

Effect of flavor-dependent partonic transverse momentum on the determination of the W boson mass in hadronic collisions

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Within the framework of transverse-momentum-dependent factorization, we investigate for the first time the impact of a flavor-dependent intrinsic transverse momentum of quarks on the production of W^\pm bosons in proton-proton collisions at $\sqrt{s} = 7$ TeV. We study the transverse-mass and the lepton transverse-momentum distributions of the W -decay products by means of a template-fit technique and we estimate the shift in the W boson mass induced by different choices of flavor-dependent parameters for the intrinsic quark transverse momentum. We obtain $-11 \leq \Delta M_{W^+} \leq 4$ MeV and $-6 \leq \Delta M_{W^-} \leq 2$ MeV with a statistical uncertainty of ± 4 MeV. Our findings call for more detailed investigations of flavor-dependent nonperturbative effects linked to the proton structure at hadron colliders.

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Introduction and motivation.

Nonperturbative effects in transverse-momentum-dependent (TMD) phenomena are a central topic in the hadronic physics community with potentially important applications to high-energy physics. The study of nonperturbative corrections originates from the work of Parisi and Petronzio [1] and Collins, Soper, and Sterman [2], which focused on the role of the hard scale of the process compared to the infrared scale of QCD. TMD factorization and evolution have been extensively studied in the literature [3–6], together with the matching to collinear factorization [2, 7–12]. Despite the limited amount of data available and the many open theoretical questions, in the past years we started gaining phenomenological information about TMD parton distribution functions (TMD PDFs) with increasing level of accuracy. Recently, the unpolarized TMD PDF was extracted for the first time from a global fit of data for semi-inclusive deep-inelastic scattering (SIDIS) and production of Drell-Yan lepton pairs and Z bosons [13]. Nonetheless, many theoretical and phenomenological questions are still open [14].

In this paper, we demonstrate that if we want to determine the free parameters of the Standard Model with very high precision, then the effects of a possible flavor dependence of the intrinsic partonic transverse momentum should not be neglected even in the kinematic region where nonperturbative effects are expected to be small [9, 13, 15–19] ($\Lambda_{\text{QCD}} \ll Q \ll \sqrt{s}$: W boson production at the LHC lies in this kinematic region). In particular, we focus on the impact of the simplest TMD PDF, the unpolarized one, on the determination of the W boson mass at hadron colliders.

Experimental measurements and uncertainties.

The determination of the W boson mass, M_W , from the global electroweak fit ($M_W = 80.356 \pm 8$ MeV) [20] features a very small uncertainty that sets a goal for the precision of the experimental measurements at hadron colliders.

Precise determinations of M_W have been extracted from $p\bar{p}$ collisions at D0 [21] and at CDF [22], and from pp collisions at ATLAS [23] with a total uncertainty of 23 MeV, 19 MeV and 19 MeV, respectively. The current world average, based on these measurements and the ones performed at LEP, is $M_W = 80.379 \pm 12$ MeV [24]. The experimental analyses are based on a template-fit procedure on the differential distributions of the decay products: in particular, the transverse momentum of the final lepton, $p_{T\ell}$, the transverse momentum of the neutrino $p_{T\nu}$ (only at the Tevatron), and the transverse mass m_T of the lepton pair (where $m_T = \sqrt{2 p_{T\ell} p_{T\nu} (1 - \cos(\phi_\ell - \phi_\nu))}$, with $\phi_{\ell,\nu}$ being the azimuthal angles of the lepton and the neutrino, respectively).

In a template-fit procedure, several histograms are generated with the highest available theoretical accuracy and the best available description of detector effects, letting the fit parameter (M_W , in this case) vary in a range: the histogram best describing experimental data selects the measured value for M_W . The details of the theoretical calculations used to compute the templates (choice of scales, PDFs, perturbative order, all-order resummation of divergent contributions, nonperturbative effects, ...) affect the result of the fit and define the theoretical systematics [25]. In this work we focus only on the impact of nonperturbative effects and, in particular, on those coming from the transverse momentum of the initial-state partons. These intrinsic- k_T effects modify the spectrum

of the W transverse momentum, q_T^W , subsequently inducing a nonnegligible shift in the extracted value of M_W .

The three experimental collaborations D0, CDF, and ATLAS typically fit the Z data to obtain an estimate for the nonperturbative parameters. Then, assuming the parameters to be flavor independent, they use these estimates to predict the q_T^W distribution. The uncertainty on M_W due to the modelling of q_T^W via template fits for the distributions in $(m_T, p_{T\ell}, p_{T\nu})$ are, respectively, $\delta M_W = (3, 9, 4)$ MeV for CDF [22], $\delta M_W = (2, 5, 2)$ MeV for D0 [21] and $\delta M_W = (3, 3)$ MeV for ATLAS [23] (the ATLAS analysis did not include $p_{T\nu}$ in the template fit).

It is well known that one of the largest sources of error in determining M_W comes from the uncertainty in the choice of the collinear PDFs [26–29]. Nevertheless, one can see that the uncertainty propagating from the q_T^W spectrum via $p_{T\ell}$ can be likewise comparably large (except for ATLAS, because of the narrow range used for the $p_{T\ell}$ fit with respect to the m_T one). This does not come as a surprise, since the $p_{T\ell}$ distribution is extremely sensitive to the modelling of q_T^W , i.e., the $p_{T\ell}$ shape gets more distorted by all-order resummation and nonperturbative contributions than the m_T shape (which, in turn, is dominated by detector resolution).

At present, neither analyses at the Tevatron and at the LHC included information on the flavor dependence of the intrinsic transverse momentum of the incoming partons participating in the hard scