DOUBLE SPIN ASYMMETRY IN $\vec{d}(\vec{e}, e'p)n$

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ABSTRACT

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Using the CLAS detector at Jefferson Lab, double spin asymmetries ($A_{||}$) for quasi-elastic electron scattering off the deuteron have been measured at several beam energies. The data were collected during the EG1 experiment, which scattered longitudinally polarized electrons with energies from 1.6 to 5.8 GeV off a longitudinally polarized cryogenic ND₃ target. The double spin asymmetries were measured as a function of photon virtuality Q^2 (0.13–3.17 (GeV/c)²), missing momentum (0.0–0.5 GeV/c), and the angle between the (inferred) “spectator” neutron and the momentum transfer direction (θ_{nq}). The results from EG1b are compared with a recent model that includes final state interactions using a complete parameterization of nucleon-nucleon scattering as well as a simplified model using the plane wave impulse approximation. From this comparison, it is concluded that the inclusion of final-state interactions are needed at high missing momenta, while the PWIA description works rather well at lower missing momenta. The asymmetry changes sign as a function of the missing momentum, as expected within PWIA from the presence of a D -state component in the deuteron wavefunction.

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CHAPTER 1

INTRODUCTION

The double spin asymmetry $A_{||}$ for the $\vec{d}(\vec{e}, e'p)n$ reaction in the quasi-elastic region provides a wealth of information. This dissertation presents the methods used to extract the double spin asymmetry from the EG1b data set collected at the Thomas Jefferson National Accelerator Facility. The results of this measurement are used to test state-of-the-art theoretical descriptions of the deuteron structure and of the reaction mechanism including final-state interactions (FSIs).

The first part of this dissertation gives a formal introduction to the physics needed to understand $A_{||}$. Comparative data from other $\vec{d}(\vec{e}, e'p)n$ experiments are referenced. A complete description of the facility used in data collection is discussed. All relevant runs, files, cuts, and corrections used in data selection are given and explained. The procedure used to calculate the final asymmetries, accounting for background and radiative corrections is described. The dissertation concludes with a discussion of the results.

1.1 BRIEF HISTORY OF NUCLEAR PHYSICS

In the late 19th century, the field of nuclear physics started to emerge. In 1896, Henri Becquerel discovered radioactivity by observing that uranium salts darkened photographic plates. Unknown to Becquerel at the time, this form of radiation involved the decay of nuclei and was unlike the recently discovered X-ray, which is a form of electromagnetic radiation. At the time, the atom was believed to be the most fundamental building block of nature, and this discovery hinted at the existence of the nucleus. It took another three decades to discover all the elements that constitute the atom.

In 1897, Joseph John Thomson was able to deflect a stream of negatively charged particles in a cathode ray tube using electric and magnetic fields[1]. Using the radius of the circular path of the particles, J. J. Thomson was able to determine the charge to mass (q/m) ratio of these particles. If the charge of these particles was the discrete

electric charge called an “electron,”¹ then the mass of these particles would be less than the mass of the hydrogen atom. Although the q/m ratio for the electron was thus known, it was not until 1908 that the charge of the electron was measured by Robert Millikan[2]. He determined the charge of the electron as $1.592 \cdot 10^{-19}$ C, which is close to the accepted value of $1.602 \cdot 10^{-19}$ C. Using the measured value, the mass of the electron could be calculated. The accepted value of the mass of the electron is $0.510 \text{ MeV}/c^2$.

Thereafter, the nucleus and the proton were discovered by Ernest Rutherford and his students H.W. Geiger and E. Marsden by scattering α -particles off gold foil in 1909[3]. The results of this experiment were interpreted two years later by Rutherford[4] with the conclusion that at the center of each atom there was a concentration of positively charged particles with a relatively large mass which produced the deflections observed in the experiment. The center of the atom is called the nucleus, whereas the positively charged particles are called protons. The mass of the proton is $938.27 \text{ MeV}/c^2$, and the charge is equal in magnitude but opposite in charge to the electron.

The final unknown particle of the atom and nucleus, the neutron, was not discovered until 1932. A former student of Rutherford’s and Geiger’s, James Chadwick, investigated why the atomic number (the number of protons in an atom) was less than the atomic mass. Since the electron has very little mass relative to the proton, it was postulated that the nucleus contained extra protons and electrons to cancel out the charge but retain the mass. Chadwick observed the neutral radiation produced from beryllium when it was bombarded with α -particles. Based on these observations, he concluded that this radiation consisted of particles very close in mass of the proton, yet neutral in charge (i.e., the “neutron”)[5]. The mass of the neutron is $939.56 \text{ MeV}/c^2$ which is approximately 0.1% heavier than the proton. The existence of the neutron helped explain the presence of isotopes and later allowed Werner Heisenberg to construct a model of the nucleus consisting of the protons and neutrons[6]. The term “nucleon” is a collective name for the proton and the neutron. With this discovery, all the constituent particles were accounted for in the atom, thus giving rise to the study of the behavior of the nucleus and the interactions of the nucleons.

¹Michael Faraday determined there is a minimum electric charge. This was later dubbed an electron by George Stoney, who calculated this value to be 10^{-20} C.

Even after all the particles of the atom were known, describing the nuclear structure was difficult. The nucleus of an atom is extremely small relative to the entire size of the atom.² Dealing with particles of this size requires the use of Quantum Mechanics to describe the structure and interactions of matter. Using Albert Einstein's concept that a photon is both a particle and a wave, Louis de Broglie proposed that all matter can be treated with the same wave-particle duality (this concept will be further discussed in the next section). Independently, Werner Heisenberg and Erwin Schrödinger developed different but equivalent theories to mathematically describe the electron and the atom.³ The works of many physicists including Paul Dirac and Wolfgang Pauli helped develop the field of Quantum Mechanics.

Using Quantum Mechanics, the stable orbits of electrons surrounding the nucleus can be described by four quantum numbers. The principle quantum number, n , gives the energy level of the electron, and is a nonzero integer ($n=1,2,3,\dots$). The orbital angular momentum of the electron, ℓ , is also an integral number ($\ell=0,1,2,\dots,n-1$). The magnetic quantum number, m , has a range from $-\ell, -\ell+1, \dots, +\ell$. The last quantum number is the projection of the electron spin, m_s , and can be $+\frac{1}{2}$ or $-\frac{1}{2}$ for a single electron.⁴ Since electrons are fermions, they must obey the Pauli exclusion principle, which states that no two identical fermions can have the same quantum numbers. This implies that for each energy level n only $2n^2$ electrons can occupy that region or "shell." The shells are filled from the lowest energy to the highest energy. The outermost shell is called the valence shell and is responsible for characteristics of the atom.

Describing the nucleus that the electrons surround is even more difficult than modeling the electrons. A widely used model describing the properties of nuclei and their internal motion is the Nuclear Shell Model developed by Maria Goeppert-Mayer and Hans Jensen[7]. Protons and neutrons are also fermions and obey the Pauli exclusion principle. Analogous to the treatment of electrons in shells, the nucleons also fill shells based on quantum numbers. When a shell has become full of nucleons, an especially stable nucleus is formed. For the most part, the nuclear properties such as angular momentum, magnetic moment, and shape are also determined by the last

²The size of an atom is of the order of 10^{-10} m, while the nucleus is of the order 10^{-14} m.

³Heisenberg developed Matrix Mechanics to describe frequencies and intensities of hydrogen. Schrödinger developed Wave Mechanics by treating an electron as a wave and was able to calculate energy levels of hydrogen.

⁴The electron is a fermion and therefore has spin $\frac{1}{2}$.

valence level.

Although the Shell Model accurately predicts measured values, the interaction of the nucleons with each other is still not well understood from first principles. The hydrogen atom with only one proton can be completely described using the Schrödinger equation by using the Coulomb force as the potential. However, the simplest system of two nucleons, the deuteron, is not so easily understood, since the interaction potential between two nucleons is very complicated. The interaction binding the nucleons is called the strong interaction and is independent of the charge of the nucleon. The strong interaction is a short-ranged and saturated force, meaning individual nucleons only interact with a limited number of other nucleons. One of the first theories to describe this force was suggested by H. Yukawa in 1935 through the exchange of an undiscovered particle called a meson[8]. This particle was discovered twenty years later and was named the π meson. At low energies, the strong interactions are instantaneous and a potential must be used to describe the nucleon-nucleon interaction. Using data from the study of the deuteron and from nucleon-nucleon scattering experiments, a phenomenological approach can be taken to modeling the nucleon-nucleon interaction. By studying the simplest nuclei and their interactions with scattering experiments, more information about the nucleon-nucleon force and nuclear structure can be obtained to develop better theories.

CHAPTER 2

PHYSICS OVERVIEW

2.1 ELECTRON SCATTERING

One of the fundamental principles in Quantum Mechanics was proposed in 1924 by a graduate student named Louis de Broglie. He proposed that the wave-particle dualism of photons suggested by Einstein is a characteristic also of matter including electrons[9]. At the time, the electron was assumed to have a point-like structure and to be a fundamental particle. The hypothesis of de Broglie was that that electron traveled in waves with a wavelength of

$$\lambda = \frac{h}{p} \quad (1)$$

where p is the momentum of the electron and h is Planck's constant. Therefore, the higher the momentum the electron obtained, the smaller the wavelength. Nuclear Physics exploits this property of matter to use electrons (and other particles) to probe the nucleus and the nucleons. In the 1950s, Robert Hofstadter and collaborators used high energy electrons (200-500 MeV) to study the structure of nuclei and nucleons[10] by scattering them off the nucleus and nucleons.

The electron's interaction with a nucleus or nucleon is well understood in Quantum Electrodynamics (QED) as mediated by the exchange of a virtual photon.¹ According to QED, one photon exchange (OPE) or Born approximation is favored because the transition amplitude is proportional to the fine structure constant ($\alpha \approx 1/137$) while higher order processes contribute additional powers of α . The virtual photon's energy is given by

$$\nu = E - E' \quad (3)$$

¹A virtual particle does not obey the energy-momentum equation of

$$E^2 - |\vec{p}|^2 c^2 = m^2 c^2, \quad (2)$$

where E is the energy of the particle, $\vec{p}$ is the momentum of the particle, m is the mass of the particle, and c is the speed of light. For the photon, $m = 0$ and $E \leq pc$. Virtual photons have $p \geq pc$.

where E and E' are the energy of the electron in the initial and final state, respectively. (Final state properties of an unchanged particle are denoted by a prime.) The photon's momentum is given as

$$\vec{q} = \vec{k} - \vec{k}' \quad (4)$$

where $\vec{k}$ and $\vec{k}'$ are the momentum of the electron in the initial and final state, respectively. The virtuality of the photon is expressed in the quantity

$$Q^2 = q^2 c^2 - \nu^2 > 0. \quad (5)$$

Electron scattering can be classified into several different categories. There are two main types of scattering based on the final state: elastic and inelastic scattering. In elastic scattering, the target nucleus or nucleon remains intact. A related category of scattering, called quasi-elastic scattering, is when the electron scatters off an individual nucleon in the nucleus and the struck nucleon emerges intact. In the simplest approximation (called Plane Wave Impulse Approximation), the scattered nucleon is knocked out of the nucleus without interacting with another nucleon. Finally, inelastic scattering is when the nucleon is fractured into additional particles. This fracturing occurs when additional energy (beyond recoil energy) is transferred to the hadron.

2.2 THE DEUTERON

Hydrogen is the simplest and most common element found in the universe. The most common type of hydrogen contains one proton and no neutrons and accounts for over 99% of all hydrogen. The other stable² isotope of hydrogen contains only a proton and a neutron in its nucleus and accounts for 0.015%³ of hydrogen. This form of hydrogen with two nucleons is called deuterium and its nucleus is called the deuteron.

The deuteron is the only stable two-body nucleus. As stated previously, the nucleon interaction potential cannot yet be derived from first principles (QCD) and a phenomenological approach is usually taken modeling the strong nuclear force. The deuteron provides the ideal environment to study nucleon-nucleon interactions and provides a testing ground to compare theoretical approaches to experiment.

²A third form of hydrogen exists called tritium with one proton and two neutrons, but the nucleus is unstable and has a half-life of ~ 12 years.

³For every 6700 ^1H isotopes there is one ^2H (isotopic abundance in terrestrial H, e.g., H_2O).

2.2.1 PROPERTIES OF THE DEUTERON

Mass

The mass of the deuteron (M_d) is known to be 1875.61 MeV[11]. This mass is less than the sum of the mass of its constituent nucleons. This difference in mass can be attributed to the binding energy, which is the amount of energy needed to separate the constituent nucleons or amount of energy released (in the form of γ -radiation) when combining the nucleons. For the deuteron, the binding energy is 2.2246 MeV[12]. This amount of energy is far less than the average binding energy per nucleon for other stable nuclei. Additionally, the deuteron does not have any excited states. This means that any measurement made on deuterium is a ground state measurement.

Magnetic Moment

The sum of the magnetic moments of the neutron and the proton also do not equal the measured magnetic dipole moment of the deuteron. The first measurement of the magnetic dipole moment was performed by Isidor Rabi et al. in 1934[13]. This was performed by looking at the magnetic deflection of an atomic deuteron beam in an inhomogeneous magnetic field. The methods used initially by Rabi have been greatly improved and a more precise measurement has been made for the magnetic dipole. In general, the magnetic moment for the deuteron can be calculated as

$$\mu = g(j, \ell, s)\mathbf{j} \quad (6)$$

where the gyromagnetic ratio for an eigenstate of spin, s , and orbital angular momentum, ℓ , is[14]

$$g(j, \ell, s) = \frac{1}{2j(j+1)} \{ (g^\ell + g^s)j(j+1) + (g^\ell - g^s)[\ell(\ell+1) - s(s+1)] \}, \quad (7)$$

where j is the total angular momentum.

For the simple case of $\ell=0$, the magnetic moment would be the sum of the magnetic moments of the nucleons. This does not appear to be exactly the case as

$$\mu_d = 0.857393\mu_N \neq \mu_p + \mu_n = 0.8797\mu_N, \quad (8)$$

where $\mu_N = 3.152 \cdot 10^{-8} eV/T$ is the nuclear magneton. The same calculation can be done for different values of ℓ and no single orbital angular momentum eigenstate matches the experimental value, hinting at a multi-state wavefunction.

Electric Quadrupole

An electric quadrupole moment can also be observed with a measured value of $Q_d=0.2859 \text{ e}\cdot\text{fm}^2$ [15]. The quadrupole moment was discovered by Rabi while observing resonance absorptions to determine magnetic moments[16]. Since the deuteron has an electric quadrupole moment, the charge distribution is deformed from a spherically symmetric distribution which also is incompatible with a pure $\ell = 0$ state. The discovery of the quadrupole moment also suggests that the force acting on the two nucleons is not a purely central force.⁴ The quadrupole moment can be defined using the wavefunctions by

$$Q_d = \frac{1}{e} \sum_i \int \psi_i^*(\vec{x}) q_i (3z_i^2 - r_i^2) \psi_i(\vec{x}) d^3\mathbf{x}, \quad (9)$$

where ψ_i is the wavefunction for nucleons with charge q_i .

Angular Momentum

Around the same time the magnetic dipole was measured, the total angular momentum of the deuteron was determined to be $J = 1$. This was originally determined using photographic photometry[17], but can also be determined using radio frequency and microwave methods. The ground state spin of the deuteron is the sum of the intrinsic spins of the nucleons ($\vec{S}$) and the orbital angular momentum ($\vec{L}$). Since $\vec{J} = \vec{L} + \vec{S}$, the possible solutions for the orbital angular momentum would be either $L=0$ or $L=2$.⁵ Looking at the $L=0$ state, this would imply that $S=1$. This is known as the triplet state (3S_1). Since the spin of the proton and neutron is $\frac{1}{2}$ and the spin of the deuteron is 1, this implies that the spins of the nucleons are parallel. No deuteron has been found where $J=0$, so the singlet state (1S_0) of the deuteron appears not to exist. This has implications that will be explained later in this dissertation. For $L = 2$, the nomenclature is 3D_1 . The results quoted above indicate that the deuteron wavefunction is a mixture of 3S_1 and 3D_1 states (in the non-relativistic approximation).

⁴If the force were central, the deuteron would be spherical and no quadrupole moment would have been found.

⁵ $L=1$ is excluded in non-relativistic models because of parity conservation. From scattering experiments and by looking at the photon produced when the deuteron is created, the parity of the deuteron is even or positive $\Pi=+1$.

2.2.2 DEUTERON WAVEFUNCTION

The presence of the quadrupole moment combined with the discrepancy of the magnetic moment hints at the deuteron wavefunction not being a pure state. The deuteron wavefunction can be written as a mixture of the (3S_1) and (3D_1) states as

$$|\psi_d\rangle = C_S|\psi_S\rangle + C_D|\psi_D\rangle \quad (10)$$

where $|C_S|^2$ and $|C_D|^2$ are often referred to as the S - or D -state. (These probabilities are not observables; however, typical theoretical wavefunctions yield values for $P_D = |C_D|^2$ between 0.4 and 0.6.) The wavefunction can then be written in radial form given by

$$\psi_d(\vec{r}, M) = \frac{u(r)}{r} \mathcal{Y}_{101}^M(\theta, \phi) + \frac{w(r)}{r} \mathcal{Y}_{121}^M(\theta, \phi), \quad (11)$$

where $u(r)$ and $w(r)$ are the S and D waves and

$$\mathcal{Y}_{JLS}^M(\theta, \phi) = \sum_{m_L, m_S} \langle JM|L, m_L; S, m_S\rangle Y_{LM}(\theta, \phi) |S, m_S\rangle, \quad (12)$$

with spherical harmonics Y_{LM} and Clebsh-Gordon coefficients $\langle JM|L, m_L; S, m_S\rangle$.

The radial shape and normalization of $u(r)$ and $w(r)$ can be derived from Schrödinger's equation given the nucleon-nucleon potential. While the nucleon-nucleon potential is not completely known, it can instead be parameterized (within a model) to describe nucleon-nucleon scattering and the deuteron properties. One such model, the Argonne v₁₈, can be used to look at the radial wavefunctions $u(r)$ and $w(r)$, see Figs. 13 and 2.

Similar to the plots of the wavefunction density, a three-dimensional plot of the surface of constant density of the deuteron can be constructed as illustrated in Fig. 3.

From the deuteron wavefunction, a matter radius can be obtained. This is the average matter size of the deuteron and is related to the deuteron wavefunction by

$$r_m^2 = \frac{1}{4} \int_0^\infty [u^2(r) + w^2(r)] r^2 dr. \quad (13)$$

Another radius, the charge radius (r_{ch}), can be related to the matter radius by

$$r_{ch}^2 = r_m^2 + \Delta r_m^2 + r_p^2 + r_n^2 + \frac{3}{4} \left(\frac{\hbar}{m_p} \right)^2 \quad (14)$$

where the proton charge rms-radius is $r_p=0.862(12)$ fm and the neutron charge rms-radius is $r_n^2=-0.113(5)$ fm². The contribution from the non-nucleonic degrees of

FIG. 1: The reduced deuteron radial wavefunctions $u(r)$ (solid) and $w(r)$ (dashed) using the Argonne v_{18} potential.

FIG. 2: The deuteron radial wavefunction in radial space, $u(r)/r$ (left), and momentum space, $pu(p)$ (right), using the Argonne v_{18} potential.

freedom (Δr_m) is very small and can be considered zero, and the mass of the proton is $m_p=938.272309(28)$ MeV.

Several observables can be calculated from the deuteron wavefunction, but the wavefunction and the probabilities associated with each state are not direct observables. Instead, observables that can be measured are used to try to gain some insight into the deuteron wavefunction and form more accurate theoretical descriptions of the deuteron wavefunction. The $A_{||}$ measured in this analysis is one such observable

FIG. 3: Plots of the density surface with a density of $0.24 \text{ nucleons/fm}^3$ for two values of M_J (magnetic substate)[18]. These plots show the toroidal shape and dumbbell shape of the deuteron, with a repulsive core for the nucleon-nucleon potential.

and is sensitive to the D -state part of the wavefunction. This is due to the fact that $A_{||}$ for quasi-elastic scattering, in PWIA, depends on the spin orientation of the struck nucleon, which is encoded in the m_s -dependence of the wavefunction Eqs. (11)–(12).

Several models of the deuteron wavefunction as described above use a non-relativistic wavefunction, including relativistic corrections to predict observables. With the use of high energy accelerators, a relativistic description of the deuteron wavefunction was needed describe the $\vec{d}(\vec{e}, e'p)n$ reaction at GeV energies. A fully relativistic model of the deuteron ground state, based on the spectator equation, has been developed by Gross and Van Orden[19]. This model has been combined with a description of the reaction mechanism, including final state interactions (FSI), by Sabine Jeschonnek and J.W. Van Orden[20]. This new model is described later in this dissertation.

2.3 KINEMATICS OF DEUTERON ELECTRODISINTERGRATION

In this dissertation, the extraction of the double spin asymmetry $A_{||}$ from the EG1b data sets in the exclusive quasi-elastic reaction $\vec{d}(\vec{e}, e'p)n$ is described. A graphical representation of the geometry for this reaction in the laboratory frame can be seen in Figure 4. An incoming polarized electron scatters off a longitudinally polarized deuteron at an angle θ . The deuteron breaks apart into its constituent nucleons, a proton and a neutron. The four-momentum of the incoming and scattered

FIG. 4: Kinematic plot of the $\vec{d}(\vec{e}, e'p)n$ reaction

electron are $k_\mu = (\vec{k}, E)$ and $k'_\mu = (\vec{k}', E')$ respectively. The four-momentum of the virtual photon is defined as

$$q_\mu \equiv k_\mu - k'_\mu, \quad (15)$$

where $q_\mu = (\vec{q}, \nu)$. Since the electron mass is much smaller than the electron energy, the Ultra-Relativistic approximation is used to neglect the mass of the electron. Natural units of $\hbar = c = 1$ are also used. In this picture, the one-photon exchange approximation is assumed where the virtual photon is absorbed by the target deuteron. The deuteron is initially at rest and its four-momentum can be expressed as

$$(P_T)_\mu = (\mathbf{0}, M_d), \quad (16)$$

where M_d is the known mass of the deuteron of 1.876 GeV. The final-state four-momentum of the system $(P')_\mu$ is expressed at the vertex using conservation principles as

$$(P')_\mu = q_\mu + (P_T)_\mu. \quad (17)$$

The scattered proton's four-momentum $(p'_p)_\mu$ and neutron's four-momentum $(p'_n)_\mu$ can be related to that of the system by

$$(P')_\mu = (p'_p)_\mu + (p'_n)_\mu. \quad (18)$$

Measuring p'_p allows us to infer information such as the missing (unobserved) momentum p_m of the system via

$$p_m = |\vec{p}_n| = |\vec{q} - \vec{p}_p|. \quad (19)$$

The virtuality of the photon (transferred four-momentum squared) is

$$Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}. \quad (20)$$

Finally, following the geometry from Fig. 4, the angle between the virtual photon and the neutron is defined as

$$\cos \theta_{nq} = \frac{\vec{p}'_n \cdot \vec{q}}{|\vec{q}| |\vec{p}'_n|}. \quad (21)$$

The double spin asymmetry can be defined as a function of these kinematic variables:

$$A_{||}(p_m, Q^2, \cos \theta_{nq}) = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}, \quad (22)$$

where σ is the cross section with arrows, respectively, indicating the relative spin orientations of the electron from the beam and the deuteron nucleus, relative to the beam direction.

2.4 THEORY AND PREDICTIONS

The simplest model for quasi-elastic deuteron breakup, the plane wave impulse approximation (PWIA), assumes that the virtual photon is absorbed by a single nucleon (impulse approximation) and the struck nucleon leaves the nucleus without further interaction (i.e., as a plane wave). In this model, the measured asymmetry is directly proportional to the initial polarization of the struck nucleon and can therefore

be used to extract information on the spin and momentum dependence of the nuclear wavefunctions. This, of course, is only an approximation; a more realistic description requires a model that includes the interaction between the spectator and knocked-out nucleon (FSI). One goal of this analysis is to determine the kinematic region where PWIA is no longer valid.

A comprehensive theoretical model formulated by Sabine Jeschonnek and J.W. Van Orden [21] has been developed for this purpose. This model has already been used successfully to predict values of the fifth structure function of the deuteron in the reaction $\vec{d}(\vec{e}, e'p)n$ [22]. The $\vec{d}(\vec{e}, e'p)n$ reaction was modeled by the authors to learn more about its sensitivity to the initial nuclear state, and to the reaction mechanism. They used a relativistic deuteron wavefunction as a solution of the Gross equation (see Sec 2.2.2). A current SAID parameterization [23] of the nucleon-nucleon scattering amplitude was used in their calculation to account for FSI. This nucleon-nucleon amplitude includes central, spin-orbit, and double spin-flip terms. Using their calculation, they were able to predict four target polarization observables: beam-target vector asymmetry A_{ed}^V , beam-target tensor asymmetry A_{ed}^T , target vector asymmetry A_d^V , and the target tensor asymmetry A_d^T .

These asymmetries are defined in this model as

$$\begin{aligned} A_V^d &= \frac{v_L R_L(\tilde{T}_{10}) + v_T R_T(\tilde{T}_{10}) + v_{TT} R_{TT}(\tilde{T}_{10}) + v_{LT} R_{LT}(\tilde{T}_{10})}{\tilde{T}_{10} \Sigma}, \\ A_T^d &= \frac{v_L R_L(\tilde{T}_{20}) + v_T R_T(\tilde{T}_{20}) + v_{TT} R_{TT}(\tilde{T}_{20}) + v_{LT} R_{LT}(\tilde{T}_{20})}{\tilde{T}_{20} \Sigma}, \\ A_V^{ed} &= \frac{v_{LT'} R_{LT'}(\tilde{T}_{10}) + v_{T'} R_{T'}(\tilde{T}_{10})}{\tilde{T}_{10} \Sigma}, \text{ and} \\ A_T^{ed} &= \frac{v_{LT'} R_{LT'}(\tilde{T}_{20}) + v_{T'} R_{T'}(\tilde{T}_{20})}{\tilde{T}_{20} \Sigma}, \end{aligned} \quad (23)$$

where

$$\Sigma = v_L R_L(U) + v_T R_T(U) + v_{TT} R_{TT}(U) + v_{LT} R_{LT}(U). \quad (24)$$

In these equations, $R_i(\tilde{T}_{kq})$ are polarization-dependent response functions, $R_i(U)$ are unpolarized response functions, and $\tilde{T}_{kq}$ are tensor polarization coefficients. These terms relate theoretical models to the asymmetry observables.

These polarization observables are in turn related to the deuteron disintegration polarized cross section by

$$\sigma = \sigma_0 \left\{ 1 + \sqrt{\frac{3}{2}} P_z A_d^V + \sqrt{\frac{1}{2}} P_{zz} A_d^T + h \left(\sqrt{\frac{3}{2}} P_z A_{ed}^V + \sqrt{\frac{1}{2}} P_{zz} A_{ed}^T \right) \right\} \quad (25)$$

where σ_0 is the unpolarized cross section, $P_z = -1 \dots +1$ is the vector polarization of the target, $P_{zz} = -2 \dots +1$ is the tensor polarization of the target, and h is the helicity of the electrons.

FIG. 5: The accessible region of possible vector and tensor polarizations of the deuteron is within the area of the dashed boundaries of the triangle.

Since the spin of the deuteron is 1, the magnetic quantum number m_z can be either ± 1 or 0. This allows the deuteron to be vector polarized with polarization

$$P_z = \frac{N_{+1} - N_{-1}}{N_{+1} + N_0 + N_{-1}} \quad (26)$$

and tensor polarized with the polarization

$$P_{zz} = \frac{N_{+1} + N_{-1} - 2N_0}{N_{+1} + N_0 + N_{-1}} \quad (27)$$

where N_i is the population of a magnetic substate indicated by the index. The limits of the tensor and vector polarization can be seen in Fig. 5.

Using the cross section from Eq. (25) the experimental double spin asymmetry can be written as

$$A_{||} = -\sqrt{\frac{3}{2}} \cdot \frac{P_b P_t A_{ed}^V}{1 + \sqrt{\frac{1}{2}} P_{tt} A_d^T} \quad (28)$$

where $P_b P_t$ is the product of the beam and target vector polarization and P_{tt} is the the target tensor polarization.

A finely binned table of A_{ed}^V and A_d^T from the model was provided by Van Orden. These calculated asymmetries were then integrated (weighted by the number of counts) over our rather broad kinematic bins in order to allow for direct comparison with the data. The experimental number of counts was calculated in finer bins in order to allow for proper weighting of the model.

2.5 EXISTING MEASUREMENTS

The $d(e, e'p)n$ reaction has been studied in detail, but there are only four unique measurements of the beam-vector asymmetry A_{ed}^V and tensor asymmetry A_d^T that have been done in the past. These asymmetries are directly related to the double spin asymmetry $A_{||}$ as seen in Section 2.4. The existing data are at a relatively low Q^2 and were compared to a model formulated by Arenhövel et al.[24],[25]. This chapter will highlight the results of these analyses. Finally, a slightly different test of the Jeschonnek and Van Orden model is also presented to demonstrate the accuracy of the model.

2.5.1 NIKHEF

The first measurements of the tensor asymmetry A_d^T were performed at the Dutch National Institute for Nuclear Physics and High Energy Physics (NIKHEF). The experiment at NIKHEF used a polarized gas target with a 565 MeV electron beam[26]. Scattered electrons and protons were detected by the BigBite magnetic spectrometer. The tensor asymmetry was extracted as a function of the angle θ_s between the polarization axis and the missing momenta and as a function of the missing momentum. The range of missing momentum is limited to below 150 MeV/c. The results of this measurement can be seen in Fig. 6.

Additionally, the first measurements of A_{ed}^V were performed at NIKHEF several years later. A longitudinally polarized beam of electrons of 720 MeV was scattered off a vector-polarized deuterium target. The scattered electron was measured at

FIG. 6: Plots of A_d^T from NIKHEF. Theoretical curves from Arenhövel are also shown. Short-dashed curves are results for PWBA, long-dashed curves include FSIs, and solid curves represent the full calculation[26].

a fixed angle $\theta = 40^\circ$, with a solid angle coverage of 96 millisteradians (msr) and knocked-out protons were measured at a central angle of $\theta_p = 40^\circ$ with a solid angle coverage of 250 msr. The missing momentum range was increased up to 350 MeV/c at a Q^2 of 0.21 (GeV/c)². Figure 7 shows that at momenta higher than 250 MeV/c, the vector asymmetry, A_{ed}^V , is highly sensitive to the D -state of the deuteron wave function.

FIG. 7: A_{ed}^V as a function of missing momentum at $Q^2 = 0.21 \text{ (GeV/c)}^2$ [27].

2.5.2 BLAST

Similar to the EG1 group of experiments, the Bates Large Acceptance Spectrometer Toroid (BLAST) experiment used a polarized electron beam⁶ incident upon a polarized deuterium target. An atomic beam source was used for the polarized deuterium target which allowed them to switch target polarization states between vector polarized and tensor polarized states. Two sets of deuteron data was taken with nominal spin angles of 32° and 47° to provide perpendicular $(\theta^*, \phi^*)=(\pi/2, 0)$ and parallel $(\theta^*, \phi^*)=(0, 0)$ kinematics. There are two analyses of the BLAST data; but the latest work by Adam DeGrush[29] reevaluates⁷ the work of Aaron Maschinot[30] to extract A_{ed}^V and A_d^T . These data was taken for a Q^2 range of $0.1 < Q^2 < 0.5$ (GeV/c)² and ten missing momentum bins from 0.0 to 0.5 GeV/c. The results of this measurement can be seen in Fig. 8.

In DeGrush's analysis, the inclusion of FSI is needed to predict a more negative value seen in the A_d^T measurement at large p_m . Similarly, it will be shown in this dissertation that FSIs are needed to predict an angle-dependence of $A_{||}$ at high missing momenta. At low missing momenta, the A_d^T measurements in DeGrush's analysis are close to zero for every bin. At higher missing momentum bins, the experimental values do not agree well with the predicted theoretical curve; however, A_{ed}^V is described rather well.

⁶Beam polarization was $\sim 60\%$ at 850 MeV[28].

⁷A reanalysis was performed due to improvements in detector software.

FIG. 8: Plots of A_d^T and A_{ed}^V for $0.2 < Q^2 < 0.3$ (GeV/c) 2 . Theoretical curves have been applied with meson exchange currents (MEC), isobar currents (IC), and relativistic correction (RC)[29].

2.5.3 HALL C

The E93-026 experiment in Hall C at the Thomas Jefferson National Accelerator Facility measured the neutron electric form factor G_E^n in $\vec{D}(\vec{e}, e'n)p$ quasi-elastic scattering. In this measurement, the neutron was detected instead of the proton. Hongguo Zhu extracted G_E^n from a measurement of A_{ed}^V and compared it to theoretical predictions using the calculation of Arenhövel. These predictions used various parameterizations of G_E^n from Galster et al.[31] which could then be compared to the data.

The target used in this experiment was a $^{15}\text{ND}_3$ target similar to the EG1b target. This experiment was limited to $Q^2 = 0.5 \text{ (GeV}/c)^2$ and the missing momentum was less than 180 MeV. The results are shown in Fig. 9.

2.5.4 CLAS

The model of Jeschonnek and Van Orden is not limited to providing values of A_{ed}^V and A_d^T . As part of the CLAS Approved Analysis “Out-of-Plane Measurements of the Structure Functions of the Deuteron,” the asymmetry $A_{LT'}$ was measured in the $\vec{d}(\vec{e}, e'p)n$ reaction. The asymmetry $A_{LT'}$ is associated with the fifth structure function of the deuteron $\sigma_{LT'}$ and is given by

$$A'_{LT} = \frac{\sigma_{LT'}}{\sigma_L + \sigma_T} \quad (29)$$

where the cross sections σ_i are partial cross sections from

$$\frac{d^5\sigma}{dQ^2 dp_m d\phi_{pq} d\Omega_e d\Omega_p} = \sigma_L + \sigma_T + \sigma_{LT} \cos\phi_{pq} + \sigma_{TT} \cos 2\phi_{pq} + h\sigma_{LT'} \sin\phi_{pq}. \quad (30)$$

$A_{LT'}$ is zero only in the absence of FSI. These measurements use the E5 data run and covers a Q^2 range of $0.2 < Q^2 < 5.0 \text{ (GeV}/c)^2$. Figure 10 shows the preliminary results of this analysis.

FIG. 9: A comparison of A_{ed}^V in four different kinematic variables. The theoretical curves are predicted asymmetries using different scaled values of G_E^n in a PWIA analysis[32].

FIG. 10: Preliminary measurements of A'_{LT} showing that the theoretical model provided by Jeschonnek and Van Orden (red line) agrees well with the data.[22] Additional model curves are from Arenhövel (black) and Laget (green).

CHAPTER 3

EXPERIMENTAL SETUP

3.1 CONTINUOUS ELECTRON BEAM ACCELERATOR FACILITY

The EG1 group of experiments took place at the Thomas Jefferson National Accelerator Facility¹ (TJNAF) located in Newport News, Virginia. Using the Continuous Electron Beam Accelerator Facility (CEBAF) at TJNAF, a polarized electron beam was provided to Hall B. The electron beam scattered off a dynamically polarized target, and these events were detected by the CEBAF Large Acceptance Spectrometer (CLAS). The EG1 experiments were run at two different times. The first run took data in late 1998 and was dubbed EG1a. The second experimental run was conducted between 2000–2001 for a period of seven months and called EG1b. The present analysis solely used data from the EG1b run.

The uniqueness of the Continuous Electron Beam Accelerator Facility is that it can provide a high intensity external continuous electron beam instead of a pulsed beam used by several accelerator facilities (e.g., SLAC and CERN). At the time of the experiment, CEBAF accelerated electrons through superconducting radio-frequency (srf) cavities to energies up to 6 GeV.² The srf cavities were operated in a continuous wave mode, and the beam is split in three packets to provide beam to three different halls, Hall A, B, and C. A diagram of CEBAF and the three experimental halls can be seen in Fig. 11. The beam makes five passes around the accelerator track and at any of those passes the packet allotted for a particular hall can be provided. This allows for a range of energies to be provided to the three halls at the same time. The beam was initially an unpolarized beam emitted from a DC-thermionic-gun-based injector, but since 1998, CEBAF has used a GaAs photo-cathode to provide a polarized beam between 70% and 80%[33] polarization to the experimental halls. A half-wave plate (HWP) can be used to change the polarization phase by 180°. The sign of the

¹The Thomas Jefferson National Accelerator Facility is also called Jefferson Lab or JLab.

²Initially, this facility was intended only to provide energies up to 4 GeV, but the accelerator has been exceeding performance. An upgrade to 12 GeV is currently underway at CEBAF with future plans to add an electron ion collider.

FIG. 11: The Continuous Electron Beam Accelerator Facility at Thomas Jefferson National Accelerator Facility

helicity requires an extra negative sign when the HWP is in use. The position of the HWP is alternated between in and out during an experimental run to cancel instrumental polarity-dependent asymmetries. The spin orientation of the electron beam can be controlled using a Wien filter. The Wien filter uses crossed electric and magnetic fields transverse to the beam to rotate the beam polarization without affecting the momenta of the electrons[34] in the beam. The electron spins undergo precession about the magnetic fields from the recirculation and end-station transport arc magnets. The value of the precession angle is dependent on the number of passes of the beam, injector energy, energy in the linac, and accelerator bend angles. The helicity state of the electron beam for each event must be correctly identified in order to accurately calculate the asymmetry. The electron beam's helicity was alternating pseudo-randomly at a rate of 30 Hz. The helicity of the beam was flipped between an original state (randomly chosen) and a complementary state for each pair.

3.2 HALL B BEAM LINE

Once the beam is accelerated to the specified energy for the run, the beam is released to the hall in an evacuated beam pipeline from the main beam line. It is

important to know all the details about the beam prior to the target. The details that are of interest include beam polarization, beam position, beam intensity, and beam current. These measurements are done by several devices placed along the beam's path. The position of all the components in the beam line can be seen in Fig. 13.

3.2.1 MØLLER POLARIMETER

The beam polarization is measured by a Møller Polarimeter. The Polarimeter uses an invasive measurement which relies on detecting scattered electrons in coincidence to determine the reaction kinematics, so it is not used while data are being collected. A polarimeter measurement was typically done every other day and took about a half hour to perform. A target chamber containing a magnetized permendur foil that is oriented at $\pm 20^\circ$ with respect to the beamline is used to produce saturated electron polarizations of 8% along the plane of the foil. The scattered electrons pass through two quadrupole magnets that separate the scattered electrons from the beam. Two scintillators located downstream of the target count the incoming electrons. The average beam polarization during the EG1b run was approximately 70%.

3.2.2 BEAM POSITION MONITORS

Two types of devices measure the beam position. The first type is represented by three nanoamp beam position monitors. The position monitors are noninvasive and measure at a rate of 1 Hz. The position monitors use three radiofrequency cavities to measure the position and are calibrated to the other beam position device, the Harp Beam Profile Monitor. Performing a harp scan involves moving wires through the beam and collecting scattering electrons with a photomultiplier tube. The harp scan measurement showed the width of the beam was contained within a 200 μm diameter.

3.2.3 FARADAY CUP

After the beam passes through the CLAS detector, it heads towards the Faraday Cup (FC) which is located 29 m downstream from the detector center. The FC measures the integrated current of the electron beam by collecting charge from approximately 4000 kg of lead (to stop the beam) supported by ceramic standoffs

FIG. 12: A top view of the the Möller Polarimeter used in Hall B

FIG. 13: Beamline of Hall B

inside of a vacuum chamber. The current is collected via an electrical feed-through and converted into a pulse frequency. This signal is fed into a counter which is gated with respect to the helicity of the beam and records the information as latch bits.

3.3 CEBAF LARGE ACCEPTANCE SPECTROMETER

The CEBAF Large Acceptance Spectrometer (CLAS) is used to study electron scattering and photon absorption events and can be seen in Fig. 14. Housed in Hall B at TJNAF, the CLAS detector has almost a full 4π acceptance in solid angle. The detector is divided into six wedge-shaped sectors. There are three layers of drift chambers along with a layer of scintillator counters. There are Cherenkov counters and electromagnetic calorimeters in the forward regions. In between each sector, there is a five-meter long superconducting magnet coil (see below) used for momentum analysis of charged particles. The drift chambers are used for track reconstruction, while the scintillators provide a way to measure time of flight. The Cherenkov counters are used for electron-pion discrimination and the calorimeters are used for identification of electrons and neutral particles.

3.3.1 TORUS MAGNET

Inside the CLAS detector is a large toroidal superconducting magnet (seen in Fig. 15), used to apply a magnetic field in the ϕ direction (transverse to the particle momentum) for momentum determination. Charged particles traversing the magnetic field do not deviate in the azimuthal angle but instead get bent towards or away from the beam axis. A positive torus current will bend negatively charged particles inward and is called the in-bending configuration. A negative torus current will bend negatively charged particles outward and is called the out-bending configuration. The magnet can provide a maximum magnetic field, with a current of 3860 A, of 2.5 T·m in the forward direction, while at the larger angle of 90° , the magnetic field would be 0.6 T·m. Constructed of an aluminum-stabilized NbTi/Cu conductor, the coils are cooled to 4.5 K and are typically operated at 1500-2250 A[35]. The torus magnet was constructed to provide a magnetic field-free region in the center to allow for a polarized target. A mini-torus magnet can also be placed close to the target to keep low momentum electrons produced by Møller scattering in the target or beam windows from reaching the drift chambers.

FIG. 14: A cross section view of the CLAS detector.

FIG. 15: A cross sectional view of the Hall B Torus Magnet as it sits between the detector components

FIG. 16: One superlayer of the CLAS drift chamber consists of cells of field wires surround one sense wire. As a particle travels through the drift chambers, the sense wires detect the induced charge.

3.3.2 DRIFT CHAMBERS

The drift chambers (DC) of the CLAS detector is divided into three distinct regions. The relative positions of each region designate three distinct magnetic field regions. Region 1 is close to the target and is in an area of low magnetic field. Region 2 is housed between the torus magnet coils is in an area of high magnetic field strength. Region 3 is outside of the magnet and sees a decreasing magnetic field strength.

Each region contains six sectors to record tracks of charged particles. Each sector covers approximately 60° in ϕ to provide nearly complete coverage around the target. The drift chambers are composed of several hexagonal shaped cells increasing in size as a function of distance from the target. A cell is defined as a single sense wire with six surrounding field wires which are strung parallel to the magnetic field. The cells form a honeycomb of field wires that is six cells across (“superlayer”) as seen in Fig. 16. There are two “superlayers” per region with one being tilted 6° in stereo angle to provide azimuthal information, pattern recognition, and tracking redundancy[36].

FIG. 17: A full view of the Cherenkov Counter. The PMTs were placed on in the regions of ϕ that were obscured by the magnetic coils.

FIG. 18: A cross section view of the Cherenkov Counter. Here a particle causes Cherenkov radiation and that radiation is reflected from the elliptical and hyperbolic mirror to be directed towards the PMT.

3.3.3 CHERENKOV COUNTERS

Outside of the drift chambers in the forward direction are Cherenkov Counters (CC). The initial purpose of the CC is to provide an electron trigger, but also to provide a way to distinguish pions from electrons[37]. The CC complements the drift chambers' θ coverage from 8° to 45° . Two focusing mirrors, a “Winston” light cone and a cylindrical mirror, are the main optical elements in the CC. The detectors were filled with perfluorobutane (C_4F_{10}) to give a high photon yield with an index of refraction $n = 1.00153$. The full CC detector can be seen in Fig. 17 and a cross section showing the optical elements can be seen in Fig. 18.

FIG. 19: One sector of the scintillator calorimeters showing all 57 paddles

As an electron travels through the CC, the electron will produce Cherenkov radiation³. In perfluorobutane, Cherenkov radiation for electrons is produced at a relatively low momentum, while pions have a momentum threshold of $p_\pi \sim 2.5$ GeV/c. The radiation is reflected off the optical mirrors and collected by the light cone.

3.3.4 SCINTILLATOR CALORIMETERS

The time-of-flight (TOF) system consists of Scintillator Counters (SC) and surrounds the majority of the CLAS detector as seen in Figs. 14 and 15. There are six sectors to the TOF system covering $8^\circ < \theta < 142^\circ$ and 60° in ϕ . Each sector contains 57 scintillator paddles with a photomultiplier tube located at each end[38]. One entire SC sector can be seen in Fig. 19. The first 23 paddles are called the “forward-angle” counters and are smaller in width (15 cm) compared to the remaining paddles called the “large-angle” counters (22 cm). The difference in width is due to geometrical space constraints, and particles at large angles typically have longer flight times so timing resolution needs are less stringent.

The TOF system provides timing information for particle identification and can be used in the triggering system. At the forward-angle paddles timing can be resolved up to 120 ps, and 250 ps in the large-angle paddles. The DC provides the momenta and trajectories of charged particles, while the TOF provides the end time of that trajectory. Using the data from the DC and TOF, the mass of a particle can be

³Cherenkov radiation is produced when a particle travels faster than the speed of light in that medium.

FIG. 20: One Electromagnetic Calorimeter Module showing the alternating stacking of planes with strips rotated 120° relative to the previous layer

determined allowing further particle identification.

3.3.5 ELECTROMAGNETIC CALORIMETERS

Similar to the CC, the Electromagnetic Calorimeters (EC) cover $8^\circ < \theta < 45^\circ$ and are the final layer of the CLAS detector. Divided into six sectors, each EC is composed of 39 layers of a 10 mm thick scintillator with a 2.2 mm thick lead sheet separating each layer[39]. Each scintillator layer is further divided into 36 strips parallel to one of the three sides of the calorimeter sector. Each EC sector is triangular in shape so there are three possible orientations at 120° relative to each side. These orientations are called U, V, and W. Every EC module contains 13 layers of each configuration. Fig. 20 shows how each successive layer is stacked in the EC module. The first five scintillator layers per orientation are called the inner layers and are connected to a photomultiplier tube via a fiber optic bundle connected to the strips. The remaining eight layers are called the outer layer and are connected to a different photomultiplier tube as seen in Fig. 21. The division of the layers allows for improved particle identification. The EC provides detection and triggering of electrons with energies above 0.5 GeV and photons at energies above 0.2 GeV. The CC can only discriminate against pions with momentum less than 2.8 GeV/c. With

FIG. 21: A vertical cut through the Electromagnetic Calorimeter showing Lead (Pb) - scintillator (SC) stacking. Purple scintillators represent the inner layers while the green scintillators represent the outer layers.

the total energy deposited in the EC and the trajectory provided by the DC, higher energy pions can be identified. Additionally, the EC provides detection of neutrons using time of flight information.

3.4 EG1B RUN GROUP

This analysis used data from the EG1b run period (2000–2001) with polarized electrons on dynamically polarized $^{15}\text{ND}_3$ targets. The group of EG1 experiments was measuring the following:

1. polarized structure functions of the proton[40],
2. polarized structure functions[41], and
3. and single pion electro-production from the proton[42][43][44].

Although these were the approved experiments, the data collected from EG1 were taken with an open trigger so that additional studies and measurements including this analysis of the double spin asymmetry $A_{||}$ from quasi-elastic scattering can be performed.

The EG1b group of experiments collected data using several different experimental configurations. The polarized electron beam had energies of 1.606, 1.723, 2.286, 2.561, 4.238, 5.615, 5.725, and 5.743 GeV which were grouped into nominal beam energies of 1.6, 2.5, 4.2, and 5.7 GeV. In-bending and out-bending configurations were used for the main toroidal magnet (torus) with currents of 2250 A or 1150 A to increase the Q^2 range. Additionally, several targets could be interchanged during any configuration.

3.4.1 EG1B TARGETS

During EG1b, longitudinally polarized electrons were scattered from two main targets, a longitudinally polarized proton target ($^{15}\text{NH}_3$) and a deuterium ($^{15}\text{ND}_3$) target. Additionally, three background targets were used: ^{12}C , ^4He (empty), and ^{15}N targets. Beam and target polarizations were either parallel or anti-parallel with respect to each other and the beam direction. The focus of this analysis was only on the deuterium target data collected from the EG1b run.

The two polarized targets, $^{15}\text{NH}_3$ and $^{15}\text{ND}_3$, were polarized by the dynamic nuclear polarization (DNP) method. Solid polarized targets that use DNP have been

used in several labs.⁴ A complete description of the process and theory for the DNP method on ammonia targets can be found in an article by D. G. Crabb and W. Meyer[46]. The deuterium target maintained roughly 20%-30% polarization during data collection. A detailed plot of the polarization history can be seen in Fig. 22. The targets were periodically annealed to remove extra paramagnetic radicals and restore polarization.

The EG1b deuterium target was composed of frozen deuterated ammonia granules ($^{15}\text{ND}_3$) that were approximately 1 mm in diameter. The granules were irradiated⁵ at either Jefferson Lab's Free Electron Laser (FEL) or Stanford's SUNSHINE facility for DNP. Ammonia's high resistance to radiation made it a suitable choice as a target for the CLAS detector. Since ^{15}N becomes polarized with the deuterium, 98% enriched ^{15}N was used to simplify polarized background correction[47]. The target was submerged in an approximately 1 K helium bath in a cell on a thin aluminum bar. The cell was made of a hydrogen-free plastic, polychlorotrifluoroethylene. This material was chosen because it has a high resistance to radiation damage and it does not produce a nuclear magnetic resonance (NMR) background signal. At the end of each cell, a thin foil sheet was used to hold the target material in place. A foil sheet of 25 μm aluminum was used for the entrance windows, and a sheet of 50 μm kapton was used for the exit windows to help reduce the radiation loss of scattered particles. The ^{12}C target used a 2.3 mm thick carbon disk and the final cell was left empty.

3.5 RUNNING CONDITIONS

As stated previously, several beam energies were used during the EG1b run. The beam was rastered over the surface of the targets in a inward and outward spiral pattern. This was done in order to minimize depolarization of the target due to heating and radiation damage. The beam position was measured using the beam position monitors located at three different positions upstream from the CLAS detector. Data were collected following each Level 1 trigger. Minimum signal thresholds in both the inner EC (0.5 GeV) and the CC⁶ had to be met in order for an event to be triggered. Once the trigger threshold has been met, information from all detectors is

⁴The E143 target at SLAC and the NA47 (SMC) at CERN used the DNP process in order to achieve polarization[45].

⁵The irradiation process dopes the material with approximately 10^{-4} unpaired electrons.

⁶The energy threshold for Cherenkov radiation in C_4F_{10} is $E=18.1 m_{particle}$, where m is the mass of the particle.

FIG. 22: Polarization history (NMR values) of both $^{15}\text{NH}_3$ (■) and $^{15}\text{ND}_3$ (○) targets over 2000 hours[47]

collected for a 90 ns time period[35]. The CLAS Data Acquisition (DAQ) system was designed for a 2 kHz event rate, but during the EG1b time period, the routine operation was between 3 and 4 kHz. Overall, there were approximately 40 terabytes of data collected divided into 2000 runs.

Initially, the data collected by the CLAS detector were digital values from 24 Readout Controllers associated with detector components. This data was grouped into events by the event builder process. The data were then organized into tables arranged by header banks clustering similar data together. These events were recorded to the SILO by the Event Recorder process in the form of Bank Object System (BOS) files.⁷ BOS files were written in 2 gigabyte blocks which were labeled by run number and block number. Typically, there were twenty to thirty blocks associated with a run due to the large amount of events collected. The BOS format was the lowest level off-line data format used in the analysis process and was later converted or “cooked” to obtain physical quantities such as positions, energies, and momentums.

Typically, the cooking process produces ntuple files that are read with the PAW

⁷BOS is a FORTRAN77 program system used for the CLAS detector to dynamically manage data.

or ROOT framework. Due to the large amount of data associated with the EG1b run, the data were formatted into Data Summary Tape (DST) files to limit the total disk space required for storage, which were stored on Jefferson Lab's silo tapes. Although cooking the BOS files did reduce the file size, each run in the DST format was still broken into several blocks. The DST files were categorized into two types, electron triggered events and positron triggered events referred to as DST and DSTp files, respectively. For this analysis, only electron triggered events were studied.

CHAPTER 4

DATA ANALYSIS

4.1 DST FILES

The lower-level processing (“cooking”) of EG1b data was done by the EG1b working group. At this stage in the analysis, event reconstruction and calibrations of the time of flight, drift chambers, and electromagnetic calorimeters were performed. The cooking was performed prior to the start of this analysis and more details on the cooking process can be found in the theses of the members of the EG1b working group [48], [49], [42], and [43] as well as the resulting analysis notes¹ [41][40]. Additionally, these theses and notes provided a foundation for several cuts implemented in this analysis.

As stated previously, the data were divided into DST files associated with a run number. A complete list of all runs detailing the experimental setup (i.e. torus polarization, torus current, beam energy, half-wave plate, target, and target polarization) was compiled for organizational and reference purposes during the cooking process. This list also labels the runs that passed quality checks and could be used for analysis. Every run was systematically inspected and compared with similar runs to ensure quality. Only good runs were processed from this list. Several programs access this list in the analysis process as reference so that run information did not have to be hard coded into the programs. The runs were grouped into sets based on torus polarity and beam energy. Table 1 gives the set labels of ND₃ runs used throughout this analysis.

To further organize the data, a program called “LinkDATA.pl” created soft links to the DST files on the silo. The soft links were created in the home folder on Jefferson Lab’s computers. The program organized the links by DST or DSTp file and run set. The LinkDATA program has several command-line arguments that were used in this analysis. The first argument allowed soft links to be made for either DST or DSTp files. The second argument verified all files were already cached on the Jefferson Lab

¹These analysis notes have been approved by the Deep Processes Physics Working Group.

TABLE 1: All ND₃ EG1b run sets with usable electron beam data, organized by beam energy and torus polarity. These set labels are used throughout this analysis. Further distinctions between sets were target polarizations used during the run.

Set Label	Run Number Range(s)	E_{Beam} (GeV)	I_{Torus} (A)
1.6+	25488-25559; 25669-26221	1.606	+1500
1.6−	26222-26359	1.606	−1500
1.7+	28512−28526	1.724	+1500
1.7−	27644-27798; 28527-28532	1.724	−1500
2.3+	27205-27351	2.288	+1500
2.5+	28001-28069	2.562	+1500
2.5−	27799-27924; 27942-27995	2.562	−1500
2.8−	27936-27941	2.792	−1500
4.2+	28074-28277; 28482-28494; 28506-28510	4.239	+2250
4.2−	28280-28479; 28500-28505	4.239	−2250
5.6+	27356-27364; 27386-27499	5.627	+2250
5.6−	27366-27380	5.627	−2250
5.7+	27069-27198	5.735	+2250
5.7−	26874-27068	5.735	−2250
5.7−	26468-26722; 26776-26851	5.764	−2250

silos, and if not, the missing files would be re-cached². The cache argument was always used before the DST files were analyzed to ensure that all data files were analyzed. The last argument specified the quality set of cooked EG1b data. Due to the large number of files for the EG1b run, the EG1b data was separated into **RATE** and **ASYM** categories. The **RATE** category includes runs from all targets that pass the standard EG1b quality checks, whereas the **ASYM** category contains only ammonia target runs that were considered of good enough quality for asymmetry calculations. The files in the **RATE** category had stricter cuts imposed for file selection. For this analysis, the **RATE** selection was only used because the other targets were needed for calculation of dilution factors which will be further described in Section 4.7.

4.2 SKIMMING

For the present analysis, smaller files were needed since performing analysis on unabridged DST files would be time consuming as well as occupy large amounts of hard disk space. Since this analysis only used exclusive events, only information about events of interest were saved and converted to the C++ based format of a ROOT file. Events of interest were chosen by the number of particles present during the event. All events with only two oppositely charged particles (an electron candidate and a proton candidate) were saved. Additionally, events with a third neutral particle detected were saved. This step of the analysis was called “skimming”. The `dstdump` program was used to perform the skimming because of its ability to perform all the corrections provided by the EG1 group, plot initial quality checks, and start lower level particle identification. Skimming the data decreased the total size of the data from 40 terabytes to approximately 500 gigabytes.

The EG1b group developed a standard analysis package called “`dstdump.cc`” to read and analyze the DST files. Furthermore, the `dstdump` program provided access to several important data analysis tools and was meant to be used as a foundation for analysis. The data analysis tools included several cuts and corrections that were not initially done during the cooking process. Some of the major functions and corrections³ of the `dstdump` program were:

²DST files could only be accessed on Jefferson Lab’s cached directory and periodically files would be deleted and needed to be re-cached.

³These corrections are standard EG1b corrections that can be found in approved EG1b analyses.

Momentum Corrections

The momentum correction was a final sector-by-sector calibration to fix inaccuracies in the magnetic field mapping and drift chamber geometry. These momentum corrections used the same techniques as developed for the E6 experiment [50].

Helicity Corrections

The helicity corrections fixed problems with the recording of the beam helicity. More information on this correction can be found in section 4.2.2.

Raster Corrections

The electron beam was continuously rastered in a spiraling, circular pattern. Due to the rastering, the electron beam entered the target at various positions, and the raster corrections centered the scattering event vertex position.

Energy Loss Corrections

As the scattered electron traverses through the target, energy is lost through ionization. Approximately $2.8 \text{ MeV cm}^2/\text{g}$ is lost for the typical EG1b target[51]. The true value of energy loss depends on the density and thickness of the windows and helium coolant. The energy loss correction accounts for this loss.

Beam Energy Correction

There was some beam energy loss due to the beam traveling through the target prior to the event vertex. This energy loss was typically on the order of a few MeVs for the electrons. A comparison between beam energies given by Hall A and the nominal Machine Control Center (MCC) was used to get a precise knowledge of the beam energy.

Torus Scaling Correction

During the original cooking, some tcl files⁴ were omitted causing the values of the torus current in the DST files to be incorrect. The torus scaling correction ensured the correct torus current value was used during analysis.

Multiple Scattering Correction

After an electron scatters off the target, it can experience further deflections

⁴The tcl files are configuration files used while cooking BOS files.

because of multiple scattering. These multiple scattering effects should be corrected if possible because they cause a net angular deflection and displacement of the apparent vertex position. EG1b standard analysis uses information from coincident particle vertices for this correction.

Stray Magnetic Field Correction

The fields of the target solenoid and the torus magnet could overlap in the inner components of the detector. Using GSIM to simulate this effect, EG1 calculated fit parameters in order to correct for the stray magnetic fields.

Particle Identification

Only simple particle identification was done during the dstdump program. The charge of the particle was the main tool for particle identification. Electron candidates were negatively charged particles. Proton candidates were positively charged particles. If an electron candidate and proton candidate were present with a neutral particle; this particle was considered as a neutron candidate. Further cuts were applied at a later stage in this analysis to properly identify particles.

Faraday Cup Counting

Information about the cumulative beam charge incident on the target was collected. This was done to properly normalize the data at a later stage in the analysis. The dstdump program summed the information about the lifetime-gated Faraday Cup from each DST file in order to give a full count of the beam charge per helicity state.

4.2.1 QUALITY CHECKS

In addition to the quality control performed by the EG1b data group in selecting quality run files, additional checks were performed on the EG1b data. We modified the dstdump program to output count rates at every stage of particle identification. This was performed for each run number and by each cut applied to determine the quality of each individual run. The count rate of events per run were normalized using the gated Faraday cup information. The count rates gave an indication of the normal event rate per beam energy run and per torus configuration. If a run was within 8% of the normal count rate for inclusive counts, then the file was considered

to be a good run. The 8% threshold was used to allow for statistical fluctuations at higher energies due to the momentum spread of the inclusive electrons.

Furthermore, the count rate also showed if there were changes in the experimental parameters. In the 2.5 GeV out-bending run, the deuterium target was compromised and the deuterium beads fell out of the cavity. The target had to be refilled and additional foil was applied. This change in experimental parameters was observed in the count rate, as can be seen in Figure 23b. The 2.5- GeV data set was treated as two data sets as seen in Table 2 for the calculation of the dilution factor and the raw asymmetries.

FIG. 23: Counts of events of interest normalized by Faraday Cup count for the 2.5 GeV run after proton identification cuts have been implemented. The out-bending plots shows the difference of counts rates before and after the target was refilled and additional foil was added. The scale is different on both plots and the zeros are suppressed.

TABLE 2: Run numbers for the two individual 2.5- GeV data sets. Parameters of the target had changed between the two run sets.

Set Label	Run Number Range(s)	$E_{Beam}(\text{GeV})$	$I_{Torus}(\text{A})$
2.5– Set 1	27799-27924	2.562	–1500
2.5– Set 2	27942-27995	2.562	–1500

4.2.2 HELICITY PAIRING

The helicity state of an event was determined by the beam helicity signal, half wave plate status, and spin rotation factor. The spin rotation factor is determined by the Wien filter located at the injector and the precession in the accelerator which was dependent on the beam energy as described in Section 3.1. The EG1b run group took great care to determine the correct helicity and mark only good helicity pairs (i.e. an original and complementary pair) for use. A program called “HelP.cc” was designed by the EG1b run group to help identify correct pairing sequences and flag questionable helicity bits. The HelP program was run in unison with the dstdump program and flagged helicity bits were either corrected or removed.

4.3 BINNING OF DATA

The asymmetries measured in this analysis are calculated as a function of three kinematic variables: the squared four-momentum transfer (Q^2), the cosine of the angle between the virtual photon and the neutron ($\cos \theta_{nq}$), and missing momentum (p_m). We integrated over the angle ϕ between the leptonic and hadronic plane.

The standard EG1b Q^2 binning was used as a template for our Q^2 binning scheme. The EG1b working group binned Q^2 logarithmically and each bin was calculated as

$$\text{Bin Number} = n = \text{int} \left(27.0 + 13.0 * \log_{10}(Q^2/0.9188388) \right). \quad (31)$$

These Q^2 bins were used in inclusive analyses but were too fine for this exclusive analysis because of the limited statistics. Several bins were combined to form four new Q^2 bins. Table 3 shows the new Q^2 binning used in this analysis and how it compares to the standard EG1 bins. The Q^2 range in this analysis was limited to 0.131-3.17 (GeV/c)².

TABLE 3: Comparison between Q^2 bins and Standard Q^2 bins for EG1b. Higher Standard Q^2 bins for EG1b are not shown.

Bin	Stand. EG1b Bin	Q^2_{min}	Q^2_{max}	Q^2 (arithmetic ave.)
N/A	0	0	0.00919	–
	1	0.00919	0.0110	0.010
	2	0.0110	0.0131	0.012
	3	0.0131	0.0156	0.014
	4	0.0156	0.0187	0.017
	5	0.0187	0.0223	0.020
	6	0.0223	0.0266	0.024
	7	0.0266	0.0317	0.029
	8	0.0317	0.0379	0.035
	9	0.0379	0.0452	0.042
	10	0.0452	0.0540	0.050
	11	0.0540	0.0645	0.059
	12	0.0645	0.0770	0.071
	13	0.0770	0.0919	0.084
	14	0.0919	0.110	0.10
	15	0.110	0.131	0.12
0	16	0.131	0.156	0.14
	17	0.156	0.187	0.17
	18	0.187	0.223	0.20
	19	0.223	0.266	0.24
	20	0.266	0.317	0.29
	21	0.317	0.379	0.35
1	22	0.379	0.452	0.42
	23	0.452	0.540	0.50
	24	0.540	0.645	0.59
	25	0.645	0.770	0.71
2	26	0.770	0.919	0.84
	27	0.919	1.10	1.0
	28	1.10	1.31	1.2
	29	1.31	1.56	1.4
3	30	1.56	1.87	1.7
	31	1.87	2.23	2.0
	32	2.23	2.66	2.4
	33	2.66	3.17	2.9

The data were divided further into bins of $\cos \theta_{nq}$. The $\cos \theta_{nq}$ region was roughly divided into three regions of 0.7 width from -1.0 to 1.0. The exact ranges of the $\cos \theta_{nq}$ bin are shown in Table 4.

