ meson, from the electroproduction, decays into two photons. The high energy photons create the photon-induced electron contamination through the pair-production process. Because the detected particles were electrons, this contamination could not be rejected by the PID techniques. The percentage of this contamination in the total electron events was determined by comparing the positron yield in the runs using the BigBite with reversed magnetic field (reversed-polarity runs or RP runs) and the electron yield in the production runs. The electrons in the production runs and the positrons in the RP runs had the same acceptance effect, and the yields could be directly compared. The positrons in the productions runs could not be directly used for this study, because oppositely charge
particles in the same BigBite magnetic field had different acceptance effects.
A 2D plot of $P_{b b}$ vs. channel ${ }_{P S}$, and the channel ${ }_{P S}$ distribution of the positively charged particles from the RP runs in the range $0.9<P_{b b}<1.2$ are presented in Fig. 4.16. In this plot, the $E_{T S} / P_{b b}$ cut for leptons in Fig. 4.12 was applied.


Figure 4.16: 2D plot of $P_{b b}$ vs. channel ${ }_{\text {PS }}$, and the preshower spectrum of positively charged particles in the RP runs.

Three main observations from Fig. 4.16 are listed below.

- The number of the positrons decreases very fast in increasing range of $P_{b b}$, which means the photon-induced contamination is much smaller in high $P_{b b}$ range than in low $P_{b b}$ range.
- The contamination from the $\pi^{+}$events to the positrons is much larger than the case of $\pi^{-}$and electrons.
- Due to limited number of RP runs, the statistics was limited. Statistical fluctuations and irregular shapes in channels are observed, which resulted in large fitting uncertainties when a small kinematic range was under study.

The shapes of the preshower spectra from the positively charged particles in the RP runs, and from the positively and negatively charged particles in the production
runs are compared in Fig. 4.17. All these spectra were in the momentum range of $0.9<P_{b b}<1.2 \mathrm{GeV}$. The observed agreements between the shapes of the spectra


Figure 4.17: The shape comparison between the preshower spectra from the positively charged particles in the RP runs (black line), and from the negatively (red line) and positively (blue line) charged particles in the production runs. The spectra in the right panel (five channels per bin) are re-binned from the ones in the left panel (one channel per bin). The spectra were scaled to have the same number of events in the channels between 100 and 200, while the shapes were intact.
allowed one to use the pion and lepton fitting results, with proper scaling factors, to study the $\pi^{+}$contamination to positrons in the RP runs. This method is similar to the PID study in the HRS and helped to reduce the uncertainties from fitting a spectrum with limited statistics.

A 2D plot of $P_{b b}$ vs. channel ${ }_{P S}$, and the channel ${ }_{P S}$ distribution for the positively charged particles from the production runs in the range $0.9<P_{b b}<1.2$ are presented in Fig. 4.18.

The $\pi^{+}$contamination to the positrons can be determined in the same way as in Eq. (4.11) for the case of $\pi^{-}$and electrons. Because of the similar shapes of the pion spectra shown in Fig. 4.17, another way with a scaling method can be used to avoid fitting a spectrum with limited statistics. This method is expressed as

$$
\begin{equation*}
f_{\text {contam }}(\pi, 2)=\frac{N_{\pi, h}(2)}{N_{h}(2)}=\frac{N_{h}(1)}{N_{h}(2)} \frac{N_{\text {int }}(2)}{N_{\text {int }}(1)} \times f_{\text {contam }}(\pi, 1) \tag{4.14}
\end{equation*}
$$



Figure 4.18: 2D plot of $P_{b b}$ vs. channel ${ }_{\mathrm{PS}}$, and the preshower spectrum of positively charged particles in the production runs.
where $N_{\text {int }}(i)$ is a number from integrating the $i$ th preshower spectrum around the pion peak (channel 100 to 170 , for example), $i=1$ and 2 . This method assumed the following:

$$
\begin{equation*}
\frac{N_{\pi, h}(1)}{N_{i n t}(1)}=\frac{N_{\pi, h}(2)}{N_{i n t}(2)} . \tag{4.15}
\end{equation*}
$$

This assumption is valid as long as the integration range of $N_{\text {int }}(i)$ covers clean pion samples, and the pion spectra 1 and 2 have the same shape. In this way, the fitting on spectrum 2 is avoided, and $f_{\text {contam }}(\pi, 2)$ can be obtained from $f_{\text {contam }}(\pi, 1)$ and four integrated numbers from the spectra.

The fraction of the photon-induced electron contamination can be expressed as

$$
\begin{equation*}
f_{\text {contam }}\left(\gamma_{e}\right)=\frac{N_{h}(\mathrm{RP})\left(1-\mathrm{f}_{\text {contam }}\left(\pi^{+}\right)\right)}{N_{h}(\operatorname{Prod})\left(1-\mathrm{f}_{\text {contam }}\left(\pi^{-}\right)\right)} \cdot \frac{L_{\mathrm{Prod}}}{L_{\mathrm{RP}}} \tag{4.16}
\end{equation*}
$$

where $L_{\text {Prod }}\left(L_{\mathrm{RP}}\right)$ represents the total effective luminosity of the production (RP) runs.

The fractions of the photon-induced electron contamination in the $\pi^{+}$and $\pi^{-}$ production SIDIS channel are presented in Tables 4.10 and 4.11. The statistical uncertainties in the scaling method are presented in the tables.

| $P_{b b}$ range $(\mathrm{GeV})$ | $C_{1} \pm \delta_{1}(\%)$ | $C_{2} \pm \delta_{2}(\%)$ | $C_{3} \pm \delta_{3}(\%)$ |
| :---: | :---: | :---: | :---: |
| $0.6-0.7$ | $43.3 \pm 6.36$ | $45.2 \pm 6.52$ | $46.3 \pm 6.93$ |
| $0.7-0.8$ | $28.0 \pm 4.00$ | $29.4 \pm 4.04$ | $29.2 \pm 4.22$ |
| $0.8-0.9$ | $18.5 \pm 2.83$ | $17.3 \pm 2.72$ | $18.1 \pm 2.86$ |
| $0.9-1.0$ | $11.6 \pm 2.13$ | $11.4 \pm 2.07$ | $10.4 \pm 2.02$ |
| $1.0-1.1$ | $8.68 \pm 1.78$ | $8.39 \pm 1.7$ | $8.84 \pm 1.77$ |
| $1.1-1.2$ | $3.36 \pm 1.19$ | $3.55 \pm 1.18$ | $3.89 \pm 1.26$ |
| $1.2-1.3$ | $3.94 \pm 1.25$ | $4.0 \pm 1.22$ | $4.46 \pm 1.28$ |
| $1.3-1.4$ | $3.15 \pm 1.19$ | $2.71 \pm 1.08$ | $2.72 \pm 1.06$ |
| $1.4-1.5$ | $2.3 \pm 1.06$ | $2.1 \pm 0.972$ | $2.01 \pm 0.944$ |
| $1.5-2.2$ | $0.527 \pm 0.241$ | $0.473 \pm 0.219$ | $0.514 \pm 0.225$ |

Table 4.10: The fractions of the photon-induced contamination $C_{i}$ in the $\pi^{+}$SIDIS production channel, under three different preshower cuts $i=1,2$ and 3 , which are channel ${ }_{P S}>300$, 400 and 500 , respectively. The $\delta$ s represent the statistical uncertainties of the contamination fractions using the scaling method introduced in the text.

### 4.2.4 Random coincidence subtraction

Besides the good SIDIS events, there were also a small amount of random-coincidence events in the time window of $-3<\mathrm{CT}<3 \mathrm{~ns}$. The random-coincidence background were subtracted by subtracting the events in the time windows of $-69<\mathrm{CT}<-19$ ns and $9<\mathrm{CT}<59 \mathrm{~ns}$ (with a scaling factor of $6 / 100$ ) from the events in the time window of $-3<\mathrm{CT}<3 \mathrm{~ns}$.

The spectra with and without the random-coincidence-background subtraction in the aerogel Cerenkov detector is presented in Fig. 4.19.

### 4.2.5 Corrections for shower-threshold drift

There were complicated and time-dependent drifts of the total shower energy threshold for the BigBite trigger during the experiment. The effect of the threshold drift can be seen in the $E_{T S}$ spectra in the shower blocks. As presented in Fig. 3.9, there were 189 shower blocks arranged in 27 rows and 7 columns. The $E_{T S}$ spectra in four typical blocks are presented in Fig. 4.20. In the top left and right panels of

| $P_{b b}$ range $(\mathrm{GeV})$ | $C_{1} \pm \delta_{1}(\%)$ | $C_{2} \pm \delta_{2}(\%)$ | $C_{3} \pm \delta_{3}(\%)$ |
| :---: | :---: | :---: | :---: |
| $0.6-0.7$ | $26.0 \pm 4.49$ | $29.6 \pm 5.01$ | $29.0 \pm 5.26$ |
| $0.7-0.8$ | $20.8 \pm 3.29$ | $21.8 \pm 3.41$ | $20.9 \pm 3.48$ |
| $0.8-0.9$ | $22.4 \pm 3.1$ | $20 \pm 2.91$ | $21.0 \pm 3.06$ |
| $0.9-1.0$ | $12.6 \pm 2.2$ | $12.6 \pm 2.16$ | $12.0 \pm 2.16$ |
| $1.0-1.1$ | $13.9 \pm 2.28$ | $13 \pm 2.16$ | $12.9 \pm 2.16$ |
| $1.1-1.2$ | $6.64 \pm 1.61$ | $5.99 \pm 1.52$ | $6.8 \pm 1.62$ |
| $1.2-1.3$ | $4.9 \pm 1.43$ | $4.43 \pm 1.35$ | $4.25 \pm 1.32$ |
| $1.3-1.4$ | $4.54 \pm 1.44$ | $4.77 \pm 1.45$ | $4.4 \pm 1.4$ |
| $1.4-1.5$ | $3.0 \pm 1.2$ | $2.6 \pm 1.12$ | $2.6 \pm 1.12$ |
| $1.5-2.2$ | $1.96 \pm 0.488$ | $1.33 \pm 0.391$ | $1.27 \pm 0.375$ |

Table 4.11: The fractions of the photon-induced contamination $C_{i}$ in the $\pi^{-}$SIDIS production channel. The format of the table is the same as the table for the $\pi^{+}$ channel above.

Fig. 4.20, the spectra show a step-function-like behavior, which means the effect of the threshold drifts in these two blocks were not significant. There are very complicated spectra in the bottom two panels, indicating significant effects of the threshold drifts. All of the 189 blocks experienced complicated threshold drifts to certain extent, which induced complicated effects in the trigger efficiency. In principle, if the threshold of each block was known for each run, these effects could be simulated,


Figure 4.19: The $\pi^{+}\left(\pi^{-}\right)$SIDIS production channels with and without the random-coincidence-background subtraction is presented in the left (right) panel.


Figure 4.20: The $E_{T S}$ spectra in four typical blocks. The black lines represent the spectra without any PID cut. The red lines represent the spectra with the $E_{T S} / P_{b b}$ PID cut for leptons.
and thus corrected for. However, two features of the study for the threshold drifts, as listed below, hindered this block-by-block solution on a run-by-run basis.

- In plots as shown in Fig. 4.20, many blocks do not show step-function-like spectra, therefore their thresholds cannot be simply read out. This is because the threshold of one block could change between different runs, and a plot combining several runs shows the mixed effect of threshold drift in an extended period of time.
- A number of runs need to be combined together to provide enough statistics for a spectrum plot of each block. With the data of only one run, only some
discrete events could be seen in many blocks, therefore it is not possible to determine the threshold on a run-by-run basis, even if one neglects the drift during one run.

The method adopted for solving the threshold-drift issue was to apply a cut in $E_{T S}$, high enough to override the complicated drift, but not too high to significantly reduce the kinematic range and the valuable data. In this way, the effect due to the threshold drift was manually removed. To understand and correct for the efficiency of the $E_{T S}$ cut, a Monte Carlo simulation was used, and a description of the total shower energy deposition was included in the simulation. The details of the simulation will be presented in the next section.

### 4.3 Monte Carlo simulation

For a full description of the experimental acceptance of E06-010, a model for the BigBite spectrometer used in E06-010 for electron detection has been developed and incorporated into the SIMC package [115] which was initially developed for JLab Hall C experiments and used for the semi-inclusive studies in Hall C [64]. It was adapted for this experiment [116]. It contains a realistic description of various detectors including the left HRS used in the experiment E06-010 for hadron detection. The energy loss, multiple scattering, pion decay processes have also been included in the SIMC package. The radiation length and materials in the simulation were defined based on the configuration of the experiment E06-010.

### 4.3.1 A general description and the flow of the simulation

The simulation used in this study is based on the "sample mean" method of Monte Carlo which has been used for multi-dimensional numerical integration for decades.

The 1D and 2D integrals using this method can be found in textbooks:

$$
\begin{align*}
I^{1 \mathrm{D}} & =\int_{L_{1 D}} d x f(x)=\frac{L_{1 D}}{N_{g e n}} \sum_{i}^{N_{g e n}} f\left(x_{i}\right)  \tag{4.17}\\
I^{2 \mathrm{D}} & =\int_{A_{2 D}} d x d y f(x, y)=\frac{A_{2 D}}{N_{\text {gen }}} \sum_{i}^{N_{g e n}} f\left(x_{i}, y_{i}\right), \tag{4.18}
\end{align*}
$$

where $I^{1 \mathrm{D}}\left(I^{2 \mathrm{D}}\right)$ is the result of the $1 \mathrm{D}(2 \mathrm{D})$ integration, $L_{1 D}\left(A_{2 D}\right)$ is the $1 \mathrm{D}(2 \mathrm{D})$ integration range, $N_{\text {gen }}$ is the number of the sample being generated, $x_{i}$ and $y_{i}$ are the variables in the $i$ th generation, and $f$ is the function being integrated. This method can be easily applied to integrations in $n$ dimensions ( $n$ is a positive integer):

$$
\begin{equation*}
I^{n \mathrm{D}}=\int_{\Delta P H S_{n D}} d P H S_{n D} f(v a r)=\frac{\Delta P H S_{n D}}{N_{g e n}} \sum_{i}^{N_{g e n}} f\left(v a r_{i}\right) \tag{4.19}
\end{equation*}
$$

where $\Delta P H S_{n D}$ is the n-dimensional phase space of the integration, and var is a set of $n$ variables.

The simulation for the number of events in a bin, is essentially a numerical integration process, with a complicated integrand which includes the differential cross section, the radiative effect, and the acceptance effect. The flow of the simulation is presented in Fig. 4.21. The factor named "weight" in this figure is a part of the integrand in Eq. (4.19), including the differential cross section and the internal radiative effect. The external radiative effect is included in the energy-loss process. The acceptance effect is described by the detector model and affected by the multiple scattering process. This multi-dimensional integration is complicated as some of the events were rejected (zero contribution in the integration) due to being outside the acceptance, or outside the boundaries of a chosen bin, or cut away for other reasons in the analysis. Putting the simulation result in Eq. (4.19) gives the number of events with one unit luminosity in a specific bin, and a proper scaling for the (effective)


Figure 4.21: The flow of the Monte Carlo simulation.
luminosity is needed for comparing the number of events between the simulation and the data.

The numbers of events in a specific bin of data, weighted and phase-space (nonweighted) simulations are expressed as

$$
\begin{align*}
N_{d a t a} \cdot f_{c o r r} & =\left\langle\frac{d \sigma}{d P H S}\right\rangle_{\text {data }} \cdot \Delta P H S_{d} \cdot f_{a c c, d} \cdot L_{d},  \tag{4.20}\\
N_{s i m} & =\left\langle\frac{d \sigma}{d P H S}\right\rangle_{s i m} \cdot \Delta P H S_{s} \cdot f_{a c c, s} \cdot L_{s},  \tag{4.21}\\
N_{p h s} & =1 \cdot \Delta P H S_{s} \cdot f_{a c c, s} \cdot L_{s}, \tag{4.22}
\end{align*}
$$

where $\Delta P H S_{d}$ is the phase space in the data for a specific bin, and $\Delta P H S_{s}$ is the
phase space in the simulations. The factor $f_{\text {acc, }, d}$ represents the acceptance effect in the data, and $f_{\text {acc,s }}$ represents the acceptance effect in the simulations. The factor $L_{d}$ is the total effective luminosity, and $L_{s}$ is the luminosity in the simulations. The quantities $\triangle P H S_{i}$ and $f_{a c c, i}$ are related to the acceptance, the fiducial and kinematic cuts and the boundaries of a specific bin, where $i=d(s)$ represents the quantities in the experiment (simulation). The averaged differential cross section (DXS) of the data (simulation) in this bin is denoted as $\left\langle\frac{d \sigma}{d P H S}\right\rangle_{i}$ where $i=$ data (sim). The DXS of the data is a quantity from nature, and can be extracted from the data, while in the simulation it comes from a theoretical model built in the simulation, and can be retrieved from the simulation results.

In the experiment, the measured cross sections included the internal and external radiative effects, and the simulation was constructed to include these effects. The external radiative effects included the energy loss and multiple scattering for the particles before and after scattering and were included in the SIMC package according to the defined experimental configuration. The materials in the E06-010 configuration before and after the scattering are listed in Tables 4.12 and 4.13.

| Type | Thickness $(\mathrm{cm})$ | Density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :---: | :---: |
| Be window | 0.0254 | 1.848 |
| ${ }^{4}$ He gas | 31.6 | 0.000166 |
| Glass window of target | 0.0123 | 2.76 |

Table 4.12: The materials in the E06-010 configuration for incoming beam electrons.

In the simulation, the length of the target cell was defined to be 40 cm , and the radius of the cell was defined to be 0.95 cm . The ${ }^{3} \mathrm{He}$ density was set at 0.00141 $\mathrm{g} / \mathrm{cm}$ (10.5 amg number density). The path length of a beam electron through the ${ }^{3}$ He gas was calculated based on the target geometry, the interaction vertex and the angle on an event-by-event basis. The path length through the target window or

| Type | Thickness $(\mathrm{cm})$ | Density $\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ |
| :--- | :---: | :---: |
| Glass cell wall of target | 0.166 | 2.76 |
| ${ }^{4}$ He gas | 45.7 | 0.000166 |
| Air (to the BigBite) | 104.3 | 0.00121 |
| Air (to the HRS) | 74.3 | 0.00121 |

Table 4.13: The materials in the E06-010 configuration for scattered/produced particles.
target wall was also calculated on an event-by-event basis.
The internal radiative effects are more closely related to the change of the Born cross sections, including the vacuum polarization, vertex correction and contributions from higher-order Feynman diagrams. The internal radiative effects were evaluated by using additional packages based on the process being studied.

### 4.3.2 The detector models

One of the key functions of a good detector model is providing a reliable map between the target variables and the hardware variables of the detector. This map is often called the detector "optics". The target variables include the momentum, the angles and the vertex of each event. The hardware variables are obtained directly from the information recorded in the detectors during the experiment.

The hardware variables described by the BigBite model were the focal-plane X and Y positions ( $X_{f p}$ and $Y_{f p}$ ) and the out-of-plane and in-plane angles ( $d X_{f p}$ and $\left.d Y_{f p}\right)$. The map between the focal-plane variables and the target variables in the BigBite model was provided by polynomial expansions up to the third order. The mapping from the target variables to the focal-plane variables was called the forward transport, and the reversed mapping was called the reverse transport. They are
expressed as

$$
\begin{align*}
& a_{f p}=\sum_{i, j, k, l} C_{i j k l}^{\text {forward }} \times a_{t a r}^{i} b_{t a r}^{j} c_{t a r}^{k} d_{t a r}^{l},  \tag{4.23}\\
& a_{t a r}=\sum_{i, j, k, l} C_{i j k l}^{\text {reverse }} \times a_{f p}^{i} b_{f p}^{j} c_{f p}^{k} d_{f p}^{l}, \tag{4.24}
\end{align*}
$$

where the subscript $f p$ (tar) represents the four focal-plane (target) variables $a, b$, $c$ and $d$, and $C_{i j k l}^{\text {forward }}\left(C_{i j k l}^{\text {reverse }}\right)$ represents the coefficient of the expansions in the forward (reverse) transport. The coefficients of each term in the expansion are often called the optical-matrix coefficients. The orders of the polynomials were no greater than three:

$$
\begin{equation*}
i+j+k+l \leq 3 \tag{4.25}
\end{equation*}
$$

The relatively big acceptance range of the BigBite was separated in about 200 sections, each of which has its own optical-matrix coefficients. The flow of an event going through the BigBite model is presented in Fig. 4.22.

A first check of the sectioned-expansion method was to calculate the focal-plane (target) variables from the target (focal-plane) variables using the data from the production runs, and compare these calculated variables with the original ones in the data. The 2D plots showing the correlations of $\operatorname{var}($ Data $)$ vs. var(Expansion) are presented in Fig. 4.23, where $\operatorname{var}(\operatorname{Data})$ is one of the target (focal-plane) variables from the data, and $\operatorname{var}$ (Expansion) is the same variable from the expansion method. It is observed that the 2D plots of of $\operatorname{var}($ Data $)$ vs. $\operatorname{var}$ (Expansion) is close to the exact match depicted by the black lines. For a more quantitative view, the difference between the variables from the data and the expansion $(\delta(v a r)=\operatorname{var}($ Expansion $)-$ $\operatorname{var}($ Data) ) are presented in Fig. 4.24. A test using the data from the Sieve runs was also carried out, as presented in Fig. 4.25.


Figure 4.22: The flow of an event going through the BigBite model.

The total-shower-energy deposition was built into the BigBite model in an empirical method as discussed below. Usually, the energy deposition in the calorimeter $\left(E_{\mathrm{dp}}\right)$ is related to the energy of the incoming particle $\left(E_{\text {in }}\right)$ as

$$
\begin{equation*}
\frac{E_{\mathrm{dp}}}{E_{i n}}=\mathcal{N}\left(\mu, \sigma^{2}\right) \tag{4.26}
\end{equation*}
$$

where $\mathcal{N}\left(\mu, \sigma^{2}\right)$ is a Gaussian distribution with a mean value of $\mu$ and a width of $\sigma$. The dependence of $\sigma$ on $E_{\text {in }}$ can be expressed as

$$
\begin{equation*}
\sigma=R_{E} \frac{1}{\sqrt{E_{i n} / E_{0}}}, \tag{4.27}
\end{equation*}
$$

where $R_{E}$ is a constant depending on the calorimeter, and $E_{0}$ is conventionally set to be 1 GeV . The elastic ep channel was used to provide a clean electron sample for determining the $R_{E}$ value of electrons detected by the BigBite. Neglecting the mass of the electrons detected by the BigBite, the $E_{T S} / P_{b b}$ distribution was used to


Figure 4.23: The comparisons between the out-of-plane ( $d x$ ) and in-plane ( $d y$ ) angles from the data and the expansion. In each panel the $\mathrm{X}(\mathrm{Y})$ axis represents the variable from the expansion (data). The relation of the variables $\left(\operatorname{var}_{X}=\operatorname{var}_{Y}\right)$ is represented by the black diagonal lines in each panel. The red dots represent the 2D comparisons.
determine an empirical value for $R_{E}$. The $E_{T S} / P_{b b}$ distributions and the Gaussian fits using the elastic ep data are presented in Fig. 4.26. Eqs. (4.26) and (4.27) with $R_{E}=0.085$ were built into a Monte Carlo simulation to describe the $E_{T S}$ for electrons.

A block-by-block setting of the $E_{T S}$ threshold was built in the simulation (the BigBite model), for finding a $E_{T S}$ cut high enough to override the threshold drifts in the data. The thresholds for the blocks with a clear step-function behavior were set at the sharp rising peak, as the top panels of Fig. 4.20. The thresholds of the


Figure 4.24: The difference between the out-of-plane ( $d x$ ) and in-plane ( $d y$ ) angles from the data and the expansion. In each panel, the X axis represents the difference of the variables $\delta(v a r)$, and the Y axis represents the number of events. The mean value ( $\mu$ ) and the root-mean-square (RMS) value are presented in each panel.
blocks behaving as in the bottom panels of Fig. 4.20 were set at the middle of the rising slope of the events with the $E_{T S} / P_{b b}$ PID cut for leptons (the red spectra in this figure). Two simulations were carried out for the study of the $E_{T S}$ cut. The first simulation was carried out without using the threshold setting: all the events accepted by the detector model were kept. The second simulation was carried out by using the threshold setting: besides requiring an event to be accepted by the detector model, the $E_{T S}$ of it was required to be larger than the threshold of the block it hit on. Then a series of $E_{T S}$ cuts was applied to the simulations, and the ratio of events between the first and second simulation in the momentum bins was compared as in


Figure 4.25: A 2D Sieve plot from the data is presented in the left panel. The correlations of the target variables (var(Data) vs. var(Expansion)) are presented in the right top and right bottom panels.

Fig. 4.27. Three features are observed in this study, as listed below.

- The cuts $E_{T S}>E_{\text {cut }}$ are able to override the threshold setting with at least $E_{\text {cut }}=0.8 \mathrm{GeV}$.
- With higher $E_{\text {cut }}$ values, more events with low momentum are removed.
- The ratio of events with $P_{b b}>0.9 \mathrm{GeV}$ is very stable with various $E_{T S}$ cuts.

The left HRS model was taken from the SIMC package [115]. This HRS model also uses the expansion method for the forward and reverse transports.


Figure 4.26: The $E_{T S} / P_{b b}$ distributions and the Gaussian fits using the elastic ep data. No preshower cut is applied to the data in the left panel. A preshower cut channel $l_{\text {PS }}>300$ is applied to the data in the right panel. The black lines represent the data, and the red lines represent the Gaussian fits. The Gaussian width $\sigma$ and the constant $R_{E}$ from the two fits are presented on each panel.

Several well-studied channels were used to test both the BigBite and the HRS model, which will be presented in the next subsection.

### 4.3.3 Testing the simulation

In a well-studied channel, the Born cross sections found in experiments have been described by theories/models:

$$
\begin{equation*}
\frac{d \sigma}{d P H S}_{\text {Born }}=\frac{d \sigma}{d P H S}_{\text {theory }} \tag{4.28}
\end{equation*}
$$

The simulation can be tested in these channels, using theoretical cross sections and proper algorithm(s) for the radiative effects. By comparing $N_{d a t a}$ and $N_{s i m}$ in Eqs. (4.20) and (4.21), with proper scaling factors for the luminosity, the agreement between $\triangle P H S_{s} \cdot f_{a c c, s}$ and $\Delta P H S_{d} \cdot f_{a c c, d}$ can be studied. In the following parts of this section, the comparison between the data and the simulation in several known channels are presented.

The simulation with the BigBite detector model was tested by using the calibration runs of elastic ep scattering at incident electron beam energies of 1.23 and


Figure 4.27: The ratios of events in BigBite (model) momentum bins. Each marker represents a ratio with a specific $E_{T S}$ cut in the simulations.
2.4 GeV. In these simulations, the form factors from [117] were used. The internal radiative effects were based on [118]. The results from the simulation used the same luminosity values as the total effective luminosity of the combined data. In the simulations, only the elastic ep process was included, thus only the peak of the proton was observed in both the simulation and the data while the peaks from higher resonances are only observed in the data. The comparisons between the data and the simulations are presented in Fig. 4.28.

The HRS with the setting for the production runs could not access the elastic ep scattering, as the scattered electrons were outside the acceptance range of the HRS. In order to test the acceptance description for the HRS, the inclusive DIS data from the ${ }^{3} \mathrm{He}$ production runs and the $\mathrm{H}_{2}$ reference cell runs at 5.9 GeV beam energy were used. The structure functions for the inclusive DIS channels were taken from a widely used model [119]. The model provided a good description of the


Figure 4.28: The momentum $P_{b b}$ (two left panels) and invariant mass $W_{b b}$ (two right panels) comparisons between the data and the simulations of the 1.23 (two top panels) and 2.4 GeV (two bottom panels) elastic ep scattering. The black solid circles are from the data. The red solid squares are from the simulations. The error bars represent the statistical uncertainties.
unpolarized inclusive DIS cross sections for the ${ }^{3} \mathrm{He}$ target in experiment E06-014 $[120,121]$ which had a similar kinematic range. In the kinematic range of the HRS, the contribution from the quasi-elastic radiative tail was estimated to be negligible using the simulations. The photon-induced electron contamination was estimated to be negligible by comparing the electron and positron yields in the HRS with negative and positive polarities. The comparisons between the data and the simulation in the ${ }^{3} \mathrm{He}$ DIS channel in the HRS is presented in Fig. 4.29. In this comparison, the Mo and Tsai algorithm was used for the internal radiative effect, and an overall scaling


Figure 4.29: The scattered electron momentum $\left(P_{h r s}\right)$, the polar angle $\left(\theta_{h r s}\right)$ and the azimuthal angle ( $\phi_{h r s}$ ) comparisons of the ${ }^{3} \mathrm{He}$ inclusive DIS channel in the HRS between the data and the simulation (Sim). The error bars represent statistical uncertainties. The black solid circles are from the data. The red solid squares are from the simulation.
factor of $100 \%$ was used in the simulation. This overall efficiency at $100 \%$ is close to what was found in experiment E06-014 (99.95\%) [120]. A reasonable agreement between the data and the simulation is observed.

The comparisons between the data and the simulations in the $\mathrm{H}_{2}$ DIS channel in the HRS is presented in Fig. 4.30. In this comparison, one of the simulations (Sim1) used the algorithm from Mo and Tsai [122] for the internal radiative effect, the other simulation (Sim2) used the algorithm from POLRAD [123]. In these comparisons, the difference introduced by using different methods for internal radiative effects (Mo


Figure 4.30: The scattered electron momentum $\left(P_{h r s}\right)$, the polar angle $\left(\theta_{h r s}\right)$ and the azimuthal angle ( $\phi_{h r s}$ ) comparisons of the $\mathrm{H}_{2}$ inclusive DIS channel in the HRS between the data and the simulations (Sim1 and Sim2). Different algorithms for the internal radiative effect were applied in Sim1 and Sim2, as described in the text. The error bars represent statistical uncertainties. The black solid circles are from the data. The red solid squares are from the simulation.
and Tsai [122] and POLRAD [123]) was less than $1 \%$. The overall scaling factor of $100 \%$ was used in the simulation. A reasonable agreement between the data and the simulations is observed.

In the kinematic range $P_{b b}>1.6 \mathrm{GeV}$ of the BigBite spectrometer, the contribution from the quasi-elastic radiative tail and the photon-induced electron contamination were less than $2 \%$, but both increased to large values in lower momentum ranges. This has been observed in experiment E06-014 [120, 121] as well. The determination of the photon-induced electron contamination was done by comparing the positron
yield in the BigBite with the reversed magnetic field and the electron yield in the production runs. In experiment E06-010, the photon-induced electron contamination involved larger uncertainties in the inclusive channel than in the semi-inclusive coincidence channel, as the runs for subtracting this contamination provided limited statistics for the inclusive channel (due to large prescale factors ${ }^{2}$ ) while providing high statistics for the semi-inclusive channels. In addition, we also found different methods for internal radiative effects (Mo and Tsai [122] and POLRAD [123]) and different models for the quasi-elastic cross sections (from [119] and [124]) gave quite different estimations for the quasi-elastic radiative tails in the range $P_{b b}<1.6 \mathrm{GeV}$. The estimation for the fraction of events from the quasi-elastic tail could differ up to $\sim 10 \%$ around $P_{b b}=1.0 \mathrm{GeV}$, while the differences were less than $1 \%$ in the range $P_{b b}>1.6 \mathrm{GeV}$.

