## On anisotropy of the maximum attainable speed of low-mass particles

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We show that for a non-zero photon mass,  $m_{ph}$ , the speed of light anisotropy,  $\delta c/c$ , is below  $10^{-37}$  based on an accepted bound on  $m_{ph}$ . A strong bound was also obtained in the case of the neutrino.

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The theory of Special Relativity (SR) is based on a postulate of isotropy of the speed of light, which for the round-trip case was first tested about 130 years ago [1] before SR was proposed [2] and has now been confirmed in an oscillator experiment [3] to the level of  $\delta c_2/c_2 < 10^{-18}$  and also with high accuracy ( $\sim 10^{-14}$ ) for the oneway case in the Compton laser back-scattering and the asymmetric optical ring cavity experiments [4, 5]. The photon mass, which is zero according to SR, currently has an experimental upper limit of  $m_{ph} < 10^{-18}$  eV, see the reviews [6, 7]. The reviews for the speed of light anisotropy and bounds are in Refs. [8–10].

In this letter we explore a connection between two bounds: one for the photon mass and another for the Anisotropy of the one-way Maximum Attainable Speed of a photon (AMAS). We extend a concept suggested for the investigation of the one-way maximum attainable speed of an electron [11] to the case of a photon (assuming non-zero photon mass).

One motivation for non-zero anisotropy of the maximum attainable speed comes from the theory of quantum gravity. The energy-momentum relationship of SR modified by quantum gravity has extra terms on the Planck energy scale [12, 13]:

$$E^2 = m^2 c^4 + p^2 c^2 + E_{Pl} \cdot (f_i^{(1)} \cdot p^i) c + \dots, \quad (1)$$

where E, m, and p are the energy of the particle, its mass, and momentum, respectively,  $E_{Pl}$  is the Planck energy, and f is a dimensionless constant. Within a process which conserves the energy of the particle, the absolute value of its momentum depends on the direction of particle motion.

Measurement of the momentum variation between different directions of the photon motion for a fixed photon energy allows us to put a limit on the constant f. The particle momentum, according to quantum mechanics, relates to the particle wavelength,  $\lambda$ , as  $p = h/\lambda$ , so the measurement of the wavelength allows determination of the photon momentum.

Considering the photon emission by an atom which is fixed in the lab frame, one can see that the atom energy level structure and resulting photon energy are independent of the direction of the photon emission. At the same time, the momentum of the emitted photon could have directional dependence. The first consequence of the momentum variation is an anisotropy of the photon emission probability due to the phase space factor. Indeed, the probability of the photon emission is:

$$dw = 2\pi |V_{fi}|^2 \delta(E_i - E_f - \hbar\omega) \frac{d\vec{p}}{(2\pi)^3},$$
 (2)

where  $V_{fi}$  is a matrix element of transition,  $E_i(E_f)$  and  $\omega$  are the energies of the initial (final) atomic states and the photon, and  $d\vec{p}$  is an element of the photon phase space.

For the unpolarized atom, in the absence of a magnetic field and an isotropic speed of light, the photon emission is perfectly isotropic. However, a non-zero AMAS value leads to emission anisotropy due to the phase space factor. The sensitivity of the photon momentum to the variation of a difference between the photon speed, v, and the one-way maximum attainable speed,  $c_1$ , in the direction of the photon momentum is very high due to a huge value of the Lorentz factor for the optical photon. The speed of the photon in the atom's rest frame is independent of the emission time and rotation of the Earth. As a result, only the  $c_1$  variation could be responsible for the momentum anisotropy and:

$$\left[\delta c_1/c_1\right] \approx \left[\delta p/p\right]/\gamma^2,\tag{3}$$

If the AMAS is close to the value of  $1/\gamma^2$ , the emission anisotropy becomes very large, which contradicts the common knowledge that unpolarized atoms radiate isotropically in the absence of a magnetic field and allows us to put a bound on the value of the anisotropy of the one-way maximum attainable speed of a photon. Using eq. 3 and  $\delta p/p = 1$  for the optical photon (energy  $\sim 3 \text{ eV}$ ,  $E_{ph}/m_{ph} > 3 \times 10^{18}$ ), we have  $\delta c_1/c_1 < 10^{-37}$ .

The bound on  $\delta c_1/c_1$  could also be obtained from the observation that the optical photons emitted by the atoms (e.g. He) have about the same wavelength independently of the direction of the photon emission.

In the configuration shown in Fig. 1, the experimental setup includes a source of mono-energetic photons from atomic transition and a photon wavelength meter based on diffraction. Measurement over a long ( $\sim$ 24-hour) period or, alternatively, rotation of the apparatus will allow

us to find an even stronger bound on the directional variation of the wavelength or photon momentum. For the compact apparatus needed in the photon case, a rotationing table could be used and the bound on all components of  $\delta c_1$  could be obtained.

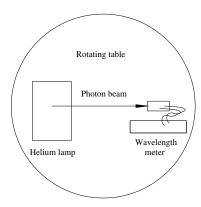


FIG. 1. The layout of the simplest experiment for the maximum attainable speed isotropy.

The photon mass, an active field by itself, was recently introduced as a possible contributor to the forces responsible for the observed "galaxy rotation curves" whose origin is usually attributed to dark matter [14]. In such a case, the estimated photon mass is  $m_{ph} \sim 10^{-23}$  eV. The corresponding bound on the one-way speed of light anisotropy is  $\delta c_1/c_1 < 10^{-47}$ .

Obviously, the bound on the one-way speed also leads to a similar stringent limit on the round-trip (two-way) speed of light anisotropy.

The considerations above were formulated for a non-zero photon mass. It is also natural to consider a transition to a smaller photon mass value. In the limit  $m_{ph} \to 0$ , the bound  $\delta c_1/c_1 \to 0$  and the conclusion is that the speed of light is absolutely isotropic for the massless photon.

The logic of the current analysis could be used for the neutrino whose mass is non-zero and estimated to be as small as 0.1 eV [7]. As a result, the 10 GeV neutrino has a Lorentz factor of  $10^{11}$ . In neutrino scattering experiments, the beam of neutrinos is typically produced in the decay of high-energy pions. Pion production is independent of the neutrino AMAS, but the pion decay angular distribution is sensitive to the anisotropy of the maximum attainable speed of the neutrino. Recent measurement of the  $\nu-n$  cross section reaches a level of 5% for absolute accuracy [15], which indicates the absence

of a strong ( $\sim 1$ ) daily variation. This leads to a tight limit:  $\delta c_{\nu}/c_{\nu} < 10^{-22}$ . Analysis of the actual data for a sidereal time effect should allow further improvement.

In summary: A stringent limit on the anisotropy of the one-way maximum attainable speed of light is found by using an analysis of the impact of  $c_{ph}$  variation on the photon emission intensity and wavelength for different directions of propagation. The anisotropy  $\delta c_{ph}/c_{ph}$  for the photon is below  $10^{-37}$ .

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