TABLE 4: The $\cos \theta_{nq}$ bins used in this analysis.

Bin	Value _{min}	Value _{max}
0	-1.0	-0.35
1	-0.35	0.35
2	0.35	1.0

The final bin was the missing momentum bin. We are interested in missing momenta from 0.0 to 0.5 GeV/ c , divided into five bins. The lowest bin was chosen from 0.0 to 0.05 GeV/ c . The next three missing momentum bins were 0.1 GeV/ c wide while the last bin was 0.15 GeV/ c wide. The exact ranges of the missing momentum bins are shown in Table 5.

TABLE 5: The missing momentum bins used in this analysis.

Bin	Value _{min} (GeV/ c)	Value _{max} (GeV/ c)
0	0.00	0.05
1	0.05	0.15
2	0.15	0.25
3	0.25	0.35
4	0.35	0.50

4.3.1 BINNING FOR THEORY/EXPERIMENTAL COMPARISON

In order to properly weight the theoretical asymmetries with our experimental count rate, much smaller binning was needed. As stated in Section 2.4, the theoretical values were provided on a grid of kinematic points at the center of these finer bins. The larger Q^2 bins were replaced with the standard EG1b bins. The $\cos \theta_{nq}$ bins were re-binned into 40 equally spaced smaller bins from -1.0 to 1.0. Forty bins were chosen so the smaller bins still aligned with the larger bins. Additionally, binning in ϕ was added. Four ϕ bins were chosen to be centered on 0, 90, 180, and 270 degrees. The new ϕ bins are shown in Table 6. The missing momentum bins remained the same and were compared with the spectator momentum in the theory calculation. All new kinematic bins were then filled by the dilution factor (see Section 4.7) corrected counts. The theoretical prediction for a given wide bin was calculated by summing the product of predicted asymmetry and experimental counts in all fine bins inside the wide bin and dividing by the total counts in the wide bins.

TABLE 6: The ϕ bins used for experimental weighting of theory.

Bin	ϕ_{min}	ϕ_{max}
0	-45.0	45.0
1	45.0	135.0
2	135.0	225.0
3	225.0	315.0

4.4 PARTICLE IDENTIFICATION

In the following, we describe how we properly identified the final state particles and the $\vec{d}(\vec{e}, e'p)n$ reaction. Particle identification began at the cooking stage of the analysis. The cooking program RECSIS performed hit-based tracking and time-based tracking to identify charged and neutral particles.

The next level of particle identification used the dstdump program described in

Section 4.2. After all corrections were applied to each event, events of interest were chosen. Negatively charged particles that have a momentum less than the beam energy are considered as electron candidates. All positively charged particles at this stage are considered to be proton candidates. Neutral particles are considered to be neutron candidates when the beta of the particle is less than 0.8 ($\beta < 0.8$). To reiterate, the purpose of the dstdump program was to apply corrections and skim all events to produce smaller ROOT files. All processing of data up to this point was performed at Jefferson Lab's Data Analysis facility.

The final particle identification and selection was performed with the ROOT files on local disks using ROOT version 5.18. Using a C++ based code, the skimmed files were analyzed to determine the final events of interest. The events chosen passed all electron, proton, and coincidence cuts that will be defined in the following sections. Furthermore, the events chosen were in the ranges of the defined bins provided in Section 4.3. These event counts were stored in text files by run to be later used to calculate asymmetries.

4.4.1 ELECTRON IDENTIFICATION

From the dstdump skimmed files there are only two charged particles detected for each event. One charged particle was a negatively charged particle and was considered to be an electron candidate. Light hadrons, like π^- , have similar time-of-flight and trajectories as electrons, and secondary reactions can additionally cause other electrons to be present. To verify an electron was observed, several cuts and conditions were therefore implemented:

Cuts during dstdump program

These cuts were implemented during the creation of the skimmed root files to identify electron candidates.

1. Particle Charge = -1
2. Only one negatively charged particle

Cuts on skimmed root files

These cuts further identified electron candidates and will be explained in the following sections.

1. Status Flag Selection

2. Trigger Bit Selection
3. Vertex Cut
4. Cherenkov Counter Cut
5. Electromagnetic Calorimeter Cut
6. Geometric and Timing Cuts
7. Fiducial Cuts

Status Flag Selection

Every particle was given a one or two digit number called a status flag during reconstruction. Single digit numbers between 0 and 5 signified that the track was time-based. Time-based tracks were accepted. Status numbers from 6 to 9 meant the track was not time based and was unacceptable. A value of 10 was added to the single digit number if the particle went through all three superlayers of the drift chambers. Therefore, the electron events kept were

$$0 \leq flag_{status} \leq 5 \text{ and} \\ 10 \leq flag_{status} \leq 15.$$

Trigger Bit Selection

Trigger words were associated with every event. The trigger word is a 16 digit binary number that contains information about trigger responses. The first 6 bits correspond to a trigger in the EC and CC in the respective sector. Trigger bit 7 and 8 correspond to only a hit in the EC with bit 8 corresponding to low EC threshold. Trigger bits 9-14 were not used, while bits 15 and 16 contained redundant information about the helicity bucket. For the event selection, a trigger word that contained trigger bits 1-6 was considered a good event because the EC and CC were both triggered.

Vertex Cuts

Events that were considered for the analysis had to originate from the target. Some of the events that were recorded by the CLAS detector originated from material surrounding the target. In order to eliminate events not coming from the target, a

cut was placed on the Z vertex position of the electron candidate. The center of the target was determined to be at $Z=-55.0$ cm and a 3σ cut was implemented. The resolution due to event reconstruction was not fine enough to differentiate between events originating from the target housing and the target material. The loose vertex cut of

$$-58.0 < Z_{vertex} < -52.0 \text{ (cm)} \quad (32)$$

was used and can be seen in Figure 24.

FIG. 24: A plot of θ_{el} in degrees vs Z_{vertex} in cm. The red lines show the boundaries of the vertex cut.

Cherenkov Counter cut

The Cherenkov Counter was used to separate electrons from pions. Pions are more massive than electrons and therefore are less likely to produce photons from Cherenkov radiation. In an exclusive analysis, there is very little pion contamination so the threshold could be set rather low. The number of photoelectrons that need to appear in the Cherenkov counter was chosen to be greater than 1.0.

Electromagnetic Calorimeter Cut

Pions and electrons show a distinctly different pattern of energy deposition in the EC. Using inclusive data, the EG1b group determined that the signals from EC_{inner} and EC_{total} divided by the reconstructed momentum reveal a clear distinction between pions and electrons.

For the present analysis, the cut for inclusive data was used. It was further determined that EC_{total}/p from pions is momentum dependent while the ratio is roughly constant for electrons. Therefore, a cut shown by Figure 25 was implemented. In conclusion, the following cuts were implemented to discriminate against pions:

1. $EC_{inner} > 0.05$
2. For $p_{electron} \leq 3.0$, $\frac{EC_{total}}{p_{electron}} > 0.15$
3. For $p_{electron} > 3.0$, $\frac{EC_{total}}{p_{electron}} > 0.20$

FIG. 25: EC_{tot}/p_{el} vs $|p_{el}|$ with the red lines showing the boundaries of the cut.

Additionally, layers of the EC may not have recorded the deposited energy information making the sum of EC_{inner} and EC_{outer} different from EC_{total} . A sum check was implemented to identify and correct the EC energies. If the EC_{inner} contained

energy information, then the total energy was corrected such that

$$EC_{total} = MAX(EC_{total}, EC_{inner} + EC_{outer}). \quad (33)$$

If the EC_{inner} recorded no information, then

$$EC_{inner} = EC_{total} - EC_{outer}. \quad (34)$$

Geometric and Timing Cuts

These cuts refer to a method proposed by M. Osipenko et al.[52] in which a CC hit is matched with a measured track in the DC. The matching was done geometrically and temporally. This work was completed by N. Guler and is only summarized here for reference.

For geometric matching, an imaginary CC projective plane is constructed behind the CC detector as in Figure 26. To construct the projective plane, the path of the Cherenkov radiation is extended behind the elliptical mirror with the initial direction of the radiation. The length of the projected radiation is equal to the distance traveled from the mirror to the PMT. The ends of all possible paths of the projected radiation form the projective plane. Two angles are formed using the normal on the projected plane. θ_p is the angle between the normal on the projective plane and the projected path of the radiation. θ_p^{center} is the polar angle of the segment center between the normal on the projective plane and the segment center. The difference between these two angles should be close to zero with an offset⁵ of θ_p^{offset} for electrons. A cut of

$$|\theta_p - \theta_p^{center} - \theta_p^{offset}| < 3\sigma_p \quad (35)$$

was used for the geometrical track matching. Further discrimination geometrically involved comparing triggers from the right and left PMT. Electrons with a $\phi > 0$ should have triggered the right PMT, while $\phi < 0$ should have triggered the left PMT. An event could have triggered both PMTs when the electron had $\phi \approx 0$. Events that did not follow this relationship were considered accidental coincidences and were cut from the analysis. Lastly, the time difference between the expected time and observed time for an electron to travel from the SC to the CC projected plane should be close to zero. This time was calculated as

$$\Delta t = t^{SC} - t^{CC} - \left(\frac{r^{SC} - r^{CC}}{c\beta} \right) \quad (36)$$

⁵The offset accounts for small shifts in the segment center position.

FIG. 26: A profile of the Cherenkov counter projection plane. The length of the projected radiation path (green lines) is equal to the length of the reflected radiation path (red lines).

where t^{SC} and t^{CC} are the recorded times for the SC and CC respectively and $r^{SC}-r^{SC}$ is the calculated track distance between the SC paddle and CC projective plane. A complete description of the parameters used as well as relevant plots detailing the cuts can be found at EG1b's web site [53].

Fiducial Cuts

Fiducial cuts were developed by the EG1b group to be used with all data sets. These fiducial cuts were designed to remove inefficient regions of the detectors where acceptance was poorly understood. These fiducial cuts were designed using a study [54] of the CC during the 1.6 GeV in-bending EG1b runs as a foundation to determine inefficient regions. The detected electrons' distribution in ϕ and θ that passed an

80% efficiency cut were plotted for each sector. Geometric cuts were applied to remove regions that contained a low population density. A complete description of the fiducial cuts can be found in the work of R. Fersch [40]. An additional cut was used to eliminate any inefficiency due to reconstruction at the edges of the DC detector. This was only implemented in θ and electrons between the angles of

$$8.0^\circ < \theta_{electron} < 49.0^\circ \quad (37)$$

were considered good electrons. The 49.0° limit was used to eliminate electrons that scattered off the target magnet coil.

Additional Cuts

A few additional cuts were also used to identify electrons. A problematic region of sector 5 of the DC was removed from analysis. Reconstruction of this region had failed during cooking [49]. The region in Sector 5 that was removed was between

$$18.0^\circ < \theta < 21.0^\circ \text{ (DC Sector 5 Only)}. \quad (38)$$

Lastly, the cuts of

- $\nu \equiv E - E' > 0.0\text{GeV}$,
- $p_{electron} \leq E_{beam}$, and
- $p_{electron} \geq 15\% * E_{beam}$

were used as a precautionary measure to eliminate erroneous electron events and reduce radiative corrections.

4.4.2 PROTON IDENTIFICATION

In addition to electron identification, proton identification was needed in order to properly identify the $\vec{d}(\vec{e}, e'p)n$ reaction. Proton candidates were chosen at the DST stage as positively charged particles. Several additional cuts were implemented to identify protons. Here is a list of the additional cuts that were applied to identify protons.

1. Particle Charge = +1
2. Electron must be present

3. Time of Flight Cuts
4. Kinematic Cuts
5. Fiducial Cuts

4.4.3 TIME OF FLIGHT CUT

Besides requiring a positive charge, the primary method used to identify a proton was the time of flight cut. This cut is based on the principle that there should be no difference between the measured and calculated time of flight. The difference between the measured and calculated time of flight was calculated as

$$\Delta t_{proton} = t_{tdc} - t_{start} - l_{path} / \sqrt{\frac{c^2 |p|^2}{p_{proton}^2 + m_{proton}^2 c^2}} \quad (39)$$

where t_{tdc} is the raw time as measured by the time-digital converter (TDC), t_{start} is the event start time, l_{path} is the length of the proton's path, p_{proton} is the proton's momentum, and m_{proton} is the mass of the proton (0.938 GeV/c²). The cut of

$$-1.0ns < \Delta t_{proton} < 2.0ns \quad (40)$$

was implemented to select protons from the proton candidates. Initially, this cut eliminated several good protons candidates due to an issue with timing. This correction is explained in section 4.6.

Kinematic Cuts

Similar to the electron vertex cut, a proton vertex cut was implemented in the analysis. This was done to make sure the detected protons originated from the target material and not material surrounding the targets. The same cut from Eq. (32) was used for the proton.

Fiducial Cuts

Similar to the electron fiducial cuts, proton fiducial cuts were implemented to remove particles on the fringe of the detector's acceptance region. Different cuts for in-bending torus and out-bending torus configurations were devised due to the proton's path curving differently in the two distinct magnetic fields. The proton's θ

angle and ϕ angle was observed in three momentum bins. The momentum bins used were:

1. Momentum Bin 1 - $p_{pr} < 0.6 \text{ GeV}/c$
2. Momentum Bin 2 - $0.6 \text{ GeV}/c \leq p_{pr} \leq 1.0 \text{ GeV}/c$
3. Momentum Bin 3 - $p_{pr} > 1.0 \text{ GeV}/c$.

Regions in the angular distribution plot with low population density on the edges of the detectors were removed. A ϕ maximum and ϕ minimum as a function of θ was determined for each momentum bin per torus configuration. The ϕ of the proton must fall between the minimum and maximum in order to pass the cuts. For in-bending runs, the minimum and maximum equations used were:

$$\phi_{max} = 60.2 - a * \theta^b \quad (41)$$

$$\phi_{min} = a * \theta^b. \quad (42)$$

While for out-bending runs, the equations used were:

$$\phi_{max} = 60.2 - a * (\theta - c)^b \quad (43)$$

$$\phi_{min} = a * (\theta - c)^b. \quad (44)$$

TABLE 7: These are parameters used for the proton fiducial cut for an in-bending torus current.

Momentum Bin	a	b
1	132.33	-0.97
2	131.19	-0.95
3	129.14	-0.96

Again, the DC Sector 5 cut and a general θ cut was applied to the proton identification. To reiterate, the cut

$$8.0 < \theta_{proton} < 49.0 \quad (45)$$

TABLE 8: These are parameters used for the proton fiducial cut for an out-bending torus current.

Momentum Bin	a	b	c
1	21.42	-0.42	19.5
2	26.24	-0.95	15.0
3	42.99	-0.67	10.0

FIG. 27: Fiducial cuts ϕ_{pr} vs θ_{pr} applied to 2.5 GeV in-bending protons where $p_{pr} > 1.0$ GeV.

was used to remove inefficiencies near the edge of the detector, and

$$18.0 < \theta < 21.0 \quad (46)$$

was removed from Sector 5 because of bad reconstruction of that area during cooking.

4.4.4 COINCIDENCE CUTS

For an event to be considered as an event of interest, a neutral particle was not required, and in most cases the neutral particle was not detected. Since information on the neutron could not be measured directly, the information was inferred from

the known kinematics of the scattering process. Using conservation of energy and momentum, we obtained information about the neutron. The missing energy (E_m) is calculated as

$$E_m = m_d + \nu - E_p, \quad (47)$$

where m_d is the mass of the deuteron, ν is the energy of the virtual photon, and E_p is the energy of the proton. The missing momentum (p_m) is calculated as

$$\vec{p}_m = \vec{q} - \vec{p}_p, \quad (48)$$

where q is the momentum of the virtual photon and p_p is the momentum of the proton. Finally, the missing mass (M_m) is calculated as

$$M_m = \sqrt{E_m^2 - p_m^2} \quad (49)$$

under the assumption that the reaction took place on a deuteron at rest. The mass of the neutron is known to be 0.939 GeV and a missing mass cut was implemented to remove multi-particle final states. The missing mass cut of

$$0.9 < M_m < 1.0 \text{ (GeV)} \quad (50)$$

was used. The upper limit of the missing mass cut is below the pion threshold, taking into account the resolution of the detectors. Additionally, cuts on missing energy and missing momentum were used to further reduce background. These cuts were

1. $E_m < 1.15 \text{ GeV}$ and
2. $0.0 < |\vec{p}_m| < 0.5 \text{ GeV}/c$.

4.5 GENERATION OF SKIM TABLES

Finally, an event was considered to be a $\vec{d}(\vec{e}, e'p)n$ event when all particles were identified by passing the cuts indicated above and it was in one of the kinematic bins used in this analysis. For that event, the helicity state was determined and a count was recorded. A multi-dimensional array of the size 5x4x3x2 was used to hold the counts of each bin. The size of the array is directly related to each kinematic bin (p_m , Q^2 , and $\cos \theta_{nq}$, respectively) and each helicity state. This array was stored into individual data files per run with bin numbers also transcribed next to each count as a redundant indicator. This allowed us to monitor count rates for every

FIG. 28: Plot of missing energy vs missing momentum. The red lines represent all three coincidence cuts applied.

run for each kinematic bin and observe overall trends of count rates to each run. A subroutine in the particle identification routine performed this operation. The skim tables were then later read by an analysis program which calculated dilution factors and asymmetries.

4.6 TIMING CORRECTION

As stated previously, there were errors found in the timing variables. These errors were discovered during particle identification. Plots of the Q^2 distribution in the 2.5+ GeV $^{15}\text{ND}_3$ data set displayed sharp peaks and valleys instead of a continuous distribution. The plots of Q^2 were per kinematic bin and are shown in Figure 29. The irregular distributions appeared in all plots suggesting that they were neither $\cos \theta_{nq}$ or missing momentum dependent.

It is important to note that these irregular distributions were not seen in any inclusive analysis. This suggested that one of the cuts implemented in this analysis was the cause. The plots of Q^2 were plotted systematically cut-by-cut to identify the

FIG. 29: Plots of Q^2 for 2.5+ GeV $^{15}\text{ND}_3$ data set. Several sharp peaks can be observed in different bins.

offending cut. During cuts that involved timing, the normal smooth distribution of Q^2 became coarse. After the proton timing identification cut was implemented, the Q^2 distribution became extremely jagged. Concurrently, we observed bands of low population density for the electron and the proton ϕ vs. θ distributions. The bands only appeared in DC sectors 1, 2, and 3 and only in the θ range of 18° to 20° .

Since the gaps and coarseness appeared only during timing cuts, the timing provided by the SC or CC was faulty. This was confirmed by plotting the Δt from Eq. (36) for the electron by SC paddle and by DC sector^{6,7}. The electron peak in Δt should be centered at 0 ns, but the plots (Fig. 30) showed the electron peak had offsets as high as 6 ns. Additional plots of Δt for the proton (Equation (39)) showed the same offset.

Next, we compared CC and SC paddles to determine where the error occurred. This was determined by looking at the CC time and SC time for every paddle and comparing information from neighboring paddles. The timing from the CC showed

⁶These paddles and sectors were triggered by an electron.

⁷Electron Δt only involves two recorded times (SC and CC) compared to the three times (SC, CC, and TDC) of Δt_{proton} .

FIG. 30: Uncorrected $\Delta t_{electron}$ for $^{15}\text{ND}_3$ 2.5 GeV In-bending. Both plots are of DC sector 6. The left plot is paddle 5, while the right plot is paddle 6.

no abnormality by comparing neighboring paddles while the SC plots displayed the offsets.

Finally, to correct the timing, the sharp electron peak in Δt was corrected to zero for every paddle in every sector using CC information. A Gaussian function was fitted to every Δt distribution for every paddle and the center of the peak was recorded in tables. The peak center value was used as an offset value to shift the electron peak back to zero. A correction subroutine was written for the dstdump program for the EG1b group. This correction eliminates the need to recook the DST files and correct the SC time at that level. Additionally, applying the correction values does not alter the timing for any other particle. This analysis only relies on two timing variables and only those variables were taken into consideration when the correction subroutine was developed. Moreover, this issue was only seen in the 2.5 GeV in-bending data set.

4.6.1 PROTON Δt

Besides correcting the Δt for the electron, Δt for the proton had to be corrected. The correction for timing above only corrected the Δt for the electron; it did not correct the underlying problem with the SC paddles. Therefore, the timing associated with the proton SC were also incorrect. Since proton identification relied heavily on timing cuts, Δt_{proton} was plotted as a whole data set (Fig. 31) and for each individual

paddle (Fig. 32) after the electron timing was corrected. The distribution using all paddles appeared as the expected distribution, but individual plots show an offset from the expected distribution. In some cases, the Δt for the proton was offset as much as 0.7 ns. The timing cut for the proton eliminated several good proton candidates due to the subtle shifts of the timing peak. The correction for the offset is to find a correction factor to shift every peak for every paddle back to zero. Using the same procedure as above, another subroutine was developed for the EG1b group to use to correct for Δt for the proton.

FIG. 31: Δt_{proton} for $^{15}\text{ND}_3$ 2.5 GeV In-bending.

4.7 DILUTION FACTOR

Given the complicated composition of $^{15}\text{ND}_3$ targets, not all events passing our cuts originated from polarized deuterium. Using the cuts described in Section 4.4, we were only able to determine the approximate region where the scattering event occurred in the target. The resolution of the CLAS detector was not precise enough to discriminate between scattering events originating from the target housing (i.e., vacuum windows, target stick windows, and mini-cup) and events from the target

FIG. 32: Uncorrected Δt_{proton} for DC sector 5 and SC paddles 6 and 7.

material. Additionally, the target material used in this analysis was deuterated ammonia submerged in a liquid helium bath. Both the nitrogen and helium contributed to this background. Fortunately, most of this material was considered to be unpolarized⁸ and did not affect the helicity-dependent difference.

With this in mind, the standard EG1b correction for unpolarized background is the use of a Dilution Factor (F_D) which is the fraction of events coming from the polarized species of interest (D). This dilution factor corrects the raw asymmetry. The raw asymmetry is defined as

$$A_{raw} = \frac{n^+ - n^-}{n^+ + n^-} \quad (51)$$

where $n^\pm$ is the normalized count per helicity state and is defined as

$$n^- = \frac{N^{\uparrow\uparrow}}{FC^{\uparrow\uparrow}} \quad \text{and} \quad n^+ = \frac{N^{\uparrow\downarrow}}{FC^{\uparrow\downarrow}} \quad (52)$$

where FC is the Faraday Cup counts. The FC counts are gated on the DAQ life time to correct for dead time effects. Introducing background counts as part of the raw asymmetry would only affect the denominator since unpolarized counts would cancel out in the numerator. Subtracting out the background (n_b) in the denominator leads to the undiluted asymmetry and is given as

$$A_{undiluted} = \frac{n^+ - n^-}{n^+ + n^- - n_b}. \quad (53)$$

⁸Polarized material is accounted for in Section 4.10.

We further define the dilution factor as

$$F_D = \frac{n^+ + n^- - n_b}{n^+ + n^-} = 1.0 - \frac{n_b}{n}. \quad (54)$$

The raw asymmetry can then be corrected for the unpolarized background by dividing out the dilution factor giving the equation for the undiluted asymmetry as

$$A_{undil} = \frac{A_{raw}}{F_D}. \quad (55)$$

Using a simplistic picture, the dilution factor for our reaction would be approximately 30% since there are three polarized protons from the three deuterons and seven unpolarized protons from ^{15}N . However, this would not take into account the scattering from the target material housing or the liquid helium bath; therefore, more refined methods were developed for EG1b analysis. The dilution factor in this analysis was extracted by modeling the background with existing data and is explained in a technical note by Sebastian Kuhn[55].

To summarize the procedure of calculating the dilution factor in this analysis, counts from each target (ammonia=A, carbon=C, and empty=MT) were obtained using all cuts and kinematic bins in this analysis. An overall count per bin per target was then calculated by

$$n(p_m, Q^2, \cos\theta_{nq})_{C,MT} = \frac{N_{C,MT}^+ + N_{C,MT}^-}{FC^+ + FC^-} \quad (56)$$

and

$$n(p_m, Q^2, \cos\theta_{nq})_A = \frac{\frac{N_A^+}{FC^+} + \frac{N_A^-}{FC^-}}{2.0} \quad (57)$$

where the raw counts (N) were normalized by the accumulated beam charge (FC).

The counts from each target can be written in terms of the contributions of entrance and exit foils (F), densities (ρ), and thicknesses (ℓ) of the liquid Helium-4 bath (He), ^{12}C , ^{15}N , and Hydrogen (H) or Deuterium (D) as

$$\begin{aligned} n_{MT} &= (\rho_F l_F \sigma_F + \rho_{He} L \sigma_{He}) F \\ &= (f \rho_C l_C \sigma_C + \rho_{He} L \sigma_{He}) F, \end{aligned} \quad (58)$$

$$n_C = ((1 + f) \rho_C l_C \sigma_C + \rho_{He} (L - l_C) \sigma_{He}) F, \quad (59)$$

and

$$\begin{aligned} n_A &= (f \rho_C l_C \sigma_C + \rho_{He} (L - \ell_A) \sigma_{He}) \\ &+ \rho_A l_A (\sigma_N + 3\sigma_{H/D}) F. \end{aligned} \quad (60)$$

Tables 9, 10, and 11 give the parameters used in these equations to calculate the dilution factor. The calculated total target length (L) for EG1b was extracted by Robert Fersch and can be found in Table 10. Further information can be found in his analysis note[40]. Two new spectra were then formed to estimate pure ^{12}C and

TABLE 9: Values of densities (ρ) and thicknesses (ℓ) used for calculation of the dilution factor.

Item	Value	Comments
$\rho_{Al}l_{Al}$	0.045 g/cm ²	
$\rho_F l_F$		$\rho_{Al}l_{AL} + \rho_K l_K$
-	0.0882 g/cm ²	before run 27997
-	0.0996 g/cm ²	after run 27997
$\rho_K l_K$		
-	0.0432 g/cm ²	before run 27997
-	0.055 g/cm ²	after run 27997
$\rho_C l_C$	0.498 g/cm ² = 0.0415 mol/cm ²	
f		$\rho_F l_F / \rho_C l_C$
-	0.177	before run 27997
-	0.200	after run 27997
ρ_{He}	0.145 g/cm ³ = 0.0362 mol/cm ³	
ρ_C	2.17 g/cm ³ = 0.180 mol/cm ³	
ℓ_C	0.23 cm	
ρ_A (ND3)	1.056 g/cm ³ = 0.0502 mol/cm ³	

^4He targets. The estimated carbon spectrum was

$$n'_{^{12}\text{C}} = \frac{L}{L + fl_C} n_C - \frac{L - l_C}{L + fl_C} n_{MT} \quad (61)$$

while the helium spectrum was

$$n'_{^4\text{He}} = \frac{1 + f}{L + fl_C} n_{MT} - \frac{f}{L + fl_C} n_C \quad (62)$$

per unit length. A scale factor is used that allowed us to predict the cross section off ^{15}N from the cross section off ^{12}C , per nucleus. The scale factor was calculated as

$$\sigma_{^{15}\text{N}} \approx \frac{7}{6}\sigma_{^{12}\text{C}} + \sigma'_n = \left(\frac{7}{6} + \frac{\sigma'_n}{\sigma_{^{12}\text{C}}}\right)\sigma_{^{12}\text{C}} \quad (63)$$

where σ'_n is the cross section on a bound neutron. The factor $\sigma'_n/\sigma_{^{12}\text{C}}$ for quasi-elastic scattering (e,e'p) is assumed to be zero⁹. Finally, using the two new spectra, the number of counts coming from the non-deuterium part of the ammonia target was

$$n_{A-D} = n_{MT} + l_A \left(\frac{\rho_A}{\rho_C l_C} \left[\frac{7}{6} \right] n'_{^{12}\text{C}} - n'_{^4\text{He}} \right) \quad (64)$$

where l_A is the packing fraction. The values of the packing fraction used in this analysis can be found in Table 11. These values were measured by Robert Fersch and can be found in his analysis note[40]. The dilution factor calculation sometimes produced unreasonable values. If a dilution factor calculation produced a negative value or if the value was 2σ from zero, then those kinematic bins were discarded. The dilution factors used in this analysis can be found in Appendix B.

4.8 BEAM AND TARGET POLARIZATION

In addition to unpolarized background, the measured asymmetry is also reduced because the beam and target were only partially polarized. The beam polarization provided by the MCC was very stable, and was intermittently measured with the Møller Polarimeter (these measurements are invasive). The target polarization was monitored by the NMR system over time. The NMR system was just used as an online monitor of the target polarization. The target material can undergo local depolarization due to radiation damage and heating due to exposure to the electron beam rendering NMR measurements somewhat unreliable. Instead, the product of the beam and target polarization ($P_b P_t$) was determined directly from the data. The values used in this analysis came from the analysis note of Nevzat Guler[41]. These values were extracted from the data by comparing a theoretical value of $A_{||}$ to either an inclusive or exclusive measurement of $A_{||}$ from quasi-elastic scattering off the deuteron. The $P_b P_t$ values were calculated as

$$P_b P_t = \frac{A_{meas}^{quasi-el}}{F_D A_{theo}^{quasi-el}} \quad (65)$$

⁹However, there could be a small contribution from charge exchange reactions which may increase the ^{15}N background by a small percentage. Such a contribution is well within the range covered by systematic errors

TABLE 10: Target Length and statistical errors for Deuterium Target.

Energy Run	Torus Pol.	L	σ_L
1.6	In-bending	1.90272	0.00383681
1.6	Out-bending	1.85267	0.00638954
1.7	In-bending	n/a	n/a
1.7	Out-bending	1.87409	0.00212571
2.3	In-bending	1.7717	0.00354486
2.3	Out-bending	n/a	n/a
2.5	In-bending	1.91712	0.00709403
2.5	Out-bending	1.85551	0.00227562
4.2	In-bending	1.9967	0.00775213
4.2	Out-bending	2.05122	0.00224023
5.6	In-bending	1.77768	0.00643883
5.6	Out-bending	n/a	n/a
5.7	In-bending	1.94808	0.00657901
5.7	Out-bending	1.86957	0.00296175
5.8	In-bending	n/a	n/a
5.8	Out-bending	1.82395	0.00466245

TABLE 11: Packing Fraction and statistical errors for Deuterium Target.

Energy Run	Torus Pol.	ℓ_A	σ_{ℓ_A}
1.6	In-bending	0.68649	0.000199225
1.6	Out-bending	0.675489	0.000524592
1.7	In-bending	n/a	n/a
1.7	Out-bending	0.626241	0.000223394
2.3	In-bending	n/a	n/a
2.3	Out-bending	n/a	n/a
2.5	In-bending	0.597433	0.000381051
2.5	Out-bending	0.63143	0.000175169
4.2	In-bending	0.597861	0.000408581
4.2	Out-bending	0.613027	0.000139086
5.6	In-bending	0.604908	0.000470047
5.6	Out-bending	n/a	n/a
5.7	In-bending	0.589663	0.000558826
5.7	Out-bending	0.570284	0.000249905
5.8	In-bending	n/a	n/a
5.8	Out-bending	0.723237	0.000305603

where $A_{theo}^{quasi-el}$ were calculated using parameterizations of the nucleon electric and magnetic form factors G_E and G_M . Table 12 lists all $P_b P_t$ values used in this analysis.

TABLE 12: All EG1b $P_b P_t$ values organized by beam energy and torus polarity.

Energy _{Beam}	I _{Torus}	Pol. _{Target}	$P_b P_t$	Error
1606	in	+	0.23178	0.01132
1606	in	-	0.17988	0.01074
1723	out	+	0.15237	0.03380
1723	out	-	0.25638	0.03515
2561	in	+	0.26164	0.01677
2561	in	-	0.20413	0.01565
2561	out	+	0.31421	0.02800
2561	out	-	0.21925	0.02473
4238	in	+	0.23679	0.02830
4238	in	-	0.17867	0.02850
4238	out	+	0.15718	0.02054
4238	out	-	0.17944	0.03564
5615	in	+	0.25389	0.04714
5615	in	+	0.27504	0.04978
5725	in	-	0.20472	0.04431
5725	in	+	0.16837	0.06136
5725	out	+	0.18639	0.06113
5725	out	-	0.12537	0.05208
5743	in	+	0.20225	0.04369
5743	in	+	0.21154	0.06641

4.9 TARGET CONTAMINATION

Solid polarized targets typically contain small amounts of polarized materials other than $\vec{d}$. Any such additional polarized materials would produce false asymmetries. Two studies[56][57] of the EG1-DVCS $^{15}\text{ND}_3$ target (which is similar to the

EG1b target) discovered a surprisingly large contribution (10-15%) from free polarized H. Unfortunately, EG1b was run several years ago and there was no possibility of doing mass spectrometry analysis on the EG1b target. For this reason, we performed a study of the EG1b target to identify any contributions from H.

The method used in this analysis relied on a comparison of exclusive ep elastic events from the proton and quasi-elastic events on the deuteron. The events were selected by applying the particle identification methods described in Section 4.4 with the additional cut of

$$||\phi_e - \phi_p| - 180.0| < 3.0, \quad (66)$$

where ϕ_e is the azimuthal angle of the scattered electron and ϕ_p is the azimuthal angle of the scattered proton.

The CLAS detector is much more precise at determining angles than momenta for detected particles. In terms of angular resolutions, the in-plane (polar) resolution is higher than the difference in azimuthal angles[50]. Using the polar component of the proton's momentum (p_θ), we can separate quasi-elastic events and elastic events. The difference between measured and expected polar component of the proton's momentum is calculated as

$$\Delta p_\theta = |p_p|(\sin(\theta_p) - \sin(\theta_Q)), \quad (67)$$

where p_p is the momentum of the proton, θ_p is the polar angle of the proton, and θ_Q is the polar angle of the virtual photon. The polar angle of the virtual photon for elastic scattering is given by

$$\tan\theta_Q = \frac{1}{\left(\frac{E_{beam}}{m_p} + 1.0\right) \tan\left(\frac{\theta_e}{2.0}\right)}, \quad (68)$$

where E_{beam} is the beam energy and m_p is the mass of the proton. We used the relationship for elastic scattering to get the sharpest possible peak for $p(e, e'p)$. For quasi-elastic scattering, a broader peak is expected due to Fermi motion. The width of the quasi-elastic peak reflects the Fermi momentum of the nucleons in the nucleus, while the elastic peak width centered at $\Delta p_\theta=0$ comes only from the (very good) CLAS resolution.

To begin with, histograms were made of Δp_θ for three targets: ^{12}C , $^{15}\text{ND}_3$, and $^{15}\text{NH}_3$ for the 4.2 GeV in-bending beam energy runs. The plots are normalized by their respective Faraday Cup counts. We use a fit to ^{12}C (Fig. 33) to emulate

the background from ^{15}N and ^4He in the ammonia target. The fit has a functional form with five parameters that were optimized for minimum χ^2 in the region around $\Delta p_\theta=0$. This functional form is scaled to match the wings of the distribution for NH_3 and then subtracted to extract the pure H peak for the full NH_3 data. Finally, using the fits from ^{12}C and NH_3 , we are able to decompose the ND_3 data into the wider, quasi-elastic (deuteron) peak, described by a Gaussian and the narrow elastic (hydrogen) peak. The results can be seen in Figs. 33–Figure 35. We were able to determine the percentage of contamination by integrating the ND_3 and NH_3 peaks. The results of integrating showed a 5-6% contamination. However, the d peak is suppressed by a factor of approximately 0.7 due to the cuts on $\phi_e - \phi_p$ so the “true” contamination is around 3.5%. This contamination may come from NH_3 impurities, frozen H_2O , or other sources. The typical value used in previous analyses (EG1a, E155) is around 1.5%, based on typical isotopical purities of $^{15}\text{ND}_3$.

In order to determine if this contamination is polarized, we looked at the double spin cross-section (DSCS). The DSCS difference is given by

$$\text{DSCS} = (n^{\downarrow\uparrow} + n^{\uparrow\downarrow}) - (n^{\uparrow\uparrow} + n^{\downarrow\downarrow}), \quad (69)$$

where n is the number of normalized counts for Δp_θ with arrows indicating the relative beam and target polarizations. The DSCS difference was plotted versus Δp_θ for both polarized targets ND_3 and NH_3 . Using the same fit parameters from before, we fitted the DSCS (Fig. 36). To check the likelihood of a significant contamination, a fixed contribution of 5% hydrogen was added to the fit for ND_3 . This leads to an increase in χ^2 by 1.22, as seen in Figure 37. This increase in χ^2 suggests that the polarized hydrogen contamination was small, with a $1\text{-}\sigma$ exclusion level of perhaps 2%. (A 2% contamination of polarizable H would yield roughly a 4% contribution to the DSCS. This is because H is more highly polarizable than deuterium according to the equal spin temperature theory and because the H peak is not suppressed by our cuts.)

Corrections due to this contamination are discussed in the next section.

FIG. 33: Results of Δp_θ distribution for $^{12}\text{C}(e,e'p)\text{X}$

FIG. 34: Results of Δp_θ for NH_3 targets.

FIG. 35: Results of Δp_θ for ND₃

FIG. 36: Double Spin Cross-Section Difference. The top panel shows the result for $^{15}\text{NH}_3$. The bottom panel is fit with a free scale parameter for both D and H; the best fit results if the scale for H is zero.

FIG. 37: Double Spin Cross-Section Difference for ND₃ with a forced 5% H contamination fit

4.10 POLARIZED BACKGROUND

As stated previously, there are other sources of polarized material in the $^{15}\text{ND}_3$ target. The dilution factor only accounted for unpolarized background. Also, the calculation of $P_b P_t$ only takes into account the polarization of deuterium in the target and the polarization of the beam, and it does not include polarized background. With this in mind, a correction was used to account for the estimated polarized background. The first source of background stemmed from the bound protons in the ^{15}N used in the $^{15}\text{ND}_3$ target; also, approximately 2% of the nitrogen was ^{14}N , with both isotopes somewhat polarized. Secondly, there are possible polarized protons coming from isotopically contaminated ammonia, for example ND_2H_1 . Lastly, as stated in Section 4.9, there are possibly some unpolarized hydrogen atoms coming from frozen H_2O .

$^{15}\text{ND}_3$ targets similar to the one used in EG1b have been used in previous experiments and corrections for polarized backgrounds have been developed. The corrections used in this analysis are derived initially from corrections developed for the E143 experiment at SLAC which used a $^{15}\text{ND}_3$ target [58]. Explanations for and values of terms used in this correction can be found in Table 13. A detailed explanation of the full derivation of this correction used in this analysis is given in a CLAS Analysis Note by Nevzat Guler[41]. The polarized background-corrected asymmetry¹⁰ was calculated as

$$A_{||}^{corr} = C_1 (A_{||} - C_2 A_p), \quad (70)$$

where the C_1 is a correction for material affecting the magnitude of the asymmetry, $C_2 A_p$ corrects for free and bound protons (i.e., a different nuclear species), and A_p is the proton asymmetry. Since the asymmetries are binned in p_m , we considered the possible differences between low p_m and high p_m .

4.10.1 CORRECTION FOR LOW P_M

For the lowest p_m bins 0 and 1, we assumed all asymmetries to be proton asymmetries, since the measured asymmetries are from quasi-elastic scattering. Therefore, the correction equation can be rewritten as

$$A_{||} = (A_{||}^{true} + C_2 A_p) / C_1 = \frac{1 + C_2}{C_1} A_p. \quad (71)$$

¹⁰Radiative corrections have not been applied yet.

As stated previously, the first correction term accounted for isotopic (hydrogen instead of deuterium) impurities and quasi-deuterons in ^{14}N . It can be written as

$$C_1 = \frac{1 + \eta_p \frac{\sigma_p}{\sigma_d}}{1 + D_n/(1 - 1.5w_D)}. \quad (72)$$

where η_p is the fraction of protons from the impurities, $\frac{\sigma_p}{\sigma_d}$ is the cross section of the free proton over the total deuteron cross section, D_n is the contribution from quasi-deuterons in ^{14}N , and w_D is the correction for the D-state contribution of the deuteron. We calculated this contribution as

$$D_n = \eta_N \frac{P_{^{14}\text{N}}}{P_d} \frac{g_{EMC}}{3} \frac{P_{d \text{ in } ^{14}\text{N}}}{P_{^{14}\text{N}}} \approx -0.0006... - 0.0010. \quad (73)$$

where the values for each term can be found in Table 13. Since the overall contribution from ^{14}N was minuscule in comparison to the other terms, it was disregarded in this analysis.

Next, we considered the second correction term which accounts for bound protons in ^{15}N and *polarized* protons due to the impurities in the target. The second term is

$$C_2 = \left(\eta_p f_{pol} \frac{P_p}{P_d} \frac{\sigma_p}{\sigma_d} + (1 - \eta_N) \frac{P_{^{15}\text{N}}}{P_d} \frac{g_{EMC}}{3} \frac{P_{p \text{ in } ^{15}\text{N}}}{P_{^{15}\text{N}}} \frac{\sigma_p^{bound}}{\sigma_d} \right), \quad (74)$$

where f_{pol} is the fraction of free protons that are polarized and the polarization of these protons is[59]

$$P_p = \begin{cases} 0.191 + 0.683P_d & \text{for } P_d > 0.16 \\ 1.875P_d & \text{for } P_d \leq 0.16. \end{cases}$$

The cross section ratio of bound proton and deuterons is highly dependent on the reaction, kinematics, and cuts chosen. For this analysis, this ratio was estimated relating the cross sections to the background n_b and the normalized counts. We knew that in the $\vec{d}(\vec{e}, e'p)n$ channel approximately 62% of the background was due to the seven bound protons in ^{15}N ,¹¹ leading to $\sigma_p^{bound} \propto 0.62n_b/7$.¹² All of the counts minus the background were due to the three deuterons in the $^{15}\text{ND}_3$ target leading to $\sigma_d \propto (n^+ + n^- - n_b)(1/3)$. Thus, the ratio is

$$\frac{\sigma_p^{bound}}{\sigma_d} \approx \frac{3}{7} \frac{0.62n_B}{n^+ + n^- - n_B}. \quad (75)$$

¹¹The remaining 38% come from the bound protons in ^4He and the windows, which are unpolarized. The percentages are estimated using Tables 9–11.

¹²According to the nuclear shell model, the unpaired proton in the $p_{1/2}$ shell in ^{15}N , while the paired protons in the other shells carry zero net polarization.

In conclusion, the correction is

$$A_{||} = \frac{n^+ - n^-}{n^+ + n^- - n_B} \times \frac{C_1}{1 + C_2} = \frac{n^+ - n^-}{a(n^+ + n^-) - bn_B} \quad (76)$$

where

$$\frac{C_1}{1 + C_2} \approx \frac{1 + \eta_p \frac{\sigma_p}{\sigma_d}}{1 + \eta_p f_{pol} \frac{P_p}{P_d} \frac{\sigma_p}{\sigma_d} + (1 - \eta_N) \frac{P_{15N}}{P_d} \frac{g_{EMC}}{3} \frac{P_{pin^{15N}}}{P_{15N}} \frac{\sigma_p^{bound}}{\sigma_d}} \quad (77)$$

becomes factored into two terms

$$a = \frac{1 + \eta_p f_{pol}(P_p/P_d)(\sigma_p/\sigma_d)}{1 + \eta_p(\sigma_p/\sigma_d)} \quad (78)$$

and

$$b = \frac{1 + \eta_p f_{pol}(P_p/P_d)(\sigma_p/\sigma_d) - 0.0886(1 - \eta_N)(P_{15N}/P_d)(P_{pin^{15N}}/P_{15N})}{1 + \eta_p(\sigma_p/\sigma_d)}. \quad (79)$$

A full derivation of this equation can be found in Ref. [41]. For the physics asymmetries in this analysis, we separately determine a and b for each bin in p_m because $\frac{\sigma_p}{\sigma_d}$ varies from greater than two for $p_m < 0.05$ GeV/ c to zero for the higher p_m bins. The values used in this analysis for a and b can be found in Table 14 and Table 15.

Correction for High p_m

For the highest p_m bins (bins 2, 3, and 4) the free protons don't contribute, but the bound protons from ^{15}N may have a different asymmetry than the bound proton in deuterium. Unlike the lower p_m bins, we do not assume that A_p is the measured asymmetry. We therefore revert back to Eq. (70) where

$$A_{||}^{corr} = C_1 (A_{||} - C_2 A_p) \quad (80)$$

and where C_1 is a correction for polarized material affecting the magnitude of the asymmetry, $C_2 A_p$ corrects for free and bound protons, and A_p is the proton asymmetry. In this analysis, the first correction term C_1 has the value 1.0 because free protons cannot contribute at large p_m . For the calculation of the asymmetry, we set C_2 to zero, but for systematic uncertainty calculation the second correction term is calculated as

$$C_2 = \frac{P_{15N}}{P_d} \frac{g_{EMC}}{3} \frac{P_{pin^{15N}}}{P_{15N}} \frac{3}{7} \frac{0.62n_B}{n^+ + n^- - n_B} \quad (81)$$

$$\approx 0.011 \left(\frac{1.0}{F_D} - 1.0 \right) \approx 0.03 \dots 0.24 \quad (82)$$

where F_D is the dilution factor calculated from Section 4.7. The value of A_p was obtained by using values from the two lowest p_m bins. Since the asymmetry coming from the bound proton from ^{15}N is unknown, we use the largest (absolute) magnitude A_p can have. This value is only used for purposes of calculating the systematic uncertainty.

TABLE 13: Description and value of parameters used in nuclear corrections. Alternative values of η are based on an analysis of EG1b data [60]. Values for ratios and polarizations are from Ref. [41] and Ref. [58].

Parameter	Typical value	Description
η_p	0.015...0.035	$\frac{\text{number of protons}}{\text{number of protons}+\text{number of deuterons}}$
η_N	0.02	$\frac{\text{number of } ^{14}\text{N}}{\text{number of } ^{14}\text{N}+\text{number of } ^{15}\text{N}}$
D_n	-0.0006...-0.0010	Correction term for quasi-deuterons
w_d	4.9%	D-state probability
$\frac{P_{^{14}\text{N}}}{P_d}$	0.4 ± 0.08	Ratio of polarization of ^{14}N and deuterium
g_{EMC}	1	Correction for the EMC effect in nitrogen ($g_{EMC}^{14} \approx g_{EMC}^{15}$)
$\frac{P_{d \text{ in } ^{14}\text{N}}}{P_{^{14}\text{N}}}$	-1/3...-0.25	Ratio of polarization of d in ^{14}N and ^{14}N [59]
$\frac{P_{p \text{ in } ^{15}\text{N}}}{P_{^{15}\text{N}}}$	-0.2...-0.33	Ratio of polarization of p in ^{15}N and ^{15}N
$\frac{P_p}{P_d}$	1.2...1.5	Ratio of polarization of protons and d
f_{pol}	$\approx 1/2$	Fraction of polarized protons from contamination[60]
$\frac{P_{^{15}\text{N}}}{P_d}$	-0.4...-0.5	Ratio of polarization of ^{15}N and d

TABLE 14: Polarization correction terms per momentum bin used in this analysis. These correction terms are derived from an assumption of a 3.5% H contamination.

p_m Bin	a	b
1	0.976	0.97
2	1.0	0.99
p_m Bin	C_1	C_2
3,4,5	1.0	0.0

TABLE 15: Polarization correction terms per momentum bin used in this analysis for systematic uncertainty calculation. These correction terms are derived from an assumption of a 1.5% H contamination.

p_m Bin	a	b
1	0.976	0.97
2	1.0	0.99

4.11 RADIATIVE CORRECTIONS

Since the theoretical predictions from Van Orden and Jeschonnek are based on the Born Approximation and does not include higher order QED processes, we must account for all radiative effects. According to Quantum Electrodynamics (QED), one photon exchange (Born Approximation) is dominant in the amplitude because higher order processes introduce terms proportional to the fine structure constant (α). Although the Born Approximation process may be favored, other higher order QED processes can occur. To illustrate sources of these, Fig. 38 shows Feynman diagrams up to α^3 -order processes.

FIG. 38: Higher Order Internal QED Processes

The processes listed in Fig. 38 are called internal since they occur during the interaction. Internal Bremsstrahlung can occur with either the incoming or scattered electron. Another internal process is vacuum polarization where a virtual electron-positron pair is created and annihilated. Lastly, a vertex correction can occur where the incoming electron emits a virtual photon which is reabsorbed after the scattering occurred. We calculated internal radiative effects using the Mo-Tsai formula for elastic ep scattering, applying them to the struck proton in the deuteron in its rest frame.

Bremsstrahlung processes can also occur on the electron before or after scattering. If a real photon is emitted while the electron is passing through the detector or target, the electron loses energy. External Bremsstrahlung was another process considered since this energy loss can be significantly higher than ionization energy loss[61]. Bremsstrahlung processes lead to a shift in kinematic variables like Q^2 , ν , and the

direction of the $\vec{q}$ vector. We calculated Bremsstrahlung loss in the target using known radiative lengths and the Mo-Tsai formulas[62].

We correct for all radiative effects by comparing a simplified PWIA simulation of the whole experiment with Born asymmetries to the same simulation with all radiative effects present. This model of the reaction was run twice for beam energies of 1.6, 2.5, 4.2, and 5.7 GeV¹³ using the in-bending configuration¹⁴. For this simulation, we used a simplified description of the scattering process: After calculating the energy and (reduced) polarization of the incoming electron¹⁵, the proton momentum and polarization is randomly generated following the wave function of the deuteron consisting of both $L=0$, $S=1$ (S -state) and $L=2$, $S=1$ (D -state) components (we used the Argonne wave function [63].) The cross section and asymmetry for the scattering process is calculated in the rest frame of the proton (observing proper 4-momentum conservation), including internal radiative effects for the second iteration. Finally, the outgoing electron kinematics is transformed back into the lab frame and (for the second iteration) further radiative energy loss is applied. Each event is passed through a simplified parametrization of the CLAS acceptance, including all cuts, and the corresponding kinematic bin is updated. We confirmed that this method without radiative corrections gives results which are very close to the full theoretical description by Van Orden et al. for their PWIA case. We assume that the effect of radiation is similar for PWIA asymmetries and those including FSI effects. The two sets of asymmetries computed by the model were then compared to one another and the difference was calculated as

$$\delta A_{||} = A_{||}^{rad} - A_{||}^{unrad}, \quad (83)$$

where $A_{||}^{rad}$ was the model with the radiative effects and $A_{||}^{unrad}$ had no radiative effects. The radiative effects observed with this method caused a suppression of the asymmetry value (i.e., moving $A_{||}$ closer to zero). The suppression appeared to have a dependence on the magnitude of the asymmetry value as well as Q^2 . Plots of $\delta A_{||}$ versus $A_{||}^{rad}$ were produced for each Q^2 bin. These plots can be found in Appendix C. A linear fit was then calculated for each Q^2 bin for each energy run. This linear

¹³Beam energies of 1.7, 5.6, and 5.8 were not modeled since they were combined with similar beam energy runs.

¹⁴Radiative effects should not vary appreciably between in-bending and out-bending runs.

¹⁵The electron was produced by external radiation in the target. This was only done for the second iteration where the radiated asymmetry was simulated.

fit had the functional form

$$\delta A_{||} = \tau A_{||}^{rad}(Q^2) + \kappa, \quad (84)$$

where τ and κ will be our correction terms. However, several values were found to not agree with the fit. These value all belonged to the backwards $\cos \theta_{nq}$ bin ($-1.0 > \cos \theta_{nq} > -0.33$). All values in the backwards $\cos \theta_{nq}$ bin were fit separately with the same functional form

$$\delta A_{||} = \tau_B A_{||}^{rad}(\text{Cos}\theta_{nq}) + \kappa_B \quad (85)$$

using only the values in the backwards bin. In conclusion, all asymmetries in the backwards $\cos \theta_{nq}$ bin were corrected in one bin, while the remaining asymmetries were corrected for each Q^2 bin. As stated before, radiative effects yield reconstructed momentum transfer ($\vec{q}$) vectors that appear more forward than they “actually” are. This means that missing momenta that appear backward, may in fact, be forward or sideways, and tend to be reconstructed as larger than they are. Hence, backward spectator angles are particularly sensitive to radiative corrections.

Similar to previous polarization experiments [64],[62], and [65] (e.g., the EG1b run group), a scaling and additive term were created for each specific kinematic bin. Using the correction terms τ and κ , an un-radiated asymmetry was formulated using Eqs. (83) and (84) as

$$A_{||}^{unradiated} = \frac{A_{||}}{\alpha} - \beta \quad (86)$$

where

$$\alpha = \frac{1}{1 - \tau} \quad \text{and} \quad \beta = \kappa \quad (87)$$

where values for τ and κ can be found in Table 16 and Table 17. The plots used to determine these values can be found in Appendix C.

TABLE 16: Values for Radiative Correction for for Q^2 bins per Beam Energy.

Beam Energy	Q^2 Bin	τ	κ
1.6	0	-0.0955	-0.0026
1.6	1	-0.0424	-0.0041
1.6	2	-0.0402	0.0001
1.6	3	n/a	n/a
2.5	0	-0.2804	-0.0286
2.5	1	-0.0445	-0.0063
2.5	2	-0.0471	0.0019
2.5	3	-0.0211	-0.0003
4.2	0	n/a	n/a
4.2	1	0.0426	0.0036
4.2	2	-0.0504	0.0005
4.2	3	-0.0632	0.0088
5.7	0	n/a	n/a
5.7	1	n/a	n/a
5.7	2	0.0184	0.0008
5.7	3	-0.0063	-0.0021

TABLE 17: Values for Radiative Correction for the backwards $\cos \theta_{nq}$ bin per Beam Energy.

Beam Energy	τ_B	κ_B
1.6	-0.0816	0.0095
2.5	-0.149	0.0271
4.2	-0.0471	-0.0011
5.7	-0.1121	0.0144

4.12 SYSTEMATIC UNCERTAINTY

We studied the possible influence of every input and parameter on the final asymmetries to determine their uncertainties due to systematic effects. Initially, the asymmetries, $A_{||}$, were calculated for every kinematic bin with all corrections. The asymmetries that come from this calculation are defined to be the standard asymmetries $A_{||}^{standard}$ and are used as a baseline for all systematic uncertainty calculation. Next, one element in the analysis chain is altered either by a value change or by excluding the element. Finally, $A_{||}$ was recalculated with the alteration and the new asymmetries are called alternative asymmetries $A_{||}^i$ with index i to differentiate which element was changed. Generally, for input parameters with a given statistical error, the element a is altered by

$$a_i = a_0 + \sigma_a^{statistical}, \quad (88)$$

and for elements without a statistical error, like corrections to the data, the asymmetry is calculated without the correction applied. The difference between the standard and alternative asymmetry is calculated as

$$\delta A_{||}^i = (A_{||}^{standard} - A_{||}^i) \quad (89)$$

The resulting differences in asymmetries are added in quadrature to form a total systematic uncertainty given by

$$\sigma_{systematic}^{total} = \sqrt{\sum (\delta A_{||}^{(i)})^2}. \quad (90)$$

This procedure is automated in the program that calculates asymmetries. The asymmetry calculation is run fourteen times, once for each variation in Table 18. The resulting asymmetries are stored in arrays. The values of $A_{||}^i$, $\delta A_{||}^i$, and $\sigma_{systematic}^{total}$ are written to tables to study individual affects on the total systematic uncertainty.

The systematic effects for this analysis can be categorized into three general categories with several elements in each category for this analysis. These categories are

1. Dilution Factor,
2. Corrections, and
3. Polarizations.

TABLE 18: Index key used for calculating systematic uncertainties.

Error Index	Varied Input	Quantity Affected
0	Standard Analysis	n/a
1	L (Ammoniated Deuterium Target Length)	F_D
2	ℓ_a (Packing Fraction)	F_D
3	$\rho_c l_c$	F_D
4	ρ_a	F_D
5	Thickness of Kapton	F_D
6	Density of Kapton	F_D
7	Thickness of Aluminum	F_D
8	$P_b P_t$ –In-bending torus;+target polarization	$P_b P_t$
9	$P_b P_t$ –In-bending torus;-target polarization	$P_b P_t$
10	$P_b P_t$ –Out-bending torus;+target polarization	$P_b P_t$
11	$P_b P_t$ –Out-bending torus;-target polarization	$P_b P_t$
12	Radiative Correction	a,b,C ₁ ,C ₂
13	Polarized Background Correction	τ, κ

4.12.1 DILUTION FACTOR

The largest systematic uncertainty comes from the calculation of the dilution factor. As seen in Section 4.7, the calculation of the dilution factor involves several measured quantities. Every variable that was used in the dilution factor was accounted for in the systematic uncertainty. Additionally, the statistical error on measured counts was propagated to the statistical error of $A_{||}$.

Target Length and Packing Fraction

The target length could vary somewhat from run to run and therefore affect the dilution factor. The nominal value for the target length was 1.9 cm. This value included the target cell, the liquid Helium, and the target windows. The target shape could distort slightly. Varying pressure distorted the shape of the window material affecting the target length. Liquid Helium overflow could also cause the target length to change. Lastly, the beam position entering the target would cause the target length to change because the target windows were not flat and had curvature due to

pressure. The packing fraction ℓ_a which is the percentage of volume occupied in the target by deuterated ammonia is also somewhat uncertain.

The target length and packing fraction were calculated separately for different data sets. The lengths were calculated by the EG1b group from two different methods [48]. The first method was a data driven method. This method used assumptions that neglected the EMC effect. Cuts to exclude regions of W where the EMC effect could be large were required for this method, and at low energies this cut excluded a significant amount of data. The second method used the radiated cross section model to determine the target length. This method could be used in all W regions. Both methods produced similar target lengths, but the value produced from the radiated cross section method was used for this analysis. To calculate the systematic uncertainty due to the target length and packing fraction, the alternative values from the data driven model were used. These alternative values are found in Table 19 and Table 20.

TABLE 19: Target Length and statistical errors for Deuterium Target for two different calculation methods.

Energy Run	Torus Pol.	$L_{cross\ section}$	σ_L	L_{data}	σ_L
1.6	In-bending	1.90	0.0038	1.93	0.0056
1.6	Out-bending	1.85	0.0063	1.82	0.0109
1.7	In-bending	n/a	n/a	n/a	n/a
1.7	Out-bending	1.87	0.0021	1.87	0.0036
2.5	In-bending	1.92	0.00709	1.93	0.0072
2.5	Out-bending	1.86	0.0023	1.84	0.0028
4.2	In-bending	2.00	0.0078	2.01	0.0060
4.2	Out-bending	2.05	0.0022	2.04	0.0021
5.6	In-bending	1.78	0.0064	1.77	0.0044
5.6	Out-bending	n/a	n/a	n/a	n/a
5.7	In-bending	1.95	0.0066	1.93	0.0044
5.7	Out-bending	1.87	0.0029	1.82	0.0023
5.8	In-bending	n/a	n/a	n/a	n/a
5.8	Out-bending	1.82	0.0047	1.79	0.0036

TABLE 20: Packing Fraction and statistical errors for Deuterium Target for two different calculation methods.

Energy Run	Torus Pol.	$\ell_A^{cross\ section}$	σ_{ℓ_A}	ℓ_A^{data}	σ_{ℓ_A}
1.6	In-bending	0.6865	0.0002	0.6611	0.0005
1.6	Out-bending	0.6755	0.0005	0.6394	0.0022
1.7	In-bending	n/a	n/a	n/a	n/a
1.7	Out-bending	0.6262	0.0002	0.5926	0.0008
2.5	In-bending	0.5974	0.0004	0.5887	0.0009
2.5	Out-bending	0.6314	0.0002	0.6179	0.0003
4.2	In-bending	0.5979	0.0004	0.5977	0.0009
4.2	Out-bending	0.6130	0.0001	0.6084	0.0003
5.6	In-bending	0.6049	0.0005	0.6045	0.0011
5.6	Out-bending	n/a	n/a	n/a	n/a
5.7	In-bending	0.5897	0.0006	0.5947	0.0013
5.7	Out-bending	0.5703	0.0002	0.5719	0.0005
5.8	In-bending	n/a	n/a	n/a	n/a
5.8	Out-bending	0.7232	0.0003	0.7226	0.0006

Materials

The model used to calculate the dilution factor used the densities and thickness of materials of the target and target housing. These values came from measurements and standard values found in literature [66]. Densities of solids are usually given at room temperature¹⁶ and depend somewhat on temperature.¹⁷ Additionally, independent measurements of material thicknesses yielded different results[67]. For example, the thickness of the carbon target (ℓ_C) had two additional measurements that gave results of 0.224 cm and 0.24 cm, thus a 0.01 cm variation was used. Similarly for the density of carbon, there were multiple measurements that yielded different results.

¹⁶The value of the densities used in this analysis note were not room temperature values.

¹⁷The LHe fluid density was taken from cryogenic property tables at the nominal temperature of 1.0 K.

Extra Kapton foils were added to the target at some point, thus increasing the thickness. The values of the densities and thicknesses were varied as shown in Table 21.

TABLE 21: Values and alterations used in calculating systematic uncertainty.

$\rho_C \ell_C$	^{12}C density $\times$ thickness	density(+2%) thickness(+0.01 cm))
ρ_a	Density of Ammoniated Target	+1%
ℓ_K	Kapton thickness	+10% (^{12}C target only)
ρ_K	Density of Kapton	+10%
ρ_{Al}	Density of Al foil	+10%

4.12.2 CORRECTIONS

There were several corrections applied to the asymmetries. These corrections were discussed previously and include the polarized background and radiative corrections.

The corrections for the polarized background varied based on the momentum bin. For p_m bins 1 and 2, originally a 3.5% contamination was assumed, but for this systematic calculation a 1.5% contamination was assumed. The correction terms a and b were varied. Table 15 gives a and b values used for systematic uncertainty calculation. For larger p_m bins, the correction values of C_1 and C_2 were changed. The second correction term had an original value of zero, but for systematic uncertainty calculation it is calculated as

$$C_2 = 0.011\left(\frac{1.0}{F_D} - 1.0\right), \quad (91)$$

where F_D is the dilution factor calculated from Section 4.7.

The radiative corrections implemented in this analysis are very small. The entire correction was treated as a systematic effect and was removed to determine the systematic uncertainty.

4.12.3 BEAM AND TARGET POLARIZATION

The product of beam and target polarization values were treated as a measured quantity. We used calculated values obtained from exclusive analysis where the main source of error was statistical in nature. We utilized the statistical error provided to calculate the systematic uncertainty with Eq. (89). Since a different $P_b P_t$ value was provided for each data set, only one $P_b P_t$ value was altered at a time while determining the systematic uncertainty.

In conclusion, all major factors outside of statistics that could contribute to the calculation of asymmetries were included in this analysis of systematic uncertainty. Each parameter that was calculated for the systematic uncertainty were independent from one another to avoid accounting for the same systematic effect multiple times.

CHAPTER 5

RESULTS

5.1 ASYMMETRY

After applying all corrections, the final physics (Born) asymmetry for p_m bins 0 and 1 are

$$A_{||}(p_m, Q^2, \cos\theta_{nq}) = \frac{\tau}{P_b P_t} \frac{n_A^+ - n_A^-}{a(n_A^+ + n_A^-) - b(n_B)} - \kappa. \quad (92)$$

where n_B is the unpolarized background, $P_b P_t$ is the product of the beam and target polarizations, τ and κ are correction terms associated with radiative corrections, and a and b are corrections terms for polarized background. The Born asymmetry for p_m bins 2, 3, and 4 is calculated as

$$A_{||}(p_m, Q^2, \cos\theta_{nq}) = C_1 \left(\frac{\tau}{P_b P_t} \frac{n_A^+ - n_A^-}{(n_A^+ + n_A^-) - (n_B)} - \kappa - C_2 A_p \right). \quad (93)$$

The Born asymmetry was calculated for every data set for every kinematic bin for the final results.

5.2 COMBINATION OF ASYMMETRIES

As stated previously, the EG1b run comprised several beam energies, target polarizations, and torus configurations. Asymmetries were calculated for each data set given by Table 1. In order to decrease statistical errors, asymmetries from several data sets were combined. The asymmetries could only be combined if they were statistically compatible. With this in mind, several steps were taken to ensure quality results.

Before any data sets could be combined, a student t-test was performed. This test illuminated any possible discrepancies between data sets. A t-value was calculated as

$$t_{A_{||}}(p_m, Q^2, \cos\theta_{nq}) = \frac{A_{||}^{G1} - A_{||}^{G2}}{\sqrt{\sigma_{A_{||}^1}^2 + \sigma_{A_{||}^2}^2}} \quad (94)$$

where asymmetries and their corresponding statistical errors σ from different sets are labeled with superscript $G1$ or $G2$. The t-test assumed that for each data set

the difference between the asymmetries is small. Therefore, the distribution of the t-values between the two sets should result in a Gaussian with a mean of zero and standard deviation of 1.0. The results of the t-test for combining opposite target

FIG. 39: A t-test from 2.5 GeV and 4.2 GeV different beam energies combining opposite target polarizations

polarization can be found in Table 22. After a t-test was performed and the asymmetries determined to be statistically compatible, combining of sets commenced. First, data sets with different target polarization but the same beam energy and the same torus polarization were combined. The data sets were combined using the equation

$$A_{\text{combined}} = \frac{\frac{A_{G1}}{\sigma_{G1}^2} + \frac{A_{G2}}{\sigma_{G2}^2}}{\frac{1}{\sigma_{G1}^2} + \frac{1}{\sigma_{G2}^2}}, \quad (95)$$

where asymmetries and their corresponding statistical errors σ from different sets are labeled with a subscripts $G1$ or $G2$. Then, the data sets with similar energies needed to be combined. This was done by combining the data sets of equal torus polarity first. For example, data sets 1.6+ and 1.7+ were combined to form the 1.x+ data set and data sets 1.6- and 1.7- were combined to form the 1.x- data set. This was also done for the 5.6, 5.7 GeV, and 5.8 GeV data sets to form the 5.x GeV data set. At

TABLE 22: Results of t-test for combining opposite target polarization

Data Set	t_{avg}	$\sigma_{t_{avg}}$	N_{bins}
1.6+	-0.115	0.164	37
1.7-	-0.034	0.196	26
2.5+	0.097	0.156	41
2.5-	0.141	0.160	39
4.2+	0.116	0.224	20
4.2-	-0.008	0.196	26
5.6+	-0.152	0.267	14
5.7+	-0.323	0.378	7
5.7-	0.375	0.354	8
5.8-	0.165	0.289	12

this point we had eight different data sets with four different beam energies: $1.x\pm$, $2.5\pm$, $4.2\pm$, and $5.x\pm$. The $2.5\pm$ asymmetries can be seen in Figure 40. For our final combined values, we combined data sets with opposite torus polarizations using Eq. (95) to obtain the four final data sets: $1.x$, 2.5 , 4.2 , and $5.x$. These values were then compared with the theoretical predictions. The theoretical predictions were similarly combined with the same weights.

5.3 FINAL ASYMMETRIES

Our final results for the double-spin asymmetry, $A_{||}$ are presented in this section and in Appendix A. We compare our data to the Jeschonnek and Van Orden model for both the FSI and the PWIA case. Two representative samples of the results can be seen in Figs. 41 and 42. Similar plots of all the final asymmetries can be found in Appendix A. Each plot of $A_{||}$ is for one Q^2 bin and for one beam energy data set. Each p_m bin is assigned a different color and symbol to differentiate between the bins. The $\cos\theta_{nq}$ bins are plotted along the x-axis. In each case, two identical plots of experimental values are overlaid with curves for the model which included FSIs and, secondly, for the PWIA model. Theoretical predictions were plotted as colored lines where each color matches the color of the corresponding p_m bin. It should be noted that for the $5.x$ GeV results, there was no model for FSI and the results are

FIG. 40: These plots show the 2.5+ GeV and 2.5- GeV $A_{||}$ before the asymmetries were combined.

only compared with the PWIA model.

In Fig. 41, it can be seen that there is very little difference between the FSI and PWIA model for the first three p_m bins. For the two highest p_m bins, the two models predict different values as a function of $\cos \theta_{nq}$. The FSI model predicts a more positive asymmetry in the forward $\cos \theta_{nq}$ bin than the PWIA model. The same observations can be made in Fig. 42. Both models describe the low p_m data well and correctly predict the sign change for $A_{||}$ with increasing p_m .

A χ^2 test for goodness of fit was used to determine which model described the data the best. The $\chi^2/d.o.f.$ values were calculated for each Q^2 bin as

$$\chi^2 = \frac{\sum_{p_m, \cos \theta_{nq}} \frac{(A_{||}^{measured} - A_{||}^{theory})^2}{\sigma_{data}^2}}{N} \quad (96)$$

where N (=degree of freedom, d.o.f.) is the number of data points summed over. Comparisons with the FSI model yielded lower $\chi^2/d.o.f.$ than those for the PWIA model. The values for $\chi^2/d.o.f.$ can be found in Table 23.

This suggests that the FSI model provides a better prediction of the asymmetries than the PWIA model. We notice that the theoretical predictions for the two lowest missing momentum bins for FSI and PWIA barely differ at all. The largest difference

FIG. 41: $A_{||}$ for Beam Energy of 1.X GeV and Q^2 Bin 1 FIG. 42: $A_{||}$ for Beam Energy of 2.5 GeV and Q^2 Bin 1

between the two predictions is seen at the higher missing momentum bin and at the most forward $\cos \theta_{nq}$ angles. The asymmetries become more positive at these values

TABLE 23: $\chi^2/d.o.f.$ of PWIA Model and FSI Model. All $\chi^2/d.o.f.$'s were calculated using all data points in the Q^2 bin. The 5.x data set could only be compared with the model using PWIA. The 2.5 GeV Q^2 bin 3 and 4.2 GeV Q^2 bin 3 data sets have little data and high statistical uncertainty above Q^2 bin 2 resulting in the low $\chi^2/d.o.f.$ value stated.

Energy _{Beam}	Q^2 Bin	FSI $\chi^2/d.o.f.$	PWIA $\chi^2/d.o.f.$
1.x	0	3.415	3.670
1.x	1	1.386	1.576
1.x	2	1.693	2.259
2.5	0	1.449	2.292
2.5	1	1.472	5.156
2.5	2	1.669	2.221
2.5	3	0.566	0.521
4.2	0	n/a	n/a
4.2	1	1.012	1.014
4.2	2	0.782	0.841
4.2	3	0.575	0.336
5.x	1	n/a	n/a
5.x	2	n/a	2.132
5.x	3	n/a	2.785

as predicted by the FSI model.

5.3.1 TEST OF $P_B P_T$

In addition to extracting double spin asymmetries, this analysis provided a check on the $P_b P_t$ values used in Section 4.8. The product of beam and target polarizations was taken from a previous analysis of quasi-elastic EG1b data as described in Section 4.10. By scaling the theoretical values from the model by Jeschonnek and Van Orden and looking for a minimum of $\chi^2/d.o.f.$ as a function of the scale parameter, we checked whether different values for $P_b P_t$ might give an even better agreement. The results are shown in Table 24 and show that the existing $P_b P_t$ values were mostly accurate within a few percent. For the 5.x GeV data set, it should be noted that

the data set had large error bars which affected the reliability of this test. Also, we covered a more limited range in Q^2 than the original $P_b P_t$ analysis; however, our results could be interpreted to indicate that $P_b P_t$ for the 5 GeV data set may be overestimated by as much as 48%.

TABLE 24: Results of minimizing $\chi^2/d.o.f.$. All $\chi^2/d.o.f.$'s were calculated using only the first three p_s bins with the inclusion of both statistical and systematic uncertainties. The 5.x data set could only be compared with the model using PWIA.

Energy _{Beam}	Original $\chi^2/d.o.f.$	Scale Factor	New $\chi^2/d.o.f.$
1.x	0.6706	1.02	0.6607
2.5	0.7198	1.09	0.6411
4.2	0.3937	0.93	0.3641
5.x	1.2402	1.48	0.7874

5.3.2 $A_{||}(\cos \theta_N)$

Asymmetries were also extracted as a function of $\cos \theta_n$, which is the angle of the spectator neutron relative to the beam direction (z -axis) and therefore the target spin direction. Theoretically, the asymmetry values are predicted not to be flat as a function of $\cos \theta_n$ at large p_m (similar to the results described in Section 2.5).

In order to extract the new asymmetries, the data were re-binned in nine $\cos \theta_n$ bins calculated as

$$\cos \theta_n = \frac{\vec{p}'_n \cdot \vec{e}}{|\vec{e}| |\vec{p}'_n|}. \quad (97)$$

with a bin width of $\Delta \cos \theta_n = 2/9$. Due to the finer bin size, statistics were limited and the asymmetries were only calculated for the 2.5 GeV data set. The Q^2 and p_m binning from the main analysis was used. The same prescription as outlined in this dissertation was used to calculate the new asymmetries. Radiative corrections were not used since the correction values could not be easily translated to the new bins. Theoretical predictions using PWIA only were calculated based on a simplified model. Figures 43–46 show the results of this measurement. Due to the limited

statistics the expectation of a non-flat asymmetry at higher missing momenta could not be verified with sufficiently high significance.

FIG. 43: Measured values of $A_{||}(\cos \theta_n)$ in Q^2 bin 0 plotted against predicted values using the PWIA model

FIG. 44: Measured values of $A_{||}(\cos \theta_n)$ in Q^2 bin 1 plotted against predicted values using the PWIA model

FIG. 45: Measured values of $A_{||}(\cos \theta_n)$ in Q^2 bin 2 plotted against predicted values using the PWIA model

FIG. 46: Measured values of $A_{||}(\cos \theta_n)$ in Q^2 Bin 3 plotted against predicted values using the PWIA model

CHAPTER 6

SUMMARY AND OUTLOOK

From the EG1b data, we extracted the double spin asymmetry $A_{||}$ in the $\vec{d}(\vec{e}, e'p)n$ reaction over a wide range in kinematics, significantly extending the reach of existing data. Data were sorted into bins of the squared four-momentum transfer Q^2 (0.13–3.17 (GeV/c)²), the cosine of the angle between the virtual photon and the neutron $\cos\theta_{nq}$ (-1.0–1.0), and missing momentum p_m (0.0–0.5 GeV/c). The raw asymmetries were corrected for unpolarized background, polarized background, and radiative effects. Final results of the asymmetries were obtained by combining similar data sets to obtain four final data sets: 1.x, 2.5, 4.2, and 5.x GeV.

We compared these results to two sets of theoretical predictions of $A_{||}$ by Sabine Jeschonnek and J.W. Van Orden. One set of theoretical predictions used PWIA to calculate $A_{||}$, while the other set included FSIs to calculate $A_{||}$. When comparing the two theoretical curves to the experimental data at lower missing momenta, there was very little difference between the theoretical values for PWIA versus FSI and the data agreed generally with both. However, at the highest missing momenta, we observed that FSIs are needed to predict a positive shift of the asymmetries at the most forward angles. Figure 42 demonstrates this observation. Asymmetries extracted at backward angles also had a tendency to be more negative than predicted by the model. This trend can be seen most in the second missing momentum bin. This observation might be due to the limited statistics or effects from the edges of the experimental acceptance. The 5.X GeV data set does not yet have predictions that include FSIs. This is due to the fact that the theoretical model is limited in p-n relative momentum. A new calculation by William Ford and J.W. Van Orden [68] will extend the energy range of the model and allow for theoretical predictions of $A_{||}$ that include FSIs at the 5.X GeV range.

Additional values of $A_{||}$ could be extracted from newer CLAS experiments. Experimental data from the CLAS EG4 experiment should allow a more precise measurement of $A_{||}$ in the lower beam energy range. The EG4 experimental run scattered polarized electrons at energies of 1.0–3.0 GeV from a longitudinally polarized ND₃

target. This experiment implemented a new Cherenkov detector to allow for detecting particles at scattering angles of 6° . Also, the EG1-DVCS experiment used a longitudinally polarized ND_3 target but scattered polarized electrons at higher energies around 5.0–6.0 GeV.

The $A_{||}$ values measured in this dissertation show an overall agreement with the Jeschonnek and Van Orden model indicating that the reaction is quite well understood microscopically. One can therefore confidently use the $\vec{d}(\vec{e}, e'p)n$ reaction to determine the polarization of deuterium targets used for measurements of other spin observables, e.g., spin structure functions in inelastic inclusive scattering.

BIBLIOGRAPHY

- [1] J. J. Thomson, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science **46**, 528 (1898)
- [2] R. A. Millikan., Phys. Rev. **2**, 109 (Aug 1913), <http://link.aps.org/doi/10.1103/PhysRev.2.109>
- [3] H. Geiger and E. Marsden, Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 495(1909)
- [4] P. E. Rutherford, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science **21**, 669 (1911)
- [5] J. Chadwick, Nature **129**, 312 (1932)
- [6] W. Heisenberg, Zeitschrift für Physik A Hadrons and Nuclei **77**, 1 (1932)
- [7] M. G. Mayer and J. H. D. Jensen, *Elementary theory of nuclear shell structure* (Wiley New York, 1955)
- [8] H. Yukawa, Proc. Phys.-Math. Soc. Japan, III. Ser. **17**, 48 (1935)
- [9] L.-V. de Broglie, *On the theory of quanta*, Ph.D. thesis (1924)
- [10] R. W. McAllister and R. Hofstadter, Phys. Rev. **102**, 851 (May 1956), <http://link.aps.org/doi/10.1103/PhysRev.102.851>
- [11] P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (Apr 2000), <http://link.aps.org/doi/10.1103/RevModPhys.72.351>
- [12] E. Kessler, Jr, M. Dewey, R. Deslattes, A. Henins, H. Börner, M. Jentschel, C. Doll, and H. Lehmann, Physics Letters A **255**, 221 (1999), ISSN 0375-9601, <http://www.sciencedirect.com/science/article/pii/S037596019900078X>
- [13] I. I. Rabi, J. M. B. Kellogg, and J. R. Zacharias, Phys. Rev. **46**, 157 (Aug 1934), <http://link.aps.org/doi/10.1103/PhysRev.46.157>
- [14] C. Bertulani, *Nuclear physics in a nutshell* (Princeton Univ Press, 2007)

- [15] R. F. Code and N. F. Ramsey, Phys. Rev. A **4**, 1945 (Nov 1971), <http://link.aps.org/doi/10.1103/PhysRevA.4.1945>
- [16] J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, Phys. Rev. **55**, 318 (Feb 1939), <http://link.aps.org/doi/10.1103/PhysRev.55.318>
- [17] G. M. Murphy and H. Johnston, Phys. Rev. **46**, 95 (Jul 1934), <http://link.aps.org/doi/10.1103/PhysRev.46.95>
- [18] R. J. Holt and R. Gilman, Reports on Progress in Physics **75**, 086301 (2012), <http://stacks.iop.org/0034-4885/75/i=8/a=086301>
- [19] F. Gross, J. W. Van Orden, and K. Holinde, Phys. Rev. C **45**, 2094 (May 1992), <http://link.aps.org/doi/10.1103/PhysRevC.45.2094>
- [20] S. Jeschonnek and J. W. Van Orden, Phys. Rev. C **78**, 014007 (Jul 2008), <http://link.aps.org/doi/10.1103/PhysRevC.78.014007>
- [21] S. Jeschonnek and J. W. Van Orden, Phys. Rev. C **80**, 054001 (Nov 2009), <http://link.aps.org/doi/10.1103/PhysRevC.80.054001>
- [22] G. G. et al. et al., *Measurements of the Fifth Structure Function fo the Deuteron*, Tech. Rep. CLAS Analysis Note Under Review (JLAB, 2012)
- [23] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C **76**, 025209 (Aug 2007), <http://link.aps.org/doi/10.1103/PhysRevC.76.025209>
- [24] H. Arenhövel, W. Leidemann, and E. Tomusiak, The European Physical Journal A-Hadrons and Nuclei **23**, 147 (2005)
- [25] H. Arenhövel, W. Leidemann, and E. L. Tomusiak, Phys. Rev. C **52**, 1232 (Sep 1995), <http://link.aps.org/doi/10.1103/PhysRevC.52.1232>
- [26] Z.-L. Zhou *et al.*, Phys. Rev. Lett. **82**, 687 (Jan 1999), <http://link.aps.org/doi/10.1103/PhysRevLett.82.687>
- [27] I. Passchier *et al.*, Phys. Rev. Lett. **88**, 102302 (Feb 2002), <http://link.aps.org/doi/10.1103/PhysRevLett.88.102302>

- [28] D. Hasell, R. Milner, R. Redwine, R. Alarcon, H. Gao, M. Kohl, and J. Calarco, Annual Review of Nuclear and Particle Science(2011)
- [29] A. DeGrush, *Single and double polarization observables in the electrodisintegration of the deuteron from BLAST*, Ph.D. thesis, Massachusetts Institute of Technology (March 2010)
- [30] Maschinot, A., *Analysis of Scattered Protons in Deuteron Electrodisintegration with a Polarized Electron Beam and an Internal Vector Tensor Polarized Target at BLAST*, Ph.D. thesis, Massachusetts Institute of Technology (2005)
- [31] S. Galster, H. Klein, J. Moritz, K. Schmidt, D. Wegener, and J. Bleckwenn, Nuclear Physics B **32**, 221 (1971), ISSN 0550-3213, <http://www.sciencedirect.com/science/article/pii/055032137190068X>
- [32] H. Zhu, *A Measurement of the Neutron Electric Form Factor in $D(e, e'n)p$ Quasi-elastic Scattering at $Q^2=0.5$ (GeV/c)²*, Ph.D. thesis, University of Virginia (2000)
- [33] C. W. Leemann, D. R. Douglas, and G. A. Krafft, Annual Review of Nuclear and Particle Science **51**, 413 (2001)
- [34] J. Grames, *Unique Electron Polarimeter Comparison and Spin-Based Energy Measurement*, Tech. Rep. JLAB-TN-01-029 (JLAB, 2001)
- [35] B. Mecking, G. Adams, S. Ahmad, E. Anciant, M. Anghinolfi, B. Asavapibhop, G. Asryan, G. Audit, T. Auger, H. Avakian, *et al.*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **503**, 513 (2003)
- [36] M. Mestayer, D. Carman, and B. Asavapibhop, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **449**, 81 (July 2000)
- [37] G. Adams, V. Burkert, R. Carl, T. Carstens, V. Frolov, L. Houghtlin, G. Jacobs, M. Kossov, M. Klusman, B. Kross, *et al.*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **465**, 414 (2001)

- [38] E. Smith, T. Carstens, J. Distelbrink, M. Eckhause, H. Egiyan, L. Elouadrhiri, J. Ficenec, M. Guidal, A. Hancock, F. Hersman, *et al.*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **432**, 265 (1999)
- [39] M. Amarian, G. Asryan, K. Beard, W. Brooks, V. Burkert, T. Carstens, A. Coleman, R. Demirchyan, Y. Efremenko, H. Egiyan, *et al.*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **460**, 239 (2001)
- [40] R. G. Fersch, *Measurement of Inclusive Proton Double-Spin Asymmetries and Polarized Structure Functions in the CLAS EG1b Experiment*, Tech. Rep. CLAS Analysis Note Under Review (JLAB, 2012)
- [41] N. Guler, *Measurements of Longitudinal Double Spin Asymmetries and Spin Structure Functions of the Deuteron in the CLAS EG1b Experiment*, Tech. Rep. CLAS Analysis Note Under Review (JLAB, 2012)
- [42] S. Careccia, *Single and Double Spin Asymmetries for π^- Electro-production from the Deuteron in the Resonance Region*, Ph.D. thesis, Old Dominion University (May 2012)
- [43] J. Pierce, *Double Spin Asymmetry in Exclusive π^+ Electroproduction with CLAS*, Ph.D. thesis, University of Virginia (January 2008)
- [44] A. Biselli, *Study of the $\Delta(1232)$ Using Double Polarization Asymmetries*, Ph.D. thesis, Rensselaer Polytechnic Institute (March 2008)
- [45] D. Crabb and D. Day, Nuclear Instruments and Methods in Physics Research A **356**, 9 (1995)
- [46] D. Crabb and W. Meyer, Annual Review of Nuclear and Particle Science **47**, 67 (1997)
- [47] C. Keith, M. Anghinolfi, M. Battaglieri, P. Bosted, D. Branford, S. Bültmann, V. Burkert, S. Comer, D. Crabb, R. De Vita, *et al.*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **501**, 327 (2003)