Considering the complications above for the inclusive DIS channel in the kinematic range $P_{b b}<1.6 \mathrm{GeV}$ in the BigBite, we only present the comparison of the data and the simulation in the range $P_{b b}>1.6 \mathrm{GeV}$ in Fig. 4.31 as an additional test for the acceptance description of the BigBite besides the elastic ep channels.

The $\mathrm{N}_{2}$ background in the ${ }^{3} \mathrm{He}$ runs was subtracted using the $\mathrm{N}_{2}$ runs. The efficiency of the PID cuts and the remaining contamination from negatively charged hadrons and the photon-induced electron after the PID cuts were corrected for, using the techniques presented in section 4.2

Besides the PID cuts on the data events, kinematic cuts were also applied to the comparisons above. In each of the comparisons, the same set of kinematic cuts was applied to the data and the simulation. In the elastic channel comparison, only the angular and vertex $z$ cuts were applied. In the DIS channel comparison the momentum cut and the DIS-selecting cuts were applied in addition. The angular,

[^1]

Figure 4.31: The scattered electron momentum $\left(P_{b b}\right)$, the polar angle $\left(\theta_{b b}\right)$ and the azimuthal angle ( $\phi_{b b}$ ) comparisons of the ${ }^{3} \mathrm{He}$ inclusive DIS channel in the BigBite between the data and the simulation (Sim). The error bars represent statistical uncertainties. The black solid circles are from the data. The red solid squares are from the simulation.
momentum and vertex $z$ cuts applied to the detectors, and the DIS-selecting cuts applied to the data and the simulations are listed in Table 4.1.

### 4.3.4 Simulating the pion decay

In the experiment, the $\mu^{ \pm}$from the decay of a $\pi^{ \pm}$could not be discriminated from the $\pi^{ \pm}$in the HRS and was assumed to be a $\pi^{ \pm}$event. When a $\pi^{ \pm}$event decays to a $\mu^{ \pm}$and a neutrino, the kinematics of this event is changed. This effect was evaluated using simulations. The SIMC package has an established component simulating the probability of the decay and the kinematic change of each $\pi^{ \pm}$event. The probability
of the decay was calculated based on the length of the track of an event and the $\pi^{ \pm}$ life time. The kinematic change was evaluated by generating the momentum and angles of the $\mu^{ \pm}$in the center-of-mass frame of the decaying $\pi^{ \pm}$, from which the kinematics of the $\mu^{ \pm}$in the lab frame were calculated. In the center-of-mass frame of the decaying $\pi^{ \pm}$, the momentum of the $\mu^{ \pm}$followed the four-momentum conservation and the angles were generated with a uniform probability distribution in the solid angle. The $\mu^{ \pm}$was recorded as a $\pi^{ \pm}$if it was in the acceptance of the HRS model, and was rejected otherwise.

The effect of the decay of $\pi^{ \pm}$was treated as one of the efficiency factors as

$$
\begin{equation*}
f_{\text {eff }}(\text { decay })=N_{\text {sim }}(\text { decay }) / N_{\text {sim }}(\text { no decay }), \tag{4.29}
\end{equation*}
$$

where $N_{\text {sim }}$ (decay) and $N_{\text {sim }}$ (no decay) are the numbers of events recorded in the simulation with decay-effect turned on and off, respectively. The value of $f_{\text {eff }}$ (decay) was found to be around $90 \%$, with a kinematic dependence.

### 4.3.5 Radiative tails from exclusive channels

The contribution from the exclusive channels $e+p \rightarrow e^{\prime}+\pi^{+}+n$ and $e+n \rightarrow e^{\prime}+\pi^{-}+p$ were evaluated by using simulations with cross section models tested in the kinematic range of this experiment [64]. The contributions from the exclusive channels were from $2 \%$ to $7.5 \%$ in the $\pi^{+}$production channel and $0.5 \%$ to $3 \%$ in the $\pi^{-}$production channel.

The contribution of the $\pi^{ \pm}$from the decay of the gluon-exchange-produced $\rho$ (diffractive $\rho$ ) is not a part of the SIDIS process and should be subtracted. The contribution of the $\pi^{ \pm}$from the decay of quark-exchange-produced $\rho$ is part of the SIDIS process and should not be subtracted. We have simulated the contribution of the $\pi^{ \pm}$from the decay of the $\rho$, in the same way as in [64]. The model for the exclusive production of $\rho$ was from PYTHIA [125] and was further tuned according to the $\rho^{0}$
cross section from a CLAS experiment at JLab as described in [64]. Comparing this simulation with the data, it was found that the fraction of the events from the $\rho$ decay was mostly less than $5 \%$ in experiment E06-010. While the level of $5 \%$ was not completely negligible, the contribution from the decay of the diffractive $\rho$ was considered to be negligible considering a recent study from CLAS [126]. In [126], the quark-exchange production of $\rho$ was found to be dominant while the diffractive $\rho$ from the gluon-exchange was found to be negligible. The fraction of the events from the decay of diffractive $\rho$ was expected to be much smaller than $5 \%$ in the kinematic range $0.45<z_{h}<0.65$ of this experiment, thus at a negligible level.

### 4.4 Extracting the differential cross sections

In order to extract the differential cross section from the data, the luminosity of the data needs to be used for the normalization process, and several additional corrections need to be applied, namely the acceptance correction, the radiative correction and the bin-centering correction.

The acceptance corrections were based on the Monte Carlo simulation with the same kinematic cuts as applied to the data. Using Eqs. (4.20), (4.21) and (4.22), the averaged differential cross section in a bin can be expressed as

$$
\begin{equation*}
\left\langle\frac{d \sigma}{d P H S}\right\rangle_{\text {data }}=\frac{N_{\text {data }} \cdot f_{\text {corr }}}{N_{\text {phs }}} \frac{L_{s}}{L_{d}}, \tag{4.30}
\end{equation*}
$$

where $L_{d}$ is the total effective luminosity, and $L_{s}$ is the luminosity in the simulations. This method requires a good description of the acceptance of the BigBite and the HRS, which was checked using the known channels as in section 4.3.3.

The radiative corrections ( RCs ) were based on the ratios of the weighted simulations with and without radiative effects. A RC coefficient $C_{r c}$ can be found in each
bin, defined as

$$
\begin{equation*}
C_{r c}=\frac{N_{s i m}^{r a d}}{N_{s i m}^{n r}} \tag{4.31}
\end{equation*}
$$

where $N_{s i m}^{r a d}$ and $N_{s i m}^{n r}$ represent the numbers of events in a specific bin of weighted simulations with and without radiative effects. Different models for the Born differential cross section would result in different $C_{r c} \mathrm{~s}$, and the proper coefficients $C_{r c} \mathrm{~s}$ can be determined when the simulation with radiative effects match the data (after the efficiency corrections and the contamination/background subtractions). The matching was found by tuning certain parameters in a certain phenomenological model until the difference between the data and the simulation in each bin was close to or less than the corresponding total experimental uncertainty. The value of $C_{r c}$ in this study was found to be between 0.85 and 1.05 with a kinematic dependence.

In each bin, the coefficient $C_{r c}$ was applied to the differential cross section from the data:

$$
\begin{equation*}
\sigma_{e x p}^{b i n}=\frac{1}{C_{r c}} \cdot\left\langle\frac{d \sigma}{d P H S}\right\rangle_{\text {data }} \tag{4.32}
\end{equation*}
$$

where $\sigma_{\text {exp }}^{b i n}$ represents the differential cross section extracted experimentally after the RCs.

The bin-centering corrections (BCCs) were evaluated in each bin. The need for the BCCs and the calculation are presented in the following paragraphs. The values of kinematic variables in a bin were determined by averaging the experimental data. For example, the value of a variable $x$ in a certain bin was determined by

$$
\begin{equation*}
\bar{x}=\frac{1}{N_{d a t a}} \sum x_{i} \tag{4.33}
\end{equation*}
$$

where $\bar{x}$ is the averaged value of $x, N_{\text {data }}$ is the total number of events in this bin and $x_{i}$ is the $x$ value of the $i$ th event. The symbol $x$ stands for any kinematic
variable. The averaged values of the kinematic variables $\left(x_{b j}, z_{h}, Q^{2}, \phi_{h}\right.$ and $\left.P_{t}\right)$ from the simulation were consistent with the data mostly within $0.5 \%$.

The differential cross section extracted from the data in a specific bin $\left(\sigma_{e x p}^{b i n}\right)$ using Eqs. (4.30) and (4.32) was an averaged value and can be directly compared with the averaged differential cross section $\sigma_{M C}^{b i n}$ from the simulation in the same bin.

The averaged differential cross section $\sigma_{M C}^{b i n}=\bar{\sigma}_{\text {model }}$ was not necessarily equal to $\sigma_{\text {theory }}=\sigma_{\text {model }}(\overline{\text { vars }})$, where $\sigma_{\text {model }}($ vars $)$ is the theoretical model for the differential cross sections, the symbol vars represents the collection of kinematic variables $\left(x_{b j}, z_{h}, Q^{2}, \phi_{h}\right.$ and $\left.P_{t}\right)$ and $\overline{v a r s}$ represents the averaged values of kinematic variables in this bin $\left(\overline{x_{b j}}, \overline{z_{h}}, \overline{Q^{2}}, \overline{\phi_{h}}\right.$ and $\left.\overline{P_{t}}\right)$. For the purpose to compare theoretical model $\sigma_{\text {theory }}$ with the data, the ratio $\sigma_{\text {theory }} / \sigma_{M C}^{\text {bin }}$ was applied to the data in each bin, based on the same tuned model of Born differential cross section as used in the radiative corrections.

The BCC for the data in one bin is defined as

$$
\begin{equation*}
\sigma_{e x p}^{B C C}=\frac{\sigma_{\text {theory }}}{\sigma_{M C}^{b i n}} \cdot \sigma_{\text {exp }}^{b i n}, \tag{4.34}
\end{equation*}
$$

where $\sigma_{\text {exp }}^{B C C}$ is the SIDIS differential cross section extracted experimentally, after the bin-centering correction with experimental central values of kinematic variables, and can be compared with the differential cross section in models evaluated at the same central values of kinematic variables. The BCC ratio $\left(\sigma_{\text {theory }} / \sigma_{M C}^{b i n}\right)$ in this study was between 1.01 and 1.1, with a kinematic dependence.

The methods to estimate the systematic uncertainties will be presented in the next section, and the results of extracted differential cross section in multi-dimensional bins will be presented in the next chapter.

### 4.5 Estimation of systematic uncertainties

The systematic uncertainties related to the PID and the acceptance can be estimated by series of PID and kinematic cuts, and will be presented first. The systematic uncertainties related to the detector resolution will be presented afterwards. Lastly, the systematic uncertainties related to the radiative corrections and exclusive tail subtraction will be discussed.

The estimation of the systematic uncertainties related to the beam charge and the target density of experiment E06-010, follows the study for the inclusive cross section extraction in experiment E06-014 [120, 121]. A summary of the main contributions of the systematic uncertainties is presented in Table 4.14.

| Source | Uncertainty (\%) |
| :--- | :---: |
| $e^{-}$identification in the BigBite | $2.0-8.0$ |
| $e^{-}$overall detection efficiency in the BigBite | $<3.0$ |
| $\pi^{ \pm}$identification in the HRS | $<1.0$ |
| Overall detection efficiency in the HRS | $<2.0$ |
| Beam charge | $<1.0$ |
| Target density | $\approx 2.0$ |
| Experimental acceptance corrections | $<10.0$ |
| Radiative corrections and exclusive tail subtraction | $2.0-6.0$ |
| Resolution-related | $<3.0$ |

Table 4.14: List of main systematic uncertainties.

### 4.5.1 Estimation by series of cuts

The detector models in the simulation were not able to perfectly describe the experimental acceptance, and the systematic uncertainties were studied by applying a series of kinematic cuts to the simulation and the data, and check the change of the ratio $R$ in each bin. The ratio $R$ is defined as

$$
\begin{equation*}
R=\frac{N_{\text {data }} \cdot f_{\text {corr }}}{N_{\text {sim }}^{\text {rad }}} . \tag{4.35}
\end{equation*}
$$

With a tuned model (and radiative effects) that matched the data reasonably well, the changes of $R$ under different cuts reflect the systematic uncertainties related to the cut. These cuts include the kinematic cuts (related to the description of the acceptance), and the $E_{T S}$ cut (related to the description of the total-shower energy deposition in the BigBite). The systematic uncertainty in each bin was estimated as

$$
\begin{equation*}
\delta_{\text {sys }}=\left[\frac{1}{N-1} \sum_{i \neq i_{0}}^{N} \frac{\left(R_{i}-R_{i_{0}}\right)^{2}}{R_{i_{0}}^{2}}-\frac{\delta_{\text {stat }}^{2}\left(R_{i_{0}}\right)}{R_{i_{0}}^{2}}\right]^{\frac{1}{2}} \tag{4.36}
\end{equation*}
$$

where $N$ is the number of the cuts in the series, $i_{0}$ represents the central cut $\left(i_{0}=1\right.$ in this study), $R_{i}$ represents the ratio with the $i$ th cut, and $\delta_{s t a t}\left(R_{i_{0}}\right)$ represents the statistical uncertainty of $R_{i_{0}}$.

The series of kinematic and $E_{T S}$ cuts were set around the cuts used in section 4.3.3 which enabled a reasonable comparison between the data and the simulation in the known channels. The relative ratio $R_{R}=R_{i} / R_{i_{0}}$ will be presented for the relative changes of $R$ and the estimation of the related systematic uncertainties.

The $R_{R}$ changes due to series of cuts in the polar (azimuthal) angles of the detectors will be presented in bin sets of the azimuthal (polar) angles. The $R_{R}$ changes due to series of cuts in the momenta of the detectors will be presented in the angular bin sets. The total systematic uncertainty related to the acceptance was estimated by combining the effects due to all types of kinematic cut series, and was found to be less than $10 \%$ with a kinematic dependence.

The PID cuts were also varied around the central values, and the related efficiencies and contaminations were re-evaluated. As the PID cuts are not related to the simulation, the numbers of data events (after correction) in bins will be presented.

## Series of HRS-angle cuts

A series of HRS-angle cuts around the central values were used to study the system-
atic uncertainties. The changes of the ratio $\left(R_{R}\right)$ by changing the cut in $\theta_{\text {hrs }}\left(\phi_{h r s}\right)$ are presented in the $\phi_{\text {hrs }}\left(\theta_{\text {hrs }}\right)$ bins, as in Figs. 4.32 and 4.33. The angle cuts are listed in Table 4.15.


Figure 4.32: The ratio $R_{R}$ in the $\theta_{\text {hrs }}$ bins, with different angle cuts applied to the HRS. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.


Figure 4.33: The ratio $R_{R}$ in the $\phi_{h r s}$ bins, with different angle cuts applied to the HRS. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

Series of momentum cuts in the HRS
A series of momentum cuts were carried out similarly to the angle cuts. The changes

| Cut number | $\theta_{\text {hrs }}$ cut | $\phi_{\text {hrs }}$ cut |
| :---: | :---: | :---: |
| 1 | $13.5^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |
| 2 | $13.5^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $80^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |
| 3 | $13.5^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $82^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |
| 4 | $13.5^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<98^{\circ}$ |
| 5 | $13.5^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<100^{\circ}$ |
| 6 | $14.0^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |
| 7 | $14.5^{\circ}<\theta_{\text {hrs }}<18.5^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |
| 8 | $13.5^{\circ}<\theta_{\text {hrs }}<18.0^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |
| 9 | $13.5^{\circ}<\theta_{\text {hrs }}<17.5^{\circ}$ | $81^{\circ}<\phi_{\text {hrs }}<99^{\circ}$ |

Table 4.15: The series of HRS-angle cuts.
of the ratio $\left(R_{R}\right)$ are presented in the $\theta_{h r s}$ and $\phi_{h r s}$ bins, as in Figs. 4.34 and 4.35.


Figure 4.34: The ratio $R_{R}$ in the $\theta_{\text {hrs }}$ bins, with different momentum cuts applied to the HRS. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

The momentum cuts are listed in Table 4.16. The systematic uncertainties were evaluated using Eq. (4.36).

Series of HRS-PID cuts
A series of HRS-PID-cut sets (Table 4.17) were applied to the data besides the central set (Table 4.4), for the systematic uncertainty study. The efficiencies and


Figure 4.35: The ratio $R_{R}$ in the $\phi_{h r s}$ bins, with different momentum cuts applied to the HRS. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

| Cut number | $P_{\text {hrs }}$ cut |
| :---: | :---: |
| 1 | $2.26<P_{\text {hrs }}<2.41 \mathrm{GeV}$ |
| 2 | $2.25<P_{\text {hrs }}<2.41 \mathrm{GeV}$ |
| 3 | $2.27<P_{\text {hrs }}<2.41 \mathrm{GeV}$ |
| 4 | $2.26<P_{\text {hrs }}<2.42 \mathrm{GeV}$ |
| 5 | $2.26<P_{\text {hrs }}<2.43 \mathrm{GeV}$ |

Table 4.16: The series of HRS momentum cuts.
contaminations were re-evaluated using different sets of PID cuts. The comparisons between the $P_{h r s}$ spectra under the series of PID-cut sets and the corresponding data corrections are presented in Fig. 4.36. It was found that the systematic uncertainty of the HRS PID is less than $0.5 \%$ (relative).

Series of BigBite-angle cuts
A series of BigBite-angle cuts around the central values were used to study the systematic uncertainties. The changes of the ratio $\left(R_{R}\right)$ by changing the cuts in $\phi_{b b}$ $\left(\theta_{b b}\right)$ are presented in the $\theta_{b b}\left(\phi_{b b}\right)$ bins, as in Figs. 4.37 and 4.38. The cuts are listed in Table 4.18.

| PID-cut set | Aerogel Cerenkov cut | Gas Cerenkov cut | Lead-glass calorimeter cut |
| :---: | :---: | :---: | :---: |
| 1 | channel $_{\mathrm{AC}}>150$ | channel $_{\mathrm{GC}}<250$ | $E_{\text {tot }} / P_{h r s}<0.6$ |
| 2 | channel $_{\mathrm{AC}}>180$ | channel $_{\mathrm{GC}}<250$ | $E_{\text {tot }} / P_{h r s}<0.6$ |
| 3 | channel $_{\mathrm{AC}}>120$ | channel $_{\mathrm{GC}}<250$ | $E_{\text {tot }} / P_{h r s}<0.6$ |
| 4 | channel $_{\mathrm{AC}}>150$ | channel $_{\mathrm{GC}}<300$ | $E_{\text {tot }} / P_{h r s}<0.6$ |
| 5 | channel $_{\mathrm{AC}}>150$ | channel $_{\mathrm{GC}}<200$ | $E_{\text {tot }} / P_{h r s}<0.6$ |
| 6 | channel $_{\mathrm{AC}}>150$ | channel $_{\mathrm{GC}}<250$ | $E_{\text {tot }} / P_{h r s}<0.7$ |
| 7 | channel $_{\mathrm{AC}}>150$ | channel $_{\mathrm{GC}}<250$ | $E_{\text {tot }} / P_{h r s}<0.5$ |

Table 4.17: A series of HRS-PID-cut sets. Set 1 is the central set.


Figure 4.36: The comparisons between the $P_{h r s}$ spectra under a series of HRS-PID-cut sets and the corresponding data corrections.

Series of momentum cuts in the BigBite
The low momentum range of the BigBite had issues related to the shower-threshold drifts, and relatively large uncertainties related to the photon-induced contamination. For a clear illustration, the effects from the series of momentum cuts in the high momentum range are discussed first. The series of momentum cuts in the high momentum range are listed in Table 4.19. The changes of the ratio $\left(R_{R}\right)$ are presented in the $\theta_{b b}$ and $\phi_{b b}$ bins, as in Figs. 4.39 and 4.40.

The series of momentum cuts in the low momentum range are listed in Table 4.20. The changes of the ratio $\left(R_{R}\right)$ are presented in the $\theta_{b b}$ and $\phi_{b b}$ bins, as in Figs.

| Cut number | $\theta_{b b}$ cut | $\phi_{b b}$ cut |
| :---: | :---: | :---: |
| 1 | $25.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $250^{\circ}<\phi_{b b}<290^{\circ}$ |
| 2 | $25.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $245^{\circ}<\phi_{b b}<290^{\circ}$ |
| 3 | $25.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $255^{\circ}<\phi_{b b}<290^{\circ}$ |
| 4 | $25.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $250^{\circ}<\phi_{b b}<285^{\circ}$ |
| 5 | $25.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $250^{\circ}<\phi_{b b}<295^{\circ}$ |
| 6 | $26.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $250^{\circ}<\phi_{b b}<290^{\circ}$ |
| 7 | $28.0^{\circ}<\theta_{b b}<37.0^{\circ}$ | $250^{\circ}<\phi_{b b}<290^{\circ}$ |
| 8 | $25.0^{\circ}<\theta_{b b}<35.0^{\circ}$ | $250^{\circ}<\phi_{b b}<290^{\circ}$ |
| 9 | $25.0^{\circ}<\theta_{b b}<33.0^{\circ}$ | $250^{\circ}<\phi_{b b}<290^{\circ}$ |

Table 4.18: The series of BigBite-angle cuts: same $\theta_{b b}$ cut, and different $\phi_{b b}$ cuts.


Figure 4.37: The ratio $R_{R}$ in the $\theta_{b b}$ bins, with different $\phi_{b b}$ cuts applied to the BigBite. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.
4.41 and 4.42 .

The systematic uncertainties ( $\delta_{\text {sys }}$ s) were evaluated using Eq. (4.36). The low-end-momentum cuts gave larger $\delta_{\text {sys }}$ than the high-end-momentum cuts. In Figs. 4.39, 4.40, 4.41 and 4.42 , a total-shower energy cut $E_{T S}>0.9 \mathrm{GeV}$ was applied to the data and the simulations. The systematic uncertainties from different $E_{T S}$ cuts are presented in the following part of the section.


Figure 4.38: The ratio $R_{R}$ in the $\phi_{b b}$ bins, with different $\theta_{b b}$ cuts applied to the BigBite. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

| Cut number | $P_{b b}$ cut |
| :---: | :---: |
| 1 | $0.9<P_{b b}<2.5 \mathrm{GeV}$ |
| 2 | $0.9<P_{b b}<2.2 \mathrm{GeV}$ |
| 3 | $0.9<P_{b b}<2.1 \mathrm{GeV}$ |
| 4 | $0.9<P_{b b}<2.0 \mathrm{GeV}$ |
| 5 | $0.9<P_{b b}<1.9 \mathrm{GeV}$ |
| 6 | $0.9<P_{b b}<1.8 \mathrm{GeV}$ |
| 7 | $0.9<P_{b b}<1.7 \mathrm{GeV}$ |

Table 4.19: The series of BigBite high momentum cuts.

Series of total-shower-energy cuts
As discussed in sections 4.2.5 and 4.3.2, a $E_{T S}$ cut is needed to remove the threshold drift issue in the data, while the effect of the cut needs to be described by the simulation. The spectra of $R_{R}$ in the $P_{b b}$ bins, with a series of $E_{T S}$ cuts are presented in Fig. 4.43. When the $E_{T S}$ cut was changed, all the corrections for the data efficiency and contamination were re-evaluated.

The systematic uncertainty ( $\delta_{\text {sys }}$ ) in each bin was estimated as in Eq. (4.36), and is listed in Table 4.21.


Figure 4.39: The ratio $R_{R}$ in the $\theta_{b b}$ bins, with different momentum cuts, in the high momentum range, applied to the BigBite. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

The large systematic uncertainties in the low momentum range would affect multiple bins of other variables, e.g. $x_{b j}$, thus a cut of $P_{b b}>0.9 \mathrm{GeV}$ was applied when the SIDIS differential cross sections were extracted.


Figure 4.40: The ratio $R_{R}$ in the $\phi_{b b}$ bins, with different momentum cuts, in the high momentum range, applied to the BigBite. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

| Cut number | $P_{b b}$ cut |
| :---: | :---: |
| 1 | $0.9<P_{b b}<2.5 \mathrm{GeV}$ |
| 2 | $0.6<P_{b b}<2.5 \mathrm{GeV}$ |
| 3 | $0.7<P_{b b}<2.5 \mathrm{GeV}$ |
| 4 | $0.8<P_{b b}<2.5 \mathrm{GeV}$ |
| 5 | $1.0<P_{b b}<2.5 \mathrm{GeV}$ |
| 6 | $1.1<P_{b b}<2.5 \mathrm{GeV}$ |
| 7 | $1.2<P_{b b}<2.5 \mathrm{GeV}$ |

Table 4.20: The series of BigBite momentum cuts.


Figure 4.41: The ratio $R_{R}$ in the $\theta_{b b}$ bins, with different momentum cuts applied to the BigBite. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

## Series of BigBite-preshower cuts

The $P_{b b}$ spectra under BigBite-preshower cuts channel ${ }_{P S}>300$, 400 and 500, and the corresponding data corrections are compared in Fig. 4.44. In this figure, no total-shower-energy cut was applied to the data, and the momentum cut was $0.6<$ $P_{b b}<2.5 \mathrm{GeV}$. Similar comparisons have been done with various total-shower-energy cuts and kinematic cuts. It was found that the systematic uncertainties related to the BigBite-preshower cuts was less than 3\%.


Figure 4.42: The ratio $R_{R}$ in the $\phi_{b b}$ bins, with different momentum cuts applied to the BigBite. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

### 4.5.2 Resolution-related systematic uncertainties

The resolutions of the detectors would cause uncertainties of the reconstructed kinematic variables, thus lead to systematic uncertainties. The resolutions of the basic kinematic variables (angles and momentum) were estimated based on the calibrations, and are summarized in Table 4.22.


Figure 4.43: The ratio $R_{R}$ in the $P_{b b}$ bins, with different $E_{T S}$ cuts applied to the data and the simulation. The left (right) panel represents the $\pi^{+}\left(\pi^{-}\right)$SIDIS production channel. The error bars in the plots represent the statistical uncertainties.

| $P_{b b}$ range $(\mathrm{GeV})$ | $\delta_{\text {sys }}\left(\pi^{+}\right)$ | $\delta_{\text {sys }}\left(\pi^{-}\right)$ |
| :---: | :---: | :---: |
| $0.7-0.8$ | 0.412 | 0.120 |
| $0.8-0.9$ | 0.240 | 0.180 |
| $0.9-1.0$ | 0.079 | 0.059 |
| $1.0-1.1$ | 0.021 | 0.0 |
| $>1.1$ | 0.0 | 0.0 |

Table 4.21: Systematic uncertainties from the series of $E_{T S}$ cuts. The value of $\delta_{s y s}$ was set to 0 if the estimated total uncertainty is smaller or equal to the statistical uncertainty.


Figure 4.44: The comparisons between the $P_{b b}$ spectra under a series of BigBitepreshower cuts and the corresponding data corrections.

The resolutions of composite/derived kinematic variables (derived from the basic ones) were studied by calculating the (smeared) composite variables with the smeared basic variables. The basic variables were smeared as

$$
\begin{align*}
P_{\text {smear }} & =P_{\text {orig }} \times \mathcal{N}\left(1, \sigma_{P}(\text { rela })^{2}\right)  \tag{4.37}\\
\theta_{\text {smear }} & =\theta_{\text {orig }}+\mathcal{N}\left(0, \sigma_{\theta}^{2}\right)  \tag{4.38}\\
\phi_{\text {smear }} & =\phi_{\text {orig }}+\mathcal{N}\left(0, \sigma_{\phi}^{2}\right) \tag{4.39}
\end{align*}
$$

where $\mathcal{N}\left(\mu, \sigma^{2}\right)$ is a Gaussian distribution with a mean value of $\mu$ and a width of $\sigma$, the subscript "smear" and "orig" represent the smeared and original variables, respectively. The quantities $\sigma_{P}($ rela $), \sigma_{\theta}$ and $\sigma_{\phi}$ represent the relative momentum

| Type of variable | Resolution |
| :--- | :---: |
| HRS momentum $P_{h r s}$ | $<10^{-4}$ (relative) |
| HRS angle $\theta_{\text {hrs }}$ | $\approx 0.07^{\circ}$ |
| HRS angle $\phi_{\text {hrs }}$ | $\approx 0.6^{\circ}$ |
| BigBite momentum $P_{b b}$ | $\approx 1 \%$ (relative) |
| BigBite angle $\theta_{b b}$ | $\approx 0.25^{\circ}$ |
| BigBite angle $\phi_{b b}$ | $\approx 0.6^{\circ}$ |

Table 4.22: Resolutions of basic kinematic variables.
resolution, the $\theta$ resolution and the $\phi$ resolution, respectively.
The comparisons between the original and smeared $x_{b j}, z_{h}$ and $\phi_{h}$ are presented in Fig. 4.45. The comparisons for the other composite variables are very similar.

The finite resolution of a variable causes the bin-migration effect, which means some events in one bin could be mis-identified and placed in other bins. This effect can be quantified by using a 2D matrix based on the type of 2D plots in Fig. 4.45. The elements in the 2D matrix is defined as

$$
\begin{equation*}
N_{i j}=\sum \operatorname{event}\left(v a r_{\text {orig }} \in b i n_{i} \& v a r_{\text {smear }} \in b i n_{j}\right) \tag{4.40}
\end{equation*}
$$

where $N_{i j}$ is the number of events with the original variable ( $v a r_{\text {orig }}$ ) in the $i$ th bin and the smeared variable $\left(v a r_{\text {smear }}\right)$ in the $j$ th bin. The estimation for the fraction $\left(f_{i j}\right)$ of the events placed in the $i$ th bin but originated from the $j$ th bin can be expressed as

$$
\begin{equation*}
f_{i j}=\frac{N_{i j}}{\sum_{j} N_{i j}} . \tag{4.41}
\end{equation*}
$$

The center of the $i$ th bin $\left(\langle v a r\rangle_{i}\right)$ is calculated using Eq. (4.33), which has an uncertainty due to the resolution. This uncertainty can be estimated by fitting the quantity $\delta v a r=v a r_{\text {smear }}-v a r_{\text {orig }}$ with a Gaussian function. It can also be estimated


Figure 4.45: The 2D comparisons between the original and smeared $x_{b j}, z_{h}$ and $\phi_{h}$.
using the probability array in Eq. (4.41) as

$$
\begin{equation*}
\delta_{i}=\left[\sum_{j}\left(\langle v a r\rangle_{i}-\langle v a r\rangle_{j}\right)^{2} \times f_{i j}\right]^{1 / 2} \tag{4.42}
\end{equation*}
$$

where $\delta_{i}$ is the uncertainty of the variable var in the $i$ th bin. The estimation from Eq. (4.42) was found to be similar to the Gaussian width from the fitting of $\delta v a r$.

