Few-Body Systems

Covariance Matrix for Helicity Couplings

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Covariance Matrix for Helicity Couplings

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Abstract The helicity couplings at $Q^2 = 0$ for excited baryonic states have been determined in the past, but no information is available regarding their correlations that are relevant for comparison to theory. We present here our calculation of such correlations between the helicity couplings. They contain information for quantitative comparisons with theoretical values, they can be used to quantify the impact of polarization observables, and can help design new experiments.

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1 Introduction

Helicity couplings are the fundamental electromagnetic properties of resonances allowing to test theories and models of excited baryons independently of their hadronic properties. Helicity couplings for excited baryons have been predicted in quark models [1–5], chiral unitary approaches [6–9], and Dyson-Schwinger calculations [10–12], and they start to emerge in lattice QCD simulations [13–15].

New data on polarization observables from polarized targets as FROST [16, 17] or from ELSA [18,19] and MAMI [20,21] provide important constraints on the helicity couplings and lead, in general, to better agreement in the analyses by different groups [22]. The photoproduction of η mesons is a particularly interesting reaction because only isospin $I = 1/2$ excitations of the nucleon

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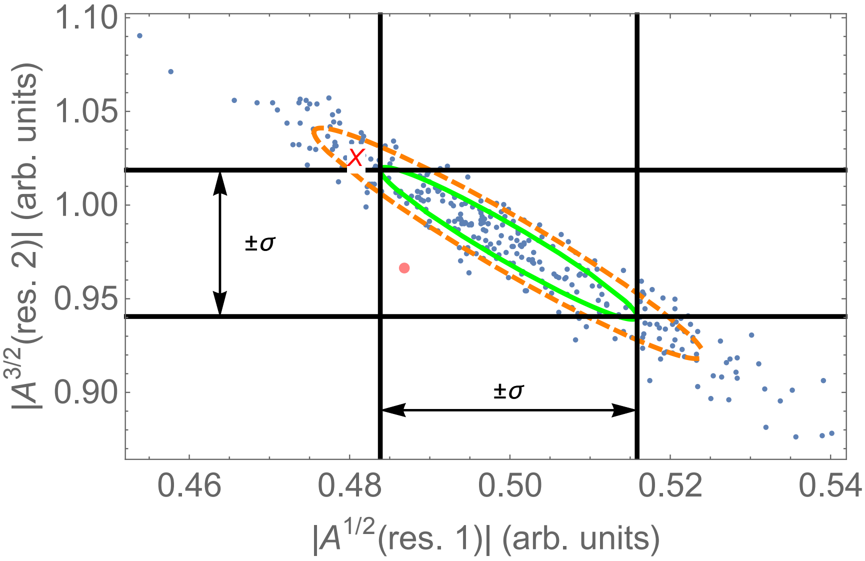

Fig. 1 Illustration for the relevance of correlations, here between $A^{1/2}$ and $A^{3/2}$ of a given resonance (or two different resonances). The black lines indicate the individual uncertainties given by the extensions of the $\Delta\chi^2 = 1$ error ellipse (green solid line); a theory prediction within uncertainties (orange filled circle) can have a very small p-value while another one outside both uncertainties (red cross) can even lie within the 68% confidence region (orange dashed ellipse).

are present, simplifying the determination of the spectrum of resonances and of their electromagnetic properties. Yet, resonances in the light-quark sector are generally broad and overlapping such that the isolation of a resonance remains a challenge.

So far, the analysis of data has usually been carried out by formulating a model for the amplitude and fitting the undetermined coefficients to the data. The relevance of certain partial waves, given certain data sets, has been determined recently [23]. One benefit of that works consists in the determination of participating Legendre coefficients which in principle narrows down the space of partial waves that has to be considered (see also [24]).

Yet, an open question is, how certain data sets affect resonance properties. Obviously, a given new measurement of a (polarization) observable that is included in the joint analysis of data will lead to reduced uncertainties in the helicity couplings. Yet, one needs to establish a figure of merit to quantify this improvement. Helicity couplings are correlated quantities, and a reasonable measure of overall uncertainties is the volume of the error ellipse that can provide such a figure of merit. Establishing this and related quantities is one aim of this study.