- [48] R. G. Fersch, *Measurement of Inclusive Proton Double-Spin Asymmetries and Polarized Structure Functions*, Ph.D. thesis, The College of William and Mary (August 2008)
- [49] N. Guler, *Spin Structure of the Deuteron*, Ph.D. thesis, Old Dominion University (December 2009)
- [50] A. Klimenko and S. Kuhn, *Momentum corrections for E6*, Tech. Rep. CLAS-NOTE-2003-005 (JLAB, 2003)
- [51] W. Yao *et al.*, Journal of Physics G: Nuclear and Particle Physics **33**, 1 (2006)
- [52] M. Osipenko, A. Vlassov, and M. Taiuti, *Matching between the electron candidate track and the Cherenkov counter hit*, Tech. Rep. CLAS-NOTE 2004-020 (JLAB, 2004)
- [53] N. Guler, “Studies for Geometrical Cut Parameters for EG1B Experiment ,” http://www.jlab.org/Hall-B/secure/eg1/EG2000/nevzat/GEOM_CC_CUTS/
- [54] A. Vlassov, “CLAS Cerenkov detector,” <https://userweb.jlab.org/~vlassov/cc/index.html>
- [55] S. Kuhn, *Dilution Factor for Exclusive*, Tech. Rep. (Old Dominion University, 2010) <https://userweb.jlab.org/~kuhn/EG1b/ExclusiveDilution.pdf>
- [56] P. Bosted, *NH₃ Correction for ND₃ Target: EG1-DVCS Technical Note 17*, Tech. Rep. (2011) <http://wwwold.jlab.org/Hall-B/secure/eg1-dvcs/technotes/hind/hind.pdf>
- [57] S. Koirala, *EG1-DVCS Part-C Target Contamination*, Tech. Rep. (2012) <http://www.jlab.org/Hall-B/secure/eg1-dvcs/skoirala/FilesForWiki/Contamination.pdf>
- [58] O. A. Rondon, Phys. Rev. C **60**, 035201 (Jul 1999), <http://link.aps.org/doi/10.1103/PhysRevC.60.035201>
- [59] K. Abe *et al.* ((E143 Collaboration)), Phys. Rev. D **58**, 112003 (Oct 1998), <http://link.aps.org/doi/10.1103/PhysRevD.58.112003>
- [60] M. Mayer, *Hydrogen Contamination in the ND₃ Target for EG1b*, Tech. Rep. CLAS-NOTE 2012-004 (JLAB, 2012)

- [61] G. Knoll, New York, John Wiley and Sons, Inc., 1979. 831 p. **1** (1979)
- [62] L. Mo and Y. Tsai, Reviews of Modern Physics **41**, 205 (1969)
- [63] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (Jan 1995), <http://link.aps.org/doi/10.1103/PhysRevC.51.38>
- [64] T. Kuchto and N. Shumeiko, Nuclear Physics B **219**, 412 (1983), ISSN 0550-3213, <http://www.sciencedirect.com/science/article/pii/0550321383906508>
- [65] F. Wesselmann, *Precision Measurement of the Spin Structure of the Proton and the Deuteron*, Ph.D. thesis, Old Dominion University (2000)
- [66] R. Weast, M. Astle, and W. Beyer, *CRC handbook of chemistry and physics*, Vol. 69 (CRC press Boca Raton, FL, 1988)
- [67] S. Kuhn, *Data Analysis for EG1*, Tech. Rep., <http://www.jlab.org/Hall-B/secure/eg1/AnalysisDoc/EG2000DataAnalyis.pdf>
- [68] W. P. Ford and J. W. Van Orden, Phys. Rev. C **87**, 014004 (Jan 2013), <http://link.aps.org/doi/10.1103/PhysRevC.87.014004>

APPENDIX A

ASYMMETRIES

The plots in this appendix are the results of $A_{||}$ from this analysis. The results for one beam energy set and one Q^2 bin are plotted in two separate plots as a function of the $\cos\theta_{nq}$ bins. All five p_m bins are plotted on each graph distinguished by a color and symbol. The colors and symbols for p_m bins are given in Table 25. The statistical error and systematic uncertainty are also plotted on each data point. The inner vertical error line represents that statistical error while the outer error line represents the systematic uncertainty.

TABLE 25: The symbols and colors for p_m bins used in the $A_{||}$ plots.

Bin	Value _{min} (GeV/c)	Value _{max} (GeV/c)	Symbol	Color
0	0.00	0.05	•	Red
1	0.05	0.15	■	Blue
2	0.15	0.25	▲	Purple
3	0.25	0.35	▼	Orange
4	0.35	0.50	★	Green

The theoretical predictions from Sabine Jeschonnek and J.W. Van Orden's model using FSI and PWIA are plotted on the two separate plots as solid lines. The $\chi^2/d.o.f.$ calculated from Eq. (96) is displayed in the lower left hand corner in each plot.

FIG. 47: Beam Energy of 1.X GeV and Q^2 Bin 0 FIG. 48: Beam Energy of 1.X GeV and Q^2 Bin 1

FIG. 49: Beam Energy of 1.X GeV and Q^2 Bin 2 FIG. 50: Beam Energy of 2.5 GeV and Q^2 Bin 0

FIG. 51: Beam Energy of 2.5 GeV and Q^2 Bin 1 FIG. 52: Beam Energy of 2.5 GeV and Q^2 Bin 2

FIG. 53: Beam Energy of 2.5 GeV and Q^2 Bin 3 FIG. 54: Beam Energy of 4.2 GeV and Q^2 Bin 0

FIG. 55: Beam Energy of 4.2 GeV and Q^2 Bin 1 FIG. 56: Beam Energy of 4.2 GeV and Q^2 Bin 2

FIG. 57: Beam Energy of 4.2 GeV and Q^2 Bin 3 FIG. 58: Beam Energy of 5.X GeV and Q^2 Bin 1

(a) PWIA

FIG. 59: Beam Energy of 5.X GeV and Q^2 Bin 2

(a) PWIA

FIG. 60: Beam Energy of 5.X GeV and Q^2 Bin 3

APPENDIX B

DILUTION FACTORS

Here are tables of the dilution factors that were calculated in this analysis. The dilution factors are given by kinematic bin with the associated statistical error. If the dilution factor calculation resulted in a negative number, then the entire bin for that set was discarded.

TABLE 28: Dilution Factors for 2.5 GeV. As noted in Section 4.2.1, there were two 2.5- data sets. Here, these sets are denoted by a (1) or (2).

Data Set	Q^2	$\cos\theta_{nq}$	p_m 1	error	p_m 2	error	p_m 3	error	p_m 4	error	p_m 5	error
2.5+	0	0	0	0	0.147	0.0839	0.132	0.0194	0.123	0.0202	0.0955	0.0252
2.5+	0	1	0	0	0.123	0.135	0.0978	0.0304	0.133	0.0273	0.184	0.0227
2.5+	0	2	0	0	0	0	0.15	0.0744	0.248	0.0347	0.225	0.0265
2.5+	1	0	0.467	0.338	0.183	0.0144	0.131	0.00784	0.105	0.0111	0.154	0.0164
2.5+	1	1	-0.982	1.76	0.156	0.0236	0.14	0.0121	0.18	0.0139	0.205	0.0122
2.5+	1	2	0	0	0.172	0.123	0.168	0.0298	0.218	0.0178	0.259	0.0108
2.5+	2	0	0.665	0.016	0.278	0.00824	0.104	0.01	0.123	0.0179	0.14	0.0327
2.5+	2	1	0.695	0.0161	0.292	0.00971	0.0884	0.0132	0.101	0.0209	0.0937	0.0212
2.5+	2	2	0.773	0.0152	0.416	0.0124	0.128	0.0207	0.0651	0.0312	0.144	0.0215
2.5+	3	0	0.778	0.0297	0.333	0.027	0.167	0.0446	-0.174	0.149	0.0324	0.677
2.5+	3	1	0.802	0.0272	0.356	0.0224	0.0556	0.0362	0.0763	0.0663	0.0866	0.0775
2.5+	3	2	0.768	0.0272	0.387	0.0199	0.11	0.032	0.0236	0.072	0.121	0.0744
2.5-(1)	0	0	0	0	0.23	0.0321	0.125	0.00664	0.176	0.00582	0.194	0.00734
2.5-(1)	0	1	0	0	0.236	0.0379	0.148	0.00877	0.196	0.00763	0.218	0.00733
2.5-(1)	0	2	0	0	0.315	0.449	0.211	0.024	0.241	0.0142	0.223	0.0126
2.5-(1)	1	0	-0.114	0.588	0.185	0.0147	0.153	0.00811	0.135	0.0114	0.165	0.0174
2.5-(1)	1	1	-0.663	2.16	0.19	0.0234	0.145	0.0124	0.155	0.0149	0.223	0.0128
2.5-(1)	1	2	0	0	0.125	0.12	0.192	0.0297	0.223	0.0184	0.232	0.0116

2.5-(1)	2	0	0.703	0.0151	0.341	0.00922	0.101	0.0128	0.0767	0.0233	0.18	0.0414
2.5-(1)	2	1	0.735	0.0148	0.348	0.0108	0.115	0.0164	0.137	0.0254	0.185	0.0247
2.5-(1)	2	2	0.759	0.0153	0.456	0.013	0.126	0.0253	0.148	0.0368	0.183	0.0262
2.5-(1)	3	0	0.787	0.0455	0.551	0.0494	-0.154	0.335	0	0	0	0
2.5-(1)	3	1	0.823	0.039	0.397	0.0395	0.0444	0.0775	0.0323	0.18	0.264	0.225
2.5-(1)	3	2	0.776	0.0427	0.426	0.0298	0.0875	0.0511	-0.0659	0.124	-0.28	0.164
2.5-(2)	0	0	0	0	0.102	0.0336	0.0615	0.00626	0.118	0.0055	0.129	0.00702
2.5-(2)	0	1	0	0	0.114	0.0394	0.0732	0.00841	0.134	0.00728	0.159	0.00699
2.5-(2)	0	2	0	0	0.362	0.376	0.171	0.0223	0.182	0.0136	0.158	0.0122
2.5-(2)	1	0	0.275	0.304	0.117	0.0141	0.0839	0.00775	0.081	0.0107	0.102	0.0165
2.5-(2)	1	1	0.11	0.844	0.124	0.0224	0.0752	0.0118	0.106	0.0138	0.145	0.0125
2.5-(2)	1	2	0	0	-0.107	0.136	0.14	0.0279	0.142	0.0181	0.14	0.0116
2.5-(2)	2	0	0.661	0.0165	0.294	0.00889	0.0604	0.0117	0.0538	0.0207	0.0829	0.0411
2.5-(2)	2	1	0.704	0.0159	0.317	0.0101	0.0783	0.015	0.102	0.0231	0.13	0.0233
2.5-(2)	2	2	0.747	0.0154	0.404	0.013	0.0947	0.0229	0.0657	0.0356	0.14	0.0243
2.5-(2)	3	0	0.741	0.0539	0.522	0.0485	0.0262	0.233	0	0	0	0
2.5-(2)	3	1	0.816	0.0393	0.404	0.0351	0.0582	0.0657	0.0627	0.149	0.273	0.195
2.5-(2)	3	2	0.813	0.0334	0.429	0.0267	0.0864	0.0443	-0.166	0.118	-0.18	0.125

TABLE 29: Dilution Factors for 4.2 GeV

Data Set	Q^2	$\cos \theta_{nq}$	p_m 1	error	p_m 2	error	p_m 3	error	p_m 4	error	p_m 5	error
4.2+	0	0	0	0	0	0	0	0	0	0	0	0
4.2+	0	1	0	0	0	0	0	0	0	0	0	0
4.2+	0	2	0	0	0	0	0	0	0	0	0	0
4.2+	1	0	0	0	0.119	0.0917	0.0692	0.0347	0.0769	0.0424	0.194	0.0525
4.2+	1	1	0	0	0.266	0.168	0.0503	0.0693	0.0963	0.0712	0.074	0.056
4.2+	1	2	0	0	0	0	-0.135	0.332	0.128	0.127	0.137	0.0618
4.2+	2	0	0.648	0.0267	0.22	0.0101	0.0904	0.00986	0.0461	0.0164	0.0936	0.0265
4.2+	2	1	0.724	0.0292	0.233	0.015	0.117	0.0161	0.101	0.0233	0.156	0.0199
4.2+	2	2	0.669	0.0443	0.353	0.0313	0.0836	0.0381	0.141	0.041	0.123	0.0251
4.2+	3	0	0.731	0.0162	0.319	0.0125	0.0911	0.0179	0.0328	0.0351	0.149	0.06
4.2+	3	1	0.714	0.0168	0.371	0.012	0.0862	0.0201	0.123	0.0338	0.145	0.0358
4.2+	3	2	0.775	0.0149	0.417	0.0128	0.104	0.0264	0.0789	0.0542	0.0707	0.0527
4.2-	0	0	0	0	0.0631	0.232	0.0941	0.0204	0.132	0.0151	0.0848	0.0157
4.2-	0	1	0	0	0.0563	0.519	0.0425	0.0412	0.108	0.0264	0.165	0.0188
4.2-	0	2	0	0	0	0	-0.0115	0.174	0.198	0.0516	0.142	0.0346
4.2-	1	0	0	0	0.0936	0.0216	0.0901	0.00577	0.0772	0.00659	0.105	0.00914
4.2-	1	1	0	0	0.0837	0.0464	0.0637	0.00993	0.124	0.00876	0.141	0.00755
4.2-	1	2	0	0	0	0	0.0836	0.0375	0.158	0.0129	0.132	0.00729
4.2-	2	0	0.572	0.0257	0.21	0.00762	0.0792	0.00671	0.0651	0.0111	0.0895	0.0184
4.2-	2	1	0.632	0.0295	0.225	0.0116	0.0746	0.0107	0.0937	0.0145	0.118	0.0135
4.2-	2	2	0.731	0.0333	0.334	0.0264	0.104	0.027	0.102	0.0248	0.125	0.0148
4.2-	3	0	0.746	0.0141	0.35	0.0107	0.0699	0.0162	0.0978	0.0304	0.0759	0.0605
4.2-	3	1	0.757	0.0134	0.36	0.0105	0.0906	0.0173	0.0777	0.0304	0.117	0.0316
4.2-	3	2	0.736	0.0143	0.419	0.0111	0.0796	0.0225	0.0762	0.0458	0.00177	0.0489

TABLE 30: Dilution Factors for 5.6+ GeV

Data Set	Q^2	$\cos \theta_{nq}$	p_m 1	error	p_m 2	error	p_m 3	error	p_m 4	error	p_m 5	error
5.6+	0	0	0	0	0	0	0	0	0	0	0	0
5.6+	0	1	0	0	0	0	0	0	0	0	0	0
5.6+	0	2	0	0	0	0	0	0	0	0	0	0
5.6+	1	0	0	0	0	0	0	0	0	0	0	0
5.6+	1	1	0	0	0	0	0	0	0	0	0	0
5.6+	1	2	0	0	0	0	0	0	0	0	0	0
5.6+	2	0	0.337	0.096	0.265	0.02	0.0585	0.0213	0.0754	0.0331	0.174	0.0465
5.6+	2	1	0.534	0.0996	0.215	0.0334	0.0669	0.037	0.0263	0.0562	0.0992	0.0485
5.6+	2	2	0.275	0.27	0.202	0.095	0.147	0.0887	0.0911	0.111	0.238	0.0643
5.6+	3	0	0.748	0.0177	0.367	0.0128	0.122	0.0187	0.096	0.0348	0.117	0.0634
5.6+	3	1	0.785	0.0165	0.398	0.0132	0.122	0.0225	0.143	0.0383	0.186	0.0386
5.6+	3	2	0.799	0.017	0.469	0.0155	0.0894	0.0348	0.111	0.0693	0.152	0.0598

TABLE 31: Dilution Factors for 5.7 GeV

Data Set	Q^2	$\cos \theta_{nq}$	p_m 1	error	p_m 2	error	p_m 3	error	p_m 4	error	p_m 5	error
5.7+	0	0	0	0	0	0	0	0	0	0	0	0
5.7+	0	1	0	0	0	0	0	0	0	0	0	0
5.7+	0	2	0	0	0	0	0	0	0	0	0	0
5.7+	1	0	0	0	0	0	0	0	0	0	0	0
5.7+	1	1	0	0	0	0	0	0	0	0	0	0
5.7+	1	2	0	0	0	0	0	0	0	0	0	0
5.7+	2	0	0.48	0.105	0.239	0.0315	0.0902	0.0326	0.0224	0.054	0.052	0.0847
5.7+	2	1	0.426	0.17	0.222	0.0497	0.0775	0.0571	0.21	0.0763	0.0685	0.0774
5.7+	2	2	0.517	0.314	0.227	0.138	-0.0513	0.153	-0.257	0.22	0.0909	0.113
5.7+	3	0	0.703	0.0287	0.325	0.0194	0.0604	0.0285	0.0137	0.0539	0.0716	0.0898
5.7+	3	1	0.687	0.0288	0.353	0.0197	0.0862	0.032	-0.0625	0.0609	0.0994	0.0575
5.7+	3	2	0.769	0.0256	0.42	0.0217	0.124	0.0455	0.107	0.0903	0.0829	0.0892
5.7-	0	0	0	0	0	0	0	0	0	0	0	0
5.7-	0	1	0	0	0	0	0	0	0	0	0	0
5.7-	0	2	0	0	0	0	0	0	0	0	0	0
5.7-	1	0	0	0	0.0678	0.172	0.133	0.041	0.0608	0.0419	0.0817	0.0442
5.7-	1	1	0	0	0.499	0.397	-0.0602	0.155	-0.0179	0.111	0.0565	0.0648
5.7-	1	2	0	0	0	0	0.326	0.746	0.361	0.319	0.143	0.183
5.7-	2	0	0.288	0.108	0.128	0.0165	0.0493	0.0122	0.0845	0.0183	0.0612	0.0291
5.7-	2	1	0.172	0.173	0.143	0.0284	0.054	0.0213	0.0975	0.0262	0.178	0.022
5.7-	2	2	0.73	0.2	0.249	0.0923	-0.011	0.0723	0.151	0.048	0.108	0.0282
5.7-	3	0	0.696	0.026	0.291	0.017	0.0708	0.0237	0.0493	0.0438	-0.0315	0.0857
5.7-	3	1	0.704	0.0262	0.303	0.0183	0.0474	0.0293	0.0799	0.0488	0.102	0.0492
5.7-	3	2	0.748	0.024	0.459	0.0196	0.0675	0.0429	0.0829	0.0823	0.104	0.0695

TABLE 32: Dilution Factors for 5.8- GeV

Data Set	Q^2	$\cos \theta_{nq}$	p_m 1	error	p_m 2	error	p_m 3	error	p_m 4	error	p_m 5	error
5.8-	1	0	0	0	0.174	0.185	0.0995	0.0497	0.0969	0.0462	0.136	0.0502
5.8-	1	1	0	0	0.157	0.542	0.681	0.113	0.0409	0.134	0.217	0.0711
5.8-	1	2	0	0	0	0	0	0	0.64	0.568	-0.472	0.357
5.8-	2	0	0.546	0.0908	0.167	0.019	0.0773	0.0142	0.0448	0.0222	0.0639	0.0346
5.8-	2	1	0.386	0.153	0.187	0.0315	0.0968	0.0241	0.0788	0.0317	0.102	0.0283
5.8-	2	2	0.361	0.285	0.181	0.11	0.0267	0.0767	0.124	0.059	0.14	0.0333
5.8-	3	0	0.778	0.0277	0.345	0.02	0.107	0.0282	0.0665	0.0518	0.0939	0.1
5.8-	3	1	0.741	0.0301	0.4	0.02	0.0617	0.0335	0.148	0.0512	0.175	0.0554
5.8-	3	2	0.777	0.0298	0.442	0.0239	0.19	0.0461	-0.0845	0.109	0.123	0.0875

APPENDIX C

RADIATIVE CORRECTIONS

The plots in this appendix show the results of determining the radiative correction values of τ and κ from Section 4.11.

FIG. 61: Beam Energy of 1.6 GeV and Q^2 Bin 0

FIG. 62: Beam Energy of 1.6 GeV and Q^2 Bin 1 FIG. 63: Beam Energy of 1.6 GeV and Q^2 Bin 2

FIG. 64: Beam Energy of 1.6 GeV and $\cos \theta_{nq}$ Bin 1 FIG. 65: Beam Energy of 2.5 GeV and Q^2 Bin 0

FIG. 66: Beam Energy of 2.5 GeV and Q^2 Bin 1 FIG. 67: Beam Energy of 2.5 GeV and Q^2 Bin 2

FIG. 68: Beam Energy of 2.5 GeV and Q^2 Bin 3 FIG. 69: Beam Energy of 2.5 GeV and $\cos \theta_{nq}$ Bin 1

FIG. 70: Beam Energy of 4.2 GeV and Q^2 Bin 1 FIG. 71: Beam Energy of 4.2 GeV and Q^2 Bin 2

FIG. 72: Beam Energy of 4.2 GeV and Q^2 Bin 3 FIG. 73: Beam Energy of 4.2 GeV and $\cos \theta_{nq}$ Bin 1

FIG. 74: Beam Energy of 5.7 GeV and Q^2 Bin 2 FIG. 75: Beam Energy of 5.7 GeV and Q^2 Bin 3

FIG. 76: Beam Energy of 5.7 GeV and $\cos \theta_{nq}$ Bin 1

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