The systematic uncertainty related to the central-value uncertainties of the variables can be estimated by inserting the variable uncertainties to the tuned cross
section model, similar to the bin-centering correction in Eq. (4.34). The resolutionrelated uncertainties can be evaluated for each variable in the expression of the differential cross section:

$$
\begin{equation*}
\delta \sigma^{r e s}\left(v a r_{n, i}\right)=\left[\frac{1}{2}\left(\delta^{+} \sigma_{n, i}^{2}+\delta^{-} \sigma_{n, i}^{2}\right)\right]^{1 / 2} \tag{4.43}
\end{equation*}
$$

where the set of the subscripts $(n, i)$ represents the $n$th variable in the $i$ th bin. The quantities $\delta^{ \pm} \sigma_{n, i}$ are defined as

$$
\begin{equation*}
\delta^{ \pm} \sigma_{n, i}=\left[\sigma_{\text {theory }}\left(\operatorname{var}_{n, i} \pm \delta_{n, i}\right)-\sigma_{\text {theory }}\left(\operatorname{var}_{n, i}\right)\right] \frac{\sigma_{e x p}^{b i n}}{\sigma_{M C}^{b i n}} \tag{4.44}
\end{equation*}
$$

where $\sigma_{\text {theory }}$ is the theoretical differential cross section from the tuned model, as in the BCC corrections. The total resolution-related uncertainties for the extracted differential cross section in $i$ th bin can be expressed as

$$
\begin{equation*}
\delta \sigma_{i}^{r e s}=\left[\sum_{n} \frac{1}{N} \delta \sigma^{r e s}\left(\text { var }_{n, i}\right)^{2}\right]^{1 / 2} \tag{4.45}
\end{equation*}
$$

where the sum over the $N$ variables that $\sigma_{\text {theory }}$ depends on is carried out. This type of systematic uncertainty was less than $3 \%$ with a kinematic dependence.
4.5.3 Systematic uncertainties of the radiative corrections and exclusive tail subtraction

The systematic uncertainties related to the tail subtractions from exclusive channels and the SIDIS radiative corrections have been evaluated in the same manner as [64]. Specifically, different models of the exclusive channels and the difference between the HAPRAD and the SIMC for the radiative corrections have been used to define the systematic uncertainties. The systematic uncertainties for these items are between $2 \%$ to $6 \%$ depending on the kinematics.

## Results and conclusion

In this chapter, the extracted unpolarized SIDIS differential cross sections are compared with models in different bin sets in the following sections. Fitting the extracted differential cross sections from this data demonstrates the data's constraint on the parameters describing the SIDIS process. The plane wave impulse approximation (PWIA) treatment of the ${ }^{3} \mathrm{He}$ nucleus in the SIDIS process is adopted in this study, thus the modeled SIDIS cross section from ${ }^{3} \mathrm{He}$ is the same as the sum of the modeled SIDIS differential cross sections from two protons and one neutron. The collinear PDF from CTEQ10 [127] and the collinear FF from DSS [128] were used in the modeled SIDIS differential cross sections.

In the multi-dimensional bin sets, models from three studies [4, 5, 10] are compared with the data. The modeled SIDIS differential cross sections were calculated as in Eq. (2.40) using the parameters $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ from these three studies. The parameterizations of the Boer-Mulders TMD PDF and the Collins TMD FF were taken from [5], giving a negligible (less than $2 \%$ ) contribution to the SIDIS differential cross sections. These three studies have been reviewed in section 2.5, and the parameters in Table 5.1 are used to represent them.

| Model | $\left\langle k_{\perp}^{2}\right\rangle\left(\mathrm{GeV}^{2}\right)$ | $\left\langle p_{\perp}^{2}\right\rangle\left(\mathrm{GeV}^{2}\right)$ |
| :--- | :---: | :---: |
| Anselmino2014: from [4] | 0.57 | 0.12 |
| Barone2015: from [5] | 0.037 | $0.126+0.506 z_{h}^{2}$ |
| Bacchetta2011: from [10] | 0.14 | $0.42 \cdot z_{h}^{0.54}\left(1-z_{h}\right)^{0.37}$ |

Table 5.1: The parameters from three phenomenological studies.

In all the bin sets, some of the kinematic variables were not separated in multiple ranges and were treated as in a single bin (single-binned-variables). The central values and ranges of the single-binned-variables changed in different bins due to the kinematic correlations. The central values of the variables were determined using Eq. (4.33). We kept the single-binned-variables in the definition of the differential cross sections and kept the differential phase space in the results as $d P H S=d x_{b j} d y d z_{h} d \phi_{S} d P_{t}^{2} d \phi_{h}$ in all the bin sets. The method to extract the differential cross section for a specific bin was discussed in section 4.4.

In all the bin sets the data corrections in Eqs. (4.2) and (4.32) were carried out in the same way for individual bins. The BCCs defined in Eq. (4.34) were carried out differently in pseudo-1D bins and multi-dimensional bins.

In pseudo-1D bins, the central values of all the kinematic variables were the experimental averages using Eq. (4.33). The BCCs were evaluated using these central values of the kinematics. The differential cross sections from the model were at the same central values of the kinematics.

In the multi-dimensional bin sets, the dependence of the differential cross section on one kinematic variable ( $\phi_{h}$ or $P_{t}$ ) was examined in multiple ranges of other variables. To remove the effect of kinematic correlations, BCCs were evaluated with range-by-range sets of kinematics. In each range, the corresponding set of kinematic variables was put in the model $\sigma_{\text {theory }}=\sigma_{\text {model }}($ vars $)$ of Eq. (4.34) while the ratio $\sigma_{e x p}^{b i n} / \sigma_{M C}^{b i n}$ in Eq. (4.34) was evaluated in the usual way for each bin. In
each of the range-by-range sets of kinematics, the variable of interest ( $\phi_{h}$ or $P_{t}$ ) had the experimental-averaged value for each bin, while all the other variables had the experimental-averaged value at the central bin in this range.

We did not combine different ranges of the multi-dimensional bin sets to give one distribution of the variable of interest, for minimizing the model dependence introduced in the BCCs.

In the following sections, the kinematic correlations and binning will be presented first, followed by the comparisons between extracted SIDIS differential cross sections and different models. The fitting results based on the stand-alone data of this study will be presented afterwards. The conclusion will be presented at the end of this chapter.

### 5.1 Kinematic correlations and binning

In the production run of E06-010, only one experimental configuration was used. Kinematic correlations are shown in Fig. 5.1. Due to the kinematic correlations, strict one-dimensional (1D) binning, in which only one variable changes while all the other variables stay intact, is prohibited.

In this study, a set of pseudo-one-dimensional (pseudo-1D) bins is used for presenting the results. Pseudo-1D means that when the binning is in one variable, for example, $x_{b j}$, the difference between one bin and another is not only in $x_{b j}$, but in all the variables $\left(x_{b j}, z_{h}, Q^{2}, \phi_{h}, P_{t}\right)$ due to kinematic correlations. Pseudo-1D bins in $x_{b j}$ has 10 consecutive bins with almost equal statistics. The central values of the kinematic variables in the pseudo-1D bins are presented in Table. 5.2.

As shown in Fig. 5.1 and Table. 5.2, the acceptance in this experiment had strong kinematic correlations. In increasing range of $x_{b j}, z_{h}$ and $Q^{2}$ also increased, while $P_{t}$ decreased. The kinematic range of $\phi_{h}$ centered around the angle of $\pi\left(180^{\circ}\right)$, due to the experimental configuration. The central values of $\phi_{h}$ and $x_{b j}$ were weakly


Figure 5.1: The correlations between $x_{b j}$ and other kinematic variables in experiment E06-010.
correlated. In the range of $\phi_{h}$ closer to the center more events were with smaller $x_{b j}$ compared with the range of $\phi_{h}$ farther from the center.

A set of two-dimensional (2D) bins is used to present the $P_{t}$ dependence of the cross sections. The set of 2D bins $(10 \times 10)$ consists of $10 P_{t}$ bins in 10 ranges of $x_{b j}$. The boundaries of the bins are set to make each bin contain almost equal statistics.

A set of three-dimensional (3D) bins is used to present the $\phi_{h}$ dependence of the cross sections. The data are binned into two ranges of $P_{t}$ first. In each of the $P_{t}$ ranges, five $x_{b j}$ bins are defined. In each of the $2 \times 5$ ranges of $P_{t}$ vs. $x_{b j}, 10 \phi_{h}$ bins are defined. Each bin of the $2 \times 5 \times 10$ set has almost equal statistics.

### 5.2 Cross sections in pseudo-1D bins

The comparisons of the SIDIS differential cross sections from the data and the quarkparton model in pseudo-1D $x_{b j}$ bins are shown in Fig. 5.2. The top panel in the

| $x_{b j}$ | $Q^{2} / \mathrm{GeV}^{2}$ | $z_{h}$ | $\phi_{h} / \mathrm{rad}$ | $P_{t} / \mathrm{GeV}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.163 | 1.47 | 0.476 | 3.10 | 0.437 |
| 0.188 | 1.70 | 0.484 | 3.08 | 0.411 |
| 0.208 | 1.85 | 0.491 | 3.07 | 0.392 |
| 0.228 | 2.00 | 0.499 | 3.07 | 0.371 |
| 0.249 | 2.14 | 0.508 | 3.06 | 0.350 |
| 0.272 | 2.29 | 0.519 | 3.06 | 0.325 |
| 0.297 | 2.45 | 0.530 | 3.05 | 0.299 |
| 0.325 | 2.62 | 0.543 | 3.05 | 0.271 |
| 0.358 | 2.81 | 0.557 | 3.04 | 0.239 |
| 0.393 | 3.09 | 0.562 | 3.02 | 0.216 |

Table 5.2: The central values of the kinematic variables in the pseudo-1D bins. The unit for $Q^{2}$ is $\mathrm{GeV}^{2}$. The unit for $P_{t}$ is GeV . The unit for $\phi_{h}$ is rad. The kinematic variables $x_{b j}, y$ and $z_{h}$ have no unit.
figure is for the $\pi^{+}$production channel ${ }^{3} \mathrm{He}\left(e, e^{\prime} \pi^{+}\right) X$ and the bottom panel for the $\pi^{-}$production channel ${ }^{3} \mathrm{He}\left(e, e^{\prime} \pi^{-}\right) X$. The vertical axis is the SIDIS differential cross section $d \sigma /\left(d x_{b j} d y d z_{h} d \phi_{S} d P_{t}^{2} d \phi_{h}\right)$ in unit of $\mathrm{nb} \cdot \mathrm{GeV}^{-2} \cdot \mathrm{rad}^{-2}$. The total experimental systematic uncertainties using quadrature combination of all the sources are shown in the band at the bottom of each plot.

The SIDIS differential cross section from the model is defined in Eq (2.40) and the parameterizations of the Gaussian widths of unpolarized TMD PDF and FF are in the forms as in [10], namely $\left\langle k_{\perp}^{2}\right\rangle=0.14 \mathrm{GeV}^{2}$ and $\left\langle p_{\perp}^{2}\right\rangle=a \cdot z_{h}^{0.54}\left(1-z_{h}\right)^{b} \mathrm{GeV}^{2}$, where $a=1.55$ and $b=2.2$ are tuned from the values in one set of the HERMES data analysis inherited and cited by [10]. The Boer-Mulders TMD PDF and Collins TMD FF parameterizations were taken from [5]. The effect of the Boer-Mulders terms in the total SIDIS cross sections were found to be less than $2 \%$ in magnitude and opposite in sign for the $\pi^{ \pm}$electroproduction channels. Terms with twists higher than those included in Eqs. (2.41) - (2.45) were neglected. The model calculates the sum of the cross sections from two protons and one neutron as an approximation for the ${ }^{3} \mathrm{He}$ nucleus. Agreement between the data and the model is observed.


Figure 5.2: The SIDIS differential cross sections (defined in text) comparison between the data and the simulation in pseudo-1D $x_{b j}$ bins. The red solid circles are from the data and the black open circles are from the quark-parton model. The error bar of each point represents the statistical uncertainty, mostly smaller than the markers. The error band on the bottom of each panel represents the experimental systematic uncertainty. The left and right panels are for $\pi^{+}$and $\pi^{-}$production channel, respectively.

The comparisons of the ratios (from the data and the model) of SIDIS $\pi^{+}$production cross sections to that of $\pi^{-}$in pseudo-1D $x_{b j}$ bins are shown in Fig. 5.3. The systematic uncertainties from the acceptance and efficiency of electron detection in the BigBite, are not included in the bottom systematic error band, as the electron part is the same in the SIDIS $\pi^{ \pm}$production.

In the plot, the error bars of the data points are for the statistical uncertainties of the data. The error bars of the model points are for the model uncertainties. In this study, the model uncertainties are defined by the quadrature combination of the differences of the ratios with and without the contribution from the Boer-Mulders terms, changing the width $\left\langle k_{\perp}^{2}\right\rangle$ to $2\left\langle k_{\perp}^{2}\right\rangle$ and changing $\left\langle p_{\perp}^{2}\right\rangle$ to $2\left\langle p_{\perp}^{2}\right\rangle$. The BoerMulders effects in the $\pi^{ \pm}$production channels have opposite signs, and the changes of the cross section ratios due to turning off the Boer-Mulders contributions are $1 \%$ to $4 \%$. The flavor dependence of the widths has not been included in the model, thus the widths do not differ in channels of the $\pi^{ \pm}$production. Theoretically, if the $\pi^{ \pm}$


Figure 5.3: The SIDIS differential cross section ratio $\sigma^{\pi^{+}} / \sigma^{\pi^{-}}$comparison between the data and the model in pseudo-1D $x_{b j}$ bins. The red solid circles are from the data and the black open circles are from the quark-parton model. The error bar of each point of data represents the statistical uncertainty. The error bars for the model parameterization uncertainty are smaller than the marker size. The error band on the bottom represents the systematic uncertainty of the data.

SIDIS production cross sections have the same transverse momentum dependence, their ratios at the same kinematics will be independent of the widths. Due to the very small differences between the central values of variables in the $\pi^{ \pm}$production channels, the effect of changing $\left\langle k_{\perp}^{2}\right\rangle$ to $2\left\langle k_{\perp}^{2}\right\rangle$ or $\left\langle p_{\perp}^{2}\right\rangle$ to $2\left\langle p_{\perp}^{2}\right\rangle$ was non-zero but less than $0.1 \%$.

Results from the data are consistent with the model without a flavor dependence of $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ as assumed in most of the global analysis for SIDIS $[4,5,10]$.

### 5.3 The $P_{t}$ dependence of the cross sections: in 2D bins

To present the $P_{t}$ dependence of the SIDIS cross sections, 2 D bins $(10 \times 10)$ of $x_{b j}$ vs. $P_{t}$ are used. Bin-centering corrections were used to remove the difference of all the variables except $P_{t}$ from one bin to another in each range of $x_{b j}$, therefore the $10 P_{t}$ bins in a certain range of $x_{b j}$ differ only in the values of $P_{t}$. The comparisons of the SIDIS differential cross sections from the data and the models from $[4,5,10]$ are presented in Figs. 5.4 and 5.5.