Another question is how theory approaches can make statistically meaningful comparisons to helicity couplings extracted from experiment. To provide a trivial motivation, consider the possible scenarios shown in Fig. 1.

A given theory prediction may lie within both parameter uncertainties (orange filled circle inside the black rectangle). However, the true 68% percent confidence region from experimental data is given by the orange dashed ellipse and it becomes clear that the prediction is in fact quite bad. Conversely, a prediction may lie outside both parameter uncertainties (red cross) but still within the 68% confidence region, making it a prediction compatible with data. Determining the correlation coefficients of helicity couplings from data is, thus, as relevant as determining the individual uncertainties.

Other questions to motivate the current study concern the design of new experiments, in particular the question how much the uncertainties decrease through new data, and how well resonances are disentangled through such data. The latter question implies, again, the need to determine the correlations of helicity couplings. By generating pseudo-data that can be freely chosen at different energies, angles, and for different observables, the impact of data on helicity couplings can be simulated which can help in the design of new experiments or experimental analyses.

2 Results

The methodology to extract the correlations of helicity couplings is demonstrated for η photoproduction including a large part of currently available data. For the demonstration we use the Jülich-Bonn approach [25] that provides coupled-channel fits to many final states. Here, it is reduced to the description of the reaction $\gamma p \to \eta p$.

Prominent 4-star resonances contained in the model are the $N(1535)1/2^-$, $N(1650)1/2^-$, $N(1710)1/2^+$, $N(1720)3/2^+$, $N(1520)3/2^-$, $N(1675)5/2^-$, and $N(1680)5/2^+$. The uncertainties and correlations of the helicity couplings of these states are calculated. There are other resonances in the model which are less prominent and which are, therefore, not considered in this study. For the helicity couplings at the pole we follow the definition of Ref. [26] which is the same as, e.g., in Ref. [25].

There are 11 helicity couplings that we are interested in, each one with a modulus and a phase, giving rise to 22 parameters. No information has yet been published regarding the correlation of these helicity couplings. The 22 uncertainties of these couplings have been extracted from experiment before, but the 231 correlations between them have not.

To determine the covariance the following steps were followed: 1) The existing multipole solution of the Jülich-Bonn model [17] was slightly refitted: the global minimum shifts because, here, only the η photoproduction data are considered out of the many final states included in the global fit. 2) The covariance matrix was determined from the Hessian. The latter can be estimated from the changes of the χ^2 as a function of the helicity couplings, numerically achieved through small changes in the helicity couplings. 3) It should be noted that we also vary other parameters, because the covariance of helicity couplings is in general only a sub-matrix in the (infinite-dimensional) space of the coeffients of the Laurant expansions of the multipoles. In practical calculations one can only vary a finite number of parameters which was restricted to about 100 in the present study.

2.1 Correlation matrices

Following the outline in the Introduction, several results have been presented in the talk. We highlight here only one outcome. In Fig. 2 we compare 6 different correlation matrices, each calculated from a different data set following the steps of the previous section. In particular, from the available η photoproduction data we leave out one observable at a time to observe the changes in the correlation, i.e., the impact of that observable, given the current data situation.

The observables included are: Case 1: $d\sigma/d\Omega$, Σ, E, F, T. Case 2: $d\sigma/d\Omega$, Σ, F, T. Case 3: $d\sigma/d\Omega$, Σ, E, T. Case 4: $d\sigma/d\Omega$, Σ, E, F. Case 5: $d\sigma/d\Omega$, E, F, T. Case 6: $d\sigma/d\Omega$, Σ, E, F, T, C_x, C_z. Case 1 includes data for all measured observables. Cases 2-5 each leave out the data from one polarization observable to isolate its impact. The sixth case is noteworthy because it contains synthetic data (C_x and C_z) that have not actually been measured but have been generated from the solution with comparable accuracy and angular coverage as the existing measurements of T and F. Their impact can be determined in the same way as we would determine the impact of other polarization observables and this information can help in the design of future experiments.