The comparisons show that the model from [10] compares the best with the data, while the model from [4] deviates the most from the data in most of the kinematic ranges. In the highest $x_{b j}$ ranges (corresponding to the lowest $P_{t}$ ranges), the model from [4] gives better comparison than the models from [5, 10], but still has sizable deviations from the data.


Figure 5.4: (color online). The differential cross sections in 2D bins: the $\pi^{+}$SIDIS production channel. The red circles are from the data, the black solid lines are from the model [10], the blue dashed lines are from the model [5] and the green dotted lines are from the model [4]. The error bars represent the statistical uncertainties of the data. The error band on the bottom of each panel represents the experimental systematic uncertainty. The $x_{b j}$ range of each plot is presented at the bottom of the panel.


Figure 5.5: (color online). The differential cross sections in 2D bins: the $\pi^{-}$SIDIS production channel. The definitions of the markers, the lines and the bands are the same as the figure above for $\pi^{+}$channel.

### 5.4 The $\phi_{h}$ dependence of the cross sections: in 3D bins

The differential cross sections of SIDIS were extracted in 3D bins $(2 \times 5 \times 10)$, to examine the $\phi_{h}$ dependence of the cross sections in $2 \times 5$ ranges of $P_{t}$ vs. $x_{b j}$. Bincentering corrections were used to remove the difference of all the variables except $\phi_{h}$ from one bin to another in each of the $P_{t}$ vs. $x_{b j}$ ranges, therefore the $10 \phi_{h}$ bins in a certain range of $P_{t}$ and $x_{b j}$ differ only in the values of $\phi_{h}$. The comparisons of the SIDIS differential cross sections from the data and the models from $[4,5,10]$ are presented in Figs. 5.6 and 5.7.

The comparisons show that the model from [10] compares the best with the data, while the model from [4] deviates the most from the data in most of the kinematic ranges.

In a phenomenological model, the modulations are from the structure functions $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ on top of the non-modulated part $\left(F_{U U}\right)$. The distributions from the model without modulations can be calculated by using the model parameterization while setting $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ to zero. The non-modulated SIDIS differential cross section does not depend on $\phi_{h}$, and behaves as a straight line in any $\phi_{h}$ range, when other kinematic variables are fixed. Comparisons between the data and the model from [10] with and without modulations are in Figs. 5.8 and 5.9.

Studies on the modulations and the constraints on the phenomenological parameters provided by the data in this study will be presented in the next section.


Figure 5.6: The differential cross sections in 3D bins: the $\pi^{+}$SIDIS production channel. The red circles are from the data, the black solid lines are from the model [10], the blue dashed lines are from the model [5] and the green dotted lines are from the model [4]. The error bars represent the statistical uncertainties of the data. The error band on the bottom of each panel represents the experimental systematic uncertainty. The $P_{t}$ (in unit of GeV ) and $x_{b j}$ ranges of each plot are presented at the top and the bottom of the panel, respectively.


Figure 5.7: The differential cross sections in 3D bins: the $\pi^{-}$SIDIS production channel. The definitions of the markers, the lines and the bands are the same as the figure above for the $\pi^{+}$channel.


Figure 5.8: The differential cross sections in 3D bins: the $\pi^{+}$SIDIS production channel. The red circles are from the data, the black solid lines are from the model [10], the blue dashed lines are from the model [10] with $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ setting to zero. The error bars represent the statistical uncertainties of the data. The error band on the bottom of each panel represents the experimental systematic uncertainty. The $P_{t}$ (in unit of GeV ) and $x_{b j}$ ranges of each plot are presented at the top and the bottom of the panel, respectively.



| - |
| :--- |
| $-\sigma$ (total $)$ |
| $---\sigma$ (no mod) |



Figure 5.9: The differential cross sections in 3D bins: the $\pi^{-}$SIDIS production channel. The definitions of the markers, the lines and the bands are the same as the figure above for the $\pi^{+}$channel.

### 5.5 Azimuthal modulation and stand-alone data fitting

Fitting the $\phi_{h}$ distribution in each of the $2 \times 5$ ranges of $P_{t}$ vs. $x_{b j}$ in the 3D bins $(2 \times 5 \times 10)$, with a simple function $A \cdot\left(1-B \cdot \cos \phi_{h}\right)$, provides a naive probe for the azimuthal modulation effect in the data. The parameter $B$ indicates the size of the modulation. The parameter $B$ s in all ranges are presented in Fig. 5.10. Due to a limited $\phi_{h}$ range in the data and a large number of fitting parameters being used ( $A$ and $B$ in one $P_{t}$ and $x_{b j}$ range differ from $A$ and $B$ in another range), the data do not provide good constraints on the $B \mathrm{~s}$.


Figure 5.10: Results of $B$ in 3D bins from the $A \cdot\left(1-B \cdot \cos \phi_{h}\right)$ fit. The red solid circles and black open circles represent the results in the $\pi^{+}$and $\pi^{-}$production channels, respectively.

Azimuthal modulation effects in the unpolarized SIDIS channel arise from the relative magnitudes of the structure functions $F_{U U}^{\cos \phi_{h}}, F_{U U}^{\cos 2 \phi_{h}}$ and $F_{U U}$. Substituting the expressions of the structure functions after the convolution [Eqs. (2.52), (2.53), and (2.54)] in to Eq. (2.40), one obtains a fitting function with two parameters, $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$. The 2 D bins $(10 \times 10)$ and 3 D bins $(2 \times 5 \times 10)$ data were fitted and the results are in Fig. 5.11. Three contours corresponding to $\delta \chi^{2}=1,2.3$ and 6.2 are drawn besides the central values from the fitting. The $\delta \chi^{2}=1$ contour is
conventionally the same as the one- $\sigma$ contour, which is at a confidence level of $39 \%$ for a two-parameter fit. The contours of $\delta \chi^{2}=2.3$ and 6.2 show the constraints of two-parameter fitting at confidence levels of $68 \%$ and $95 \%$, respectively. In this fitting the Boer-Mulders parts of the structure functions were set to zero.


Figure 5.11: Fitting contours with the functional form of the total unpolarized SIDIS cross section (refer to the text). The top panel is for the fitting results using the 2 D bins $(10 \times 10)$ data, the bottom panel for the 3D bins $(2 \times 5 \times 10)$. The central values of the fitting are the black crosses. The three contours from the smallest to the largest in each panel correspond to $\delta \chi^{2}=1,2.3$ and 6.2 , respectively.

The central values of the fitting in the 2D bins are

$$
\begin{align*}
& \left\langle k_{\perp}^{2}\right\rangle=0.003 \pm 0.008 \mathrm{GeV}^{2}  \tag{5.1}\\
& \left\langle p_{\perp}^{2}\right\rangle=0.2104 \pm 0.0025 \mathrm{GeV}^{2} \tag{5.2}
\end{align*}
$$

The central values of the fitting in the 3D bins are

$$
\begin{align*}
\left\langle k_{\perp}^{2}\right\rangle & =0.006 \pm 0.010 \mathrm{GeV}^{2}  \tag{5.3}\\
\left\langle p_{\perp}^{2}\right\rangle & =0.2148 \pm 0.0026 \mathrm{GeV}^{2} \tag{5.4}
\end{align*}
$$

The fitting results indicate consistent azimuthal modulation effects from the data in 3D bins with the $\phi_{h}$ information and 2D bins without the $\phi_{h}$ information.

Fitting the data with a simpler functional form, namely setting $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ to zero, was also done. The results are presented in Fig. 5.12.


Figure 5.12: Fitting contours with the functional form of the non-modulated unpolarized SIDIS cross section ( $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ set to zero: refer to the text). The top panel is for the fitting results using the 2D bins $(10 \times 10)$ data, the bottom panel for the 3D bins $(2 \times 5 \times 10)$. The central values of the fitting are the black crosses. The three contours from the smallest to the largest in each panel correspond to $\delta \chi^{2}=1,2.3$ and 6.2 , respectively.

The central values of this fitting in the 2D bins are

$$
\begin{align*}
& \left\langle k_{\perp}^{2}\right\rangle=0.090 \pm 0.097 \mathrm{GeV}^{2}  \tag{5.5}\\
& \left\langle p_{\perp}^{2}\right\rangle=0.1840 \pm 0.0276 \mathrm{GeV}^{2} \tag{5.6}
\end{align*}
$$

The central values of the fitting in the 3D bins are

$$
\begin{align*}
& \left\langle k_{\perp}^{2}\right\rangle=0.085 \pm 0.112 \mathrm{GeV}^{2}  \tag{5.7}\\
& \left\langle p_{\perp}^{2}\right\rangle=0.1901 \pm 0.0330 \mathrm{GeV}^{2} \tag{5.8}
\end{align*}
$$

The very different constraints of $\left\langle k_{\perp}^{2}\right\rangle$ vs. $\left\langle p_{\perp}^{2}\right\rangle$ using the functional form including all three structure functions (Fig. 5.11) and the functional form including only structure function $F_{U U}$ (Fig. 5.12), come from the specific model formulation, namely $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ as in Eqs. (2.53) and (2.54). These specific functional forms, when applied to the data in this study, would result in the intrinsic transverse momentum width $\left\langle k_{\perp}^{2}\right\rangle$ of the quarks in the nucleon being consistent with zero at small central values, which contradicts the results from the global analyses $[4,5,10]$. The effect of including the Boer-Mulders terms as parameterized in [5] was tested to be negligible (less than $2 \%$ in the kinematic range of this study).

To examine the data's constraint on the intrinsic widths with relaxed model formulations, two adjusted functional forms were used to do the fitting in the 3D bins with the $\phi_{h}$ information. The first one includes the structure functions $F_{U U}$ and $F_{U U}^{\cos \phi_{h}}$, with an additional fitting parameter $A$ to tune the amplitude of modulation as $A \cdot F_{U U}^{\cos \phi_{h}}$. The results of the fitting are

$$
\begin{align*}
\left\langle k_{\perp}^{2}\right\rangle & =0.078 \pm 0.1505 \mathrm{GeV}^{2}  \tag{5.9}\\
\left\langle p_{\perp}^{2}\right\rangle & =0.1925 \pm 0.0464 \mathrm{GeV}^{2}  \tag{5.10}\\
A & =0.0119 \pm 0.1971 \tag{5.11}
\end{align*}
$$

The intrinsic widths $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ are under loose constraint individually while the amplitude of $A \cdot F_{U U}^{\cos \phi_{h}}$ is suppressed by a small factor $A$.

The second one includes the structure functions $F_{U U}, F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$, with an additional fitting parameter $A$ to tune the amplitude of modulation as $A \cdot\left(F_{U U}^{\cos \phi_{h}}+\right.$ $\left.F_{U U}^{\cos 2 \phi_{h}}\right)$. The results of the fitting are

$$
\begin{align*}
\left\langle k_{\perp}^{2}\right\rangle & =0.080 \pm 0.1542 \mathrm{GeV}^{2}  \tag{5.12}\\
\left\langle p_{\perp}^{2}\right\rangle & =0.1918 \pm 0.0475 \mathrm{GeV}^{2}  \tag{5.13}\\
A & =0.0077 \pm 0.1820 \tag{5.14}
\end{align*}
$$

The intrinsic widths are under similar constraint as in the first case with a small factor $A$ suppressing the amplitude of $A \cdot\left(F_{U U}^{\cos \phi_{h}}+F_{U U}^{\cos 2 \phi_{h}}\right)$.

Without introducing specific forms of $F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$, the parameters $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ in the SIDIS channels appear as the combined quantity $\left\langle P_{t}^{2}\right\rangle$. Sensitivity to $\left\langle P_{t}^{2}\right\rangle$ is explicitly provided by the $P_{t}$ behavior of the data. The comparison between the data and the models in the two functional forms (with and without $F_{U U}^{\cos \phi_{h}}$ and $\left.F_{U U}^{\cos 2 \phi_{h}}\right)$ using the parameters from fitting in 2D bins of the data are shown in Figs. 5.13 and 5.14.

### 5.6 Conclusion

In the multi-dimensional bin sets, the data are compared with three models from $[4,5,10]$ while the ${ }^{3} \mathrm{He}$ nucleus is approximated as two protons and one neutron in a plane-wave picture. In most of the kinematic ranges, the model from [10] compares the best with the data while the model from [4] deviates the most from the data. In the highest $x_{b j}$ ranges (corresponding to the lowest $P_{t}$ ranges), the model from [4] gives the best comparison with the data.

Azimuthal modulations in unpolarized SIDIS are observed to be consistent with zero within the experimental uncertainties in this study. Using the specific functional form as in the global analysis [5], the fitting results show that the width of quark intrinsic transverse momentum $\left\langle k_{\perp}^{2}\right\rangle$ is much smaller than the results from the global analyses of other types of data $[3,4,5,10]$. With relaxed model formulation, $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ are under looser constraint individually, while the combined quantity $\left\langle P_{t}^{2}\right\rangle$ is constrained by the $P_{t}$ behavior of the data. The widths $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ in the structure functions, related to the azimuthal modulations are determined consistently using the extracted cross sections with and without the information of $\phi_{h}$.

Apparently, a simple model at the lowest twist was able to describe the main features of the data. The applicability of the simple model to semi-inclusive experi-
ments on the proton and deuteron targets in modest $Q^{2}$ ranges was also observed by other JLab experiments [64, 65]. While one might naively expect large contributions from the higher-twist terms in the modest $Q^{2}$ range, they have not been found to be significant experimentally. It is possible that the contributions of the higher-twist terms in the SIDIS process are not as large as expected. It is also possible that the higher-twist contributions have been absorbed into the lowest-twist model by changing the parameters $\left(\left\langle k_{\perp}^{2}\right\rangle\right.$ and $\left.\left\langle p_{\perp}^{2}\right\rangle\right)$. On the other hand, besides the general agreement between the simple model and the data in this study, sizable differences exist in some of the kinematic ranges. These differences might be related to the higher-twist terms. The higher-twist terms might also be responsible for the very different $\left\langle k_{\perp}^{2}\right\rangle$ values found in this study and from the studies [3, 4, 5, 10]. A discussion about the potential effect from the higher-twist terms on determining the $\left\langle k_{\perp}^{2}\right\rangle$ can be found in [5].

Clearly, high-precision data in the modest $Q^{2}$ range with a full azimuthal angular coverage will, in addition to study the leading-twist TMDs, provide opportunities to study the details of the higher-twist terms and their effects on the azimuthal angular modulations. The future 12 GeV SIDIS programs at JLab with SoLID combining high luminosities and a large acceptance including a full azimuthal angular coverage [129, 130] will provide high-precision data of the SIDIS differential cross sections as well as the azimuthal modulations in multi-dimensional bins covering a broad kinematic range. These data will significantly advance the development of the TMD phenomenology and our understanding of the TMD physics.


Figure 5.13: The differential cross sections in 2D bins: the $\pi^{+}$SIDIS production channel. The red circles are from the data. The black solid lines are from the model including the structure functions $F_{U U}, F_{U U}^{\cos \phi_{h}}$ and $F_{U U}^{\cos 2 \phi_{h}}$ with parameters $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ from stand-alone data fitting. The blue dashed lines are from the model including only the structure functions $F_{U U}$ with parameters $\left\langle k_{\perp}^{2}\right\rangle$ and $\left\langle p_{\perp}^{2}\right\rangle$ from fitting the data of this work only. The error bars represent the statistical uncertainties of the data. The error band on the bottom of each panel represents the experimental systematic uncertainty. The $x_{b j}$ range of each plot is presented at the bottom of the panel.



| - |
| :--- |
| $-\sigma$ (total) |
| $\ldots \ldots \ldots$ |
|  |



Figure 5.14: The differential cross sections in 2D bins: the $\pi^{-}$SIDIS production channel. The definitions of the markers, the lines and the bands are the same as the figure above for $\pi^{+}$channel.

## Appendix A

 Formalisms for inclusive radiative effectsIn this appendix, a formalism based on Mo and Tsai $[122,131]$ (MT) is presented. This approach removed the energy-peaking approximation which has been assumed in various other approaches. Discussions and some re-derivations of the expressions from $[122,131]$ are presented along the derivation of this approach.

There were three key points in the MT formalism as listed below.

- The vacuum polarization and vertex correction changes the Born cross section, at a basic QED level. This part was considered as an energy-loss-independent part of the internal radiative effects.
- The internal bremsstrahlung is not directly calculated using QED, but based on modified Bethe-Heitler formula [122].
- The external energy loss included the external bremsstrahlung and ionization when the particles go through materials.

Based on these three points, Mo and Tsai argued that the Born cross section firstly is changed by the energy-loss-independent radiative effects. Then, this cross
section is further changed, because of the difference between the kinematics recorded experimentally and the kinematics at the interaction point. Mo and Tsai assumed the method based on the modified Bethe-Heitler formula for internal bremsstrahlung was the "exact" calculation, and different approximate methods to integrate this formula were compared with the non-approximated one. Among the approximate methods, a most convenient one was named the equivalent radiator method (ERM). As discussed by Bjorken in [132], the internal-bremsstrahlung effects could be described by an equivalent radiator, when the angle peaking condition is assumed in the calculation based on modified Bethe-Heitler formula. The angle peaking approximation is a conventional and convenient method to compute the Bethe-Heitler integration, which assumes the bremsstrahlung photons are in the directions of the incident and scattered electrons. It is useful to mention that, the approach based on the Bethe-Heitler formula is also a type of empirical method (not full or exact QED).

The formalism from MT used probability integrations to "fold" the Born cross section with the radiative effects [131], which can be expressed as

$$
\begin{align*}
\sigma_{r a d}\left(E_{s}, E_{p}\right)= & \int_{E_{s}^{\min }}^{E_{s}} d E_{s}^{\prime} \int_{E_{p}}^{E_{p}^{\max }} d E_{p}^{\prime} F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}^{\prime}, E_{p}^{\prime}\right) \\
& \times I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{\prime}, E_{p}, T_{p}\right) \tag{A.1}
\end{align*}
$$

where $\sigma_{\text {rad }}\left(E_{s}, E_{p}\right)$ represents the radiative (measured) cross section with incident and scattered electron energies of $E_{s}$ and $E_{p}$, and $\sigma_{\text {born }}\left(E_{s}^{\prime}, E_{p}^{\prime}\right)$ represents the Born cross section with incident and scattered electron energies of $E_{s}^{\prime}$ and $E_{p}^{\prime}$ at the interaction point of the scattering. The term $I\left(E, E^{\prime}, T\right)$ represents the probability distribution related to the energy loss of $\left(E-E^{\prime}\right)$, and a total path length $T$. The quantities $T_{s}$ and $T_{p}$ represent the total path lengths for the incident and scattered electrons. The factor $F$ is the energy-loss-independent factor, which includes the
vacuum polarization and the vertex correction.
Based on ERM, the total path lengths $T_{s}$ and $T_{p}$ can be expressed as

$$
\begin{align*}
& T_{s}=T_{b}+T_{r}  \tag{A.2}\\
& T_{p}=T_{a}+T_{r} \tag{A.3}
\end{align*}
$$

where $T_{b}\left(T_{a}\right)$ represent the path length, in the unit of the radiation length, of the electron through the materials before (after) the scattering. The effects related to $T_{b}$ and $T_{a}$ are the external radiative effects. The effective internal path length $T_{r}$ was defined as [131, 133]

$$
\begin{equation*}
T_{r}=b^{-1} \frac{\alpha}{\pi}\left[\ln \left(\frac{Q^{2}}{m^{2}}\right)-1\right], \tag{A.4}
\end{equation*}
$$

where $m$ is the electron mass, and the constant $b$ depends on the materials the electrons pass through. As will be presented later, a path length $T$ always appears as $b \times T$, thus the internal radiative effects (related to $b \times T_{r}$ ) is independent of the materials. In $[131,133], b$ was defined as

$$
\begin{equation*}
b=\frac{4}{3}\left[1+\frac{Z+1}{9(Z+\eta)} \frac{1}{\ln \left(183 Z^{-1 / 3}\right)}\right], \tag{A.5}
\end{equation*}
$$

where $Z$ is the (averaged) number of proton(s) in the nuclei of the materials, and $\eta$ was defined as

$$
\begin{equation*}
\eta=\frac{\ln \left(1440 Z^{-2 / 3}\right)}{\ln \left(183 Z^{-1 / 3}\right)} \tag{A.6}
\end{equation*}
$$

The physical picture of MT can be illustrated as: the measured cross section is different from the Born cross section because of the $F$ factor, and experiments only record the electron energies $\left(E_{s}\right.$ and $\left.E_{p}\right)$ which differ from the energies at the interaction point of the scattering $\left(E_{s}^{\prime}\right.$ and $\left.E_{p}^{\prime}\right)$. Eq. (A.1) is the folding formula that connects the Born cross section to the measured cross section.

The probability distribution includes the energy loss from the external and internal bremsstrahlung, and ionization (also called straggling in [122, 131]) for the external energy loss. Following the steps leading to Eq. (4.1) in [131], it can be expressed as

$$
\begin{equation*}
I(E, E-\delta E, T)=\frac{1}{\Gamma(1+b T)}\left(\frac{\delta E}{E}\right)^{b T}\left\{\frac{b T}{\delta E} \phi\left(\frac{\delta E}{E}\right)+\frac{\xi}{\delta E^{2}}\right\} \tag{A.7}
\end{equation*}
$$

where $\Gamma$ represents the Gamma function, $T=T_{s}\left(T_{p}\right)$, and $\xi=a x_{0} T_{b}\left(a x_{0} T_{a}\right)$ for an incident (scattered) electron. The function $\phi(v)$ was defined as

$$
\begin{equation*}
\phi(v) \approx 1-v+\frac{3}{4} v^{2} . \tag{A.8}
\end{equation*}
$$

The first term in the curly bracket of Eq. (A.7) is related to the external and internal bremsstrahlung, and the second one is related to the external energy loss due to the ionization process.

The function $I\left(E, E^{\prime}, T\right)$ has a singularity at $E=E^{\prime}$, which needs a special treatment in a integration. This issue can be tackled by separating the 2D integration in Eq. (A.1) into four parts, expressed as

$$
\begin{align*}
\sigma_{r a d}\left(E_{s}, E_{p}\right)= & \left(\int_{E_{s}^{\min }}^{E_{s}-\Delta_{s}}+\int_{E_{s}-\Delta_{s}}^{E_{s}}\right) d E_{s}^{\prime}\left(\int_{E_{p}+\Delta_{p}}^{E_{p}^{\max }}+\int_{E_{p}}^{E_{p}+\Delta_{p}}\right) d E_{p}^{\prime} F\left(Q^{2}\right) \\
& \times \sigma_{\mathrm{born}}\left(E_{s}^{\prime}, E_{p}^{\prime}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{\prime}, E_{p}, T_{p}\right) \tag{A.9}
\end{align*}
$$

where $\Delta_{s}$ and $\Delta_{p}$ are two empirical small energy values. The four parts involving different types of singularities are listed below.

- One regular integral (no singularity): $\int_{E_{s}^{\min }}^{E_{s}-\Delta_{s}} d E_{s}^{\prime} \int_{E_{p}+\Delta_{p}}^{E_{p}^{\max }} d E_{p}^{\prime}$.
- Two single-singular integrals: $\int_{E_{s}-\Delta_{s}}^{E_{s}} d E_{s}^{\prime} \int_{E_{p}+\Delta_{p}}^{E_{p}^{\max }} d E_{p}^{\prime}$, and $\int_{E_{s}^{\min }}^{E_{s}-\Delta_{s}} d E_{s}^{\prime} \int_{E_{p}}^{E_{p}+\Delta_{p}} d E_{p}^{\prime}$.
- One double-singular integral: $\int_{E_{s}-\Delta_{s}}^{E_{s}} d E_{s}^{\prime} \int_{E_{p}}^{E_{p}+\Delta_{p}} d E_{p}^{\prime}$.

The integration boundaries are defined by the kinematics of the elastic scattering:

$$
\begin{align*}
E_{s}^{\min } & =\frac{E_{p}}{1-E_{p} M^{-1}(1-\cos \theta)},  \tag{A.10}\\
E_{p}^{\max }\left(E_{s}^{\prime}\right) & =\frac{E_{s}^{\prime}}{1+E_{s}^{\prime} M^{-1}(1-\cos \theta)}, \tag{A.11}
\end{align*}
$$

where $\theta$ is the polar angle of the scattered electron, and $M$ is the mass of the target nucleon or nucleus.

The singular part in the integrals can be formally expressed as

$$
\begin{equation*}
I_{s g}=\int_{E-\Delta}^{E} d E^{\prime} I\left(E, E^{\prime}, T\right) f\left(E^{\prime}\right) \tag{A.12}
\end{equation*}
$$

where $f\left(E^{\prime}\right)$ represents the other parts of the integrand besides $I\left(E, E^{\prime}, T\right)$. Assuming $f\left(E^{\prime}\right) \approx f(E)$ in the small range $(E-\Delta, E)$, Eq. (A.12) can be analytically carried out by using Eq. (A.7):

$$
\begin{equation*}
I_{s g}=\frac{f(E)}{\Gamma(1+b T)}\left(\frac{\Delta}{E}\right)^{b T}\left[1-\frac{\xi}{(1-b T) \Delta}\right] . \tag{A.13}
\end{equation*}
$$

Eq. (A.13) is the same as Eq. (B.36) of [131], in which a more convoluted derivation was presented.

The final (non-singular) formula for the folded cross section can be obtained by
substituting Eq. (A.13) into Eq. (A.9):

$$
\begin{align*}
\sigma_{r a d}\left(E_{s}, E_{p}\right)= & \int_{E_{s}^{\min }}^{E_{s}-\Delta_{s}} d E_{s}^{\prime} \int_{E_{p}+\Delta_{p}}^{E_{p}^{\max }} d E_{p}^{\prime} F\left(Q^{2}\right) \\
& \times \sigma_{\mathrm{born}}\left(E_{s}^{\prime}, E_{p}^{\prime}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{\prime}, E_{p}, T_{p}\right) \\
& +\int_{E_{p}+\Delta_{p}}^{E_{p}^{\max }} d E_{p}^{\prime} \frac{1}{\Gamma\left(1+b T_{s}\right)}\left(\frac{\Delta_{s}}{E_{s}}\right)^{b T_{s}}\left[1-\frac{\xi_{b}}{\left(1-b T_{s}\right) \Delta_{s}}\right] \\
& \times F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}, E_{p}^{\prime}\right) I\left(E_{p}^{\prime}, E_{p}, T_{p}\right) \\
& +\int_{E_{s}^{\min }}^{E_{s}-\Delta_{s}} d E_{s}^{\prime} \frac{1}{\Gamma\left(1+b T_{p}\right)}\left(\frac{\Delta_{p}}{E_{p}}\right)^{b T_{p}}\left[1-\frac{\xi_{a}}{\left(1-b T_{p}\right) \Delta_{p}}\right] \\
& \times F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}^{\prime}, E_{p}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) \\
& +\frac{1}{\Gamma\left(1+b T_{s}\right)}\left(\frac{\Delta_{s}}{E_{s}}\right)^{b T_{s}}\left[1-\frac{\xi_{b}}{\left(1-b T_{s}\right) \Delta_{s}}\right] \\
& \times \frac{1}{\Gamma\left(1+b T_{p}\right)}\left(\frac{\Delta_{p}}{E_{p}}\right)^{b T_{p}}\left[1-\frac{\xi_{a}}{\left(1-b T_{p}\right) \Delta_{p}}\right] \\
& \times F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}, E_{p}\right) . \tag{A.14}
\end{align*}
$$

This expression can be directly used in a code for numerical calculations. A few points listed below should be useful for the people who use this formula.

- The quantities $Q^{2}$ and $T_{r}$ (internal part of $T_{s}$ and $T_{p}$ ) are kinematic dependent, and should be computed inside the integrations.
- One may also use $b_{s}$ and $b_{p}$ in the product with $T_{s}$ and $T_{p}$, instead of a common $b$. The internal part $b T_{r}$ will not change when $b$ is varied, as discussed previously following Eq. (A.4).
- The small quantities $\Delta_{s}$ and $\Delta_{p}$ should be small enough so Eq. (A.13) is a good approximation. One can study the systematic uncertainties related to this approximation by varying the values of $\Delta_{s}$ and $\Delta_{p}$.

Due to the limitation of the computing power at the time Mo and Tsai worked out their algorithm, various peaking approximations were developed to accelerate the numerical integration $[122,131]$. One of the most widely used version is the energy-peaking approximation, its analytical form (for usage in code) can be found in Eq. (4.1) of [131] and Eq. (A82) of [133]. This analytical form can be expressed as

$$
\begin{align*}
\sigma_{r a d}\left(E_{s}, E_{p}\right)= & \left(\frac{\Delta_{s}}{E_{s}}\right)^{b T_{s}}\left(\frac{\Delta_{p}}{E_{p}}\right)^{b T_{p}} \frac{1}{\Gamma\left(1+b T_{s}+b T_{p}\right)} \\
& \times\left[1-\frac{\xi_{b}+\xi_{a}}{\left(1-b T_{s}+b T_{p}\right) \Delta_{p}}\right] F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}, E_{p}\right) \\
& +\int_{E_{s}^{\min }}^{E_{s}-\Delta_{s}} d E_{s}^{\prime} f_{(s)} F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}^{\prime}, E_{p}\right) I^{\prime}\left(E_{s}, E_{s}^{\prime}, T_{s}\right) \\
& +\int_{E_{p}+\Delta_{p}}^{E_{p}^{\max }} d E_{p}^{\prime} f_{(p)} F\left(Q^{2}\right) \sigma_{\mathrm{born}}\left(E_{s}, E_{p}^{\prime}\right) I^{\prime}\left(E_{p}^{\prime}, E_{p}, T_{p}\right) \tag{A.15}
\end{align*}
$$

The factors $f_{(s)}$ and $f_{(p)}$ from the peaking approximation were defined as

$$
\begin{align*}
& f_{(s)}=\left(\frac{E_{s}-E_{s}^{\prime}}{E_{p} R}\right)^{b T_{p}}  \tag{A.16}\\
& f_{(p)}=\left[\frac{R\left(E_{p}^{\prime}-E_{p}\right)}{E_{s}}\right]^{b T_{s}}, \tag{A.17}
\end{align*}
$$

where $R$ was expressed as

$$
\begin{equation*}
R=\frac{\Delta_{s}}{\Delta_{p}}=\frac{M+2 E_{s} \sin ^{2}(\theta / 2)}{M-2 E_{p} \sin ^{2}(\theta / 2)} \tag{A.18}
\end{equation*}
$$

Using the ratio $R$, only $\Delta_{p}$ was empirically defined in [131, 133].
The functional form of the probability distribution $I^{\prime}$ in Eq. (A.15) was the same as $I$ in Eq. (A.7), with the following change: for an incident and scattered electron, $\xi$ was commonly defined as $\xi=a x_{0}\left(T_{b}+T_{a}\right) / 2$. This difference can be traced back
to the (incomplete) update from [133] to the original formalism in [131], as listed below.

- In [131], the authors used the approximation of $T_{b}=T_{a}$ (half-path-length approximation), which means the scattering was assumed to happen at the center of the materials. Thus, the ionization-related quantity $\xi$ was defined to be $\xi=a x_{0}\left(T_{b}+T_{a}\right) / 2$. In appendix C of [131], the authors had some discussions about the uncertainty of this approximation.
- In [133], the authors intended to remove the assumption $T_{b}=T_{a}$, and changed the bremsstrahlung-related quantities in the folding formula (with energypeaking approximation), but left the ionization-related part intact. This seems like an incomplete update, but if $T_{b}$ is close to $T_{a}$, or the radiative effects due to the bremsstrahlung is dominating, the error should be acceptable.

With the development of computers, people are now able to carry out the full 2D integrations mentioned above, and use the Monte Carlo method to simulate the energy loss effects. The external radiative effects can be simulated by defining materials before and after the scattering ( $T_{b}, T_{a}$ and $b$ ). In the formalism developed by Mo and Tsai, the internal radiative effects can also can also be simulated by calculating the effective path length $T_{r}$, and the $F$ factor, based on the kinematics of each event. Using a Monte Carlo simulation, one is able to change the descriptions of the energy loss mechanisms without the need to derive an analytical form of the combined probability distribution. Simulation tool kits, such as Geant [134], have been developed, and are able to properly describe the external radiative effects.

Mo and Tsai discussed ERM and other methods (before 1975) for the description of the internal radiative effects [122, 131]. Another widely used formalism (POLRAD) for the internal radiative effects in the inclusive scattering based on QED was developed in 1997 [123]. In POLRAD, a 2D integration has to be carried out.

A code module (RADIATE) based on [133] was developed by K. Slifer, et al. for JLab experiments, and have been used for more than 15 years. I developed a code module (XYRAD2D) based on Eq. (A.14). A comparison between the integration ranges of POLRAD, MT 2D integration (XYRAD2D), and MT peaking integrations (RADIATE) for one example event is presented in Fig A.1. The kinematics of this event is listed in Table A.1. The corresponding Born and radiative differential cross sections (DXSs) of the inclusive DIS process, using the model from [119], are listed in Table A.2. In this example, the radiative DXSs are evaluated by RADIATE, XYRAD2D and POLRAD with only the internal radiative effects.

| $E_{s}(\mathrm{GeV})$ | $E_{p}(\mathrm{GeV})$ | $\theta$ (degree) | $Q^{2}\left(\mathrm{GeV}^{2}\right)$ | $W(\mathrm{GeV})$ |
| :---: | :---: | :---: | :---: | :---: |
| 5.89 | 1.0 | 30 | 1.58 | 2.91 |

Table A.1: Kinematics of the example event.

| Type of DXS | Born | RADIATE | XYRAD2D | POLRAD |
| :---: | :---: | :---: | :---: | :---: |
| Value | 17.55 | 21.77 | 22.03 | 20.98 |

Table A.2: The Born and radiative differential cross sections evaluated by RADIATE, XYRAD2D and POLRAD, at the kinematics listed in Table A.1. The DXS $d \sigma / d \Omega d E$ is in the unit of $\left[\mathrm{nb} \mathrm{GeV}^{-1} \mathrm{Sr}^{-1}\right.$ ].

## Radiative tail from the elastic channel

The derivation from Eq. (A.1) to Eq. (A.14) can be easily used for the evaluation of the radiative tail from the elastic channel.

The cross section of the elastic channel can be expressed as

$$
\begin{equation*}
\sigma_{\mathrm{el}}\left(E_{s}, E_{p}\right)=\sigma_{\mathrm{el}}\left(E_{s}\right) \delta\left(E_{p}-E_{p}^{e l}\left(E_{s}\right)\right) \tag{A.19}
\end{equation*}
$$

where the angular dependence of $\sigma_{\mathrm{el}}$ is not explicitly expressed, and $E_{p}^{e l}\left(E_{s}^{\prime}\right)$ is defined


Figure A.1: Each step in the integrations of POLRAD (green rhombus), MT 2D integration (blue solid circles), and MT peaking integrations (red squares) for one example event. The central kinematics of the event as in Table A. 1 is represented by the black empty circle.
as

$$
\begin{equation*}
E_{p}^{e l}\left(E_{s}^{\prime}\right)=\frac{E_{s}^{\prime}}{1+E_{s}^{\prime} M^{-1}(1-\cos \theta)} . \tag{A.20}
\end{equation*}
$$

Substituting Eq. (A.19) into Eq. (A.1), the radiative cross section of the elastic
channel can be expressed as

$$
\begin{align*}
\sigma_{r a d}\left(E_{s}, E_{p}\right)= & \int_{E_{s}^{\min }}^{E_{s}} d E_{s}^{\prime} \int_{E_{p}}^{E_{p}^{\max }} d E_{p}^{\prime} F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}^{\prime}\right) \\
& \times \delta\left(E_{p}^{\prime}-E_{p}^{e l}\left(E_{s}^{\prime}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{\prime}, E_{p}, T_{p}\right)\right. \\
= & \int_{E_{s}^{\min }}^{E_{s}} d E_{s}^{\prime} F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}^{\prime}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{e l}\left(E_{s}^{\prime}\right), E_{p}, T_{p}\right) \\
= & \left(\int_{E_{s}^{\min }}^{E_{s}^{\min }+\Delta_{1}}+\int_{E_{s}^{\min }+\Delta_{1}}^{E_{s}-\Delta_{2}}+\int_{E_{s}-\Delta_{2}}^{E_{s}}\right) d E_{s}^{\prime} \\
& \times F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}^{\prime}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{e l}\left(E_{s}^{\prime}\right), E_{p}, T_{p}\right) \tag{A.21}
\end{align*}
$$

where $\Delta_{1}$ and $\Delta_{2}$ are small empirical energy values. Eq. (A.21) consists of two singular integrals and one regular integral:

- Integral $\int_{E_{s}^{\min }}^{E_{\min }}+\Delta_{1} d E_{s}^{\prime}$ (denoted as $A$ for later discussions): when $E_{s}^{\prime}=E_{s}^{\min }$, $E_{p}^{e l}\left(E_{s}^{\prime}\right)=E_{p}$ makes $I\left(E_{p}^{e l}\left(E_{s}^{\prime}\right), E_{p}, T_{p}\right)$ singular.
- Integral $\int_{E_{s}-\Delta_{2}}^{E_{s}} d E_{s}^{\prime}$ (denoted as $B$ for later discussions): when $E_{s}^{\prime}=E_{s}$, $I\left(E_{s}, E_{s}^{\prime}, T_{s}\right)$ is singular.
- Integral $\int_{E_{s}^{\min }+\Delta_{1}}^{E_{E_{1}}-\Delta_{2}} d E_{s}^{\prime}$ (denoted as $C$ for later discussions): regular 1D integral.

Integral $B$ can be calculated as

$$
\begin{align*}
B= & \frac{1}{\Gamma\left(1+b T_{s}\right)}\left(\frac{\Delta_{2}}{E_{s}}\right)^{b T_{s}}\left[1-\frac{\xi_{b}}{\left(1-b T_{s}\right) \Delta_{2}}\right] \\
& \times \frac{1}{\Gamma\left(1+b T_{p}\right)}\left(\frac{\omega_{p}}{E_{p}}\right)^{b T_{p}}\left[\frac{b T_{p}}{\omega_{p}} \phi\left(\frac{\omega_{p}}{E_{p}}\right)+\frac{\xi_{a}}{\omega_{p}^{2}}\right] \\
& \times F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}\right) . \tag{A.22}
\end{align*}
$$

Integral $A$ can be calculated as

$$
\begin{align*}
A= & \int_{E_{s}^{\min }}^{E_{s}^{\min }+\Delta_{1}} d E_{s}^{\prime} F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}^{\prime}\right) I\left(E_{s}, E_{s}^{\prime}, T_{s}\right) I\left(E_{p}^{e l}\left(E_{s}^{\prime}\right), E_{p}, T_{p}\right) \\
= & \int_{E_{p}}^{E_{p}+\Delta_{1}^{\prime}} d E_{p}^{e l} \frac{d E_{s}^{\prime}}{d E_{p}^{e l}} F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}^{\min }\right) I\left(E_{s}, E_{s}^{\min }, T_{s}\right) I\left(E_{p}^{e l}, E_{p}, T_{p}\right) \\
= & \frac{1}{\Gamma\left(1+b T_{s}\right)}\left(\frac{\omega_{s}}{E_{s}}\right)^{b T_{s}}\left[\frac{b T_{s}}{\omega_{s}} \phi\left(\frac{\omega_{s}}{E_{s}}\right)+\frac{\xi_{b}}{\omega_{s}^{2}}\right] \\
& \times \frac{1}{\Gamma\left(1+b T_{p}\right)}\left(\frac{\Delta_{1}^{\prime}}{E_{p}}\right)^{b T_{p}}\left[1-\frac{\xi_{a}}{\left(1-b T_{p}\right) \Delta_{1}^{\prime}}\right] \\
& \times F\left(Q^{2}\right) \sigma_{\mathrm{el}}\left(E_{s}^{\min }\right)\left[1-E_{p} M^{-1}(1-\cos \theta)\right]^{-2} \tag{A.23}
\end{align*}
$$

The quantities $\omega_{s}, \omega_{p}$ and $\Delta_{1}^{\prime}$ are defined as

$$
\begin{align*}
\omega_{s} & =E_{s}-E_{s}^{\min }  \tag{A.24}\\
\omega_{p} & =E_{p}^{e l}\left(E_{s}\right)-E_{p}  \tag{A.25}\\
\Delta_{1}^{\prime} & =E_{p}^{e l}\left(E_{s}^{\min }+\Delta_{1}\right)-E_{p}^{e l}\left(E_{s}^{\min }\right) \tag{A.26}
\end{align*}
$$

Using Eqs. (A.23) and (A.22), the radiative cross section in Eq. (A.21) can be explicitly expressed, and put in code.

Additional discussions on the formalism of Mo and Tsai

In the tech-notes from previous JLab experiments [135, 136], the cross section of the elastic tail $\sigma_{\text {rad }}^{e l}$ was expressed as

$$
\begin{equation*}
\sigma_{r a d}^{e l}=\left(\sigma_{i n t}^{b}+\sigma_{e x t}^{b}+\sigma_{e x t}^{i o n}\right) F, \tag{A.27}
\end{equation*}
$$

where subscript int (ext) represents internal (external), and superscript $b$ (ion) represents bremsstrahlung (ionization). The first term is the radiative cross section with
the effect of internal bremsstrahlung, evaluated based on the modified Bethe-Heitler formula. The second term is the radiative cross section with the effect of external bremsstrahlung, evaluated with the external path lengths ( $T_{b}$ and $T_{a}$ ). The third term is the radiative cross section with the effect of external ionization, evaluated with the quantity $\xi$ before and after the scattering.

Such an additive form was also presented in Eq. (3.1) of [131], with a comment: when the equivalent radiator method is used, the first term $\left(\sigma_{i n t}^{b}\right)$ should be dropped, and add the equivalent internal path length $\left(T_{r}\right)$ to the external path lengths. Two points are listed below as discussions of Eq. (A.27) [Eq. (3.1) of [131]]

- Tsai carried out the derivation of $\sigma_{i n t}^{b}$ with the energy-peaking approximation, and reached Eq. (C.13) in [131]. Then the result was generalized (without intermediate steps of derivation) to include the external bremsstrahlung and ionization, as in Eq. (3.1) of [131].
- The final form of the combined probability distribution in [131], as in Eq. (A.7), has a similar mystery. While there are extended discussions about the proper way to combine the probability distributions related to bremsstrahlung and ionization in appendix B of [131], the final expression in Eq. (4.1) of [131] simply used the combined probability distribution in Eq. (A.7), which has an additive form. The intermediate steps between Eq. (C.23) (with only the internal bremsstrahlung) and Eq. (4.1) of [131] were also omitted in [131].

Based on the two points above, this additive form, while being concise and convenient for coding, seems to have included implicit approximations in the process of combining the internal and external radiative effects.

In this appendix, using the equivalent radiator method, the expressions for folding the cross sections of the continuous and discrete (elastic) spectra are derived with
a simple approximation in Eq. (A.13). Due to the use of Eq. (A.7), the implicit approximations Tsai used for combining the probability distributions are inherited.

There is also a practice to remove the angle-peaking approximation when integrating the modified Bethe-Heitler formula in [135], which still used the energy-peaking approximation.

Various versions of folding the Born cross section have used different approximations, and their comparisons may be useful when estimating (and even suppressing) the systematic uncertainties.

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## Biography


#### Abstract

Xuefei Yan attended Huazhong University of Science and Technology in 2007, and graduated in 2011 with a B.S. degree in physics. In the fall of 2011, he started the graduate studies at Duke University.


Selected awards and honors

1. Undergraduate: National Scholarship, < 1.4 percent, October 2009.
2. Undergraduate: Top Academic Student Award, $<2$ percent, October 2009.
3. Undergraduate: Scholarship for excellence in sports, October 2009.
4. Undergraduate: first prize, physics experiment competition of college student in Hubei province, 2010.
5. Graduate: Henry W. Newson Graduate Fellowship at Duke University for outstanding contributions to research, September 2014.
6. Graduate: JSA fellowship for graduate student. May 2016.

Publications and contributions

1. Xuefei Yan, et al. Phys. Rev. C. 95, 035209 (2017). "First measurement of unpolarized semi-inclusive deep-inelastic scattering cross sections from a ${ }^{3} \mathrm{He}$ target" was published in PRC on March 24 2017. This analysis used the data from experiment E06-010 at JLab to extract the unpolarized SIDIS differential cross section. I did the detailed work for the cross section analysis (simulation for the experimental acceptance, radiative correction, and updated efficiency and contamination study with higher precision) on top of what had been done in the asymmetry studies. Phenomenological fitting using this stand-alone data was also carried out by me in this study.
2. G. Laskaris, X. Yan, et al. 750, 547-551 Phys.Lett. B (2015). "Measurement of the Doubly-Polarized ${ }^{3} \mathrm{He}(\gamma, \mathrm{n})$ pp Reaction at 16.5 MeV and Its Implications for the GDH Sum Rule": as the second author of the paper, I used my expertise in hardware especially Spin-Exchange-Optical-Pumping of a ${ }^{3} \mathrm{He}$ cell as well as NMR and EPR for the ${ }^{3} \mathrm{He}$ polarization measurements throughout the experiment, taking more than 12 hours shift every day when beam on as I was one of the only two people who could fix hardware problems if anything goes wrong, participated in all hardware setup and data analysis.
3. (Undergraduate) Xuefei Yan, et al. 31(6) PHYSICS EXPERIMENTATION (2011). "Design of fiber optic sensor for nano-scale displacement measurement": As the leader of the project, I successfully applied for a fund from the Ministry of Education of China; with only one other student, I constructed the apparatus, carried out the experiment, wrote the paper, and published it. This project won the first prize in the physics experiment competition of college student in Hubei province in 2010.
4. (Undergraduate) Xuefei Yan, 30(3) College Physics (2011). "Analysis of the
interaction between multiple conductive spheres with electric image iteration": as the only author for this paper, I extended the idea of two-sphere electric image method to one that works for multiple (three or more) spheres (while I was working out problems in Jacksons Electrodynamics for fun) and successfully described the interactions between three conductive spheres.

[^0]:    ${ }^{1}$ A channel number is a non-calibrated reading from a detector.

[^1]:    ${ }^{2}$ A prescale factor is a setting in the DAQ system controlling the ratio between the number of triggers and that of the recorded events. For example, if a prescale factor is set to 100 , only one event will be recorded while there are 100 triggers.