The entries of the correlation matrices in Fig. 2 with a darker square show stronger absolute correlations while lighter squares indicate weaker correlations. The diagonal elements are all dark because each element of the matrix correlates perfectly with itself. The off diagonal elements give us information about the correlations between different couplings.

It is not surprising that there are a lot of dark squares in the upper left corner of each matrix. The first two helicity couplings have the same quantum numbers and correspond to the two S-wave resonances that are close in energy. Thus, their values are more likely to be correlated than helicity couplings from resonances with different quantum numbers or very different masses (in relation to their widths). One example of the differences between the cases is that in case 5, we can see a correlation between the amplitude of $|A^{3/2}|\, N(1680)5/2^+$, and the phase of $|A^{3/2}|\, N(1675)5/2^-$. We do not see this correlation in the other cases meaning that the inclusion of the beam asymmetry Σ data reduces that correlation. The Σ data has the most visible impact because it is the largest of the polarization observable data sets. Its size was recently considerably increased by the addition of data published in [16].

Note that in the first five cases, there is a correlation between $|A^{1/2}|$ of the $N(1675)5/2^-$ resonance and $|A^{1/2}|$ of the $N(1520)3/2^-$ resonance. This correlation is not visible in the 6th case, the case where we include the syn-

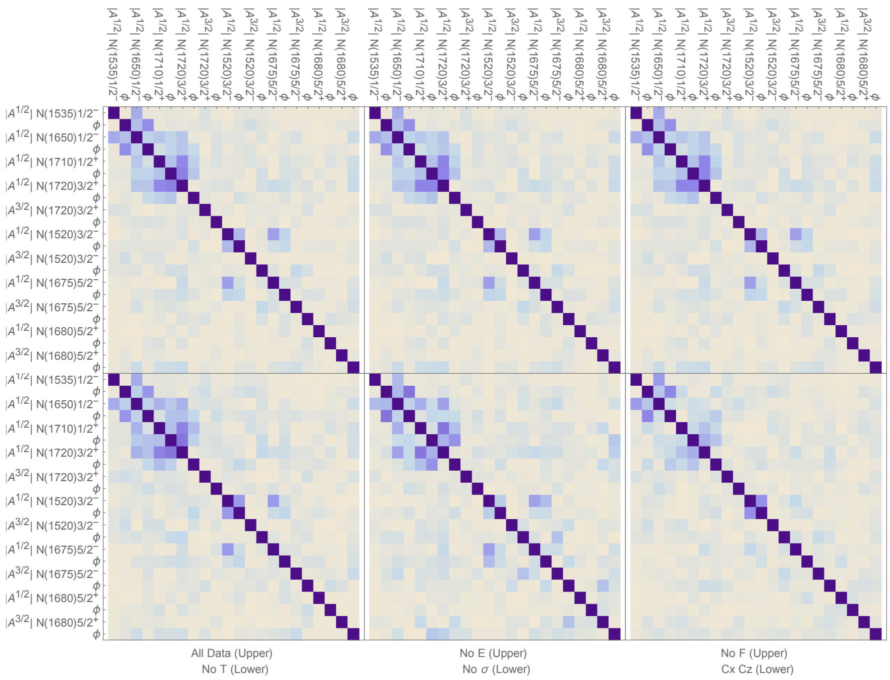

Fig. 2 Correlation matrices for the 6 defined cases. The rows and columns show the absolute values and phases of the helicity couplings at the pole following the convention of Ref. [27]. The absolute value of the correlationis color coded - the darker the square, the larger the correlation.

thetic C_x and C_z data, meaning that a measurement of such data would likely disentangle this correlation in the helicity couplings.

3 Conclusion

Determining the correlations of helicity couplings allows the quantification of the impact of polarization observables for precision and correlations of resonance properties. Furthermore, this method can be extended to give information on the impact of observables that have not yet been measured which can help with the design of future experiments. These impacts can be observed in the shown correlation matrices but they can also be seen in certain bulk properties of these matrices, like the generalized variance. We intend to publish these bulk properties and a more detailed description of our analysis shortly.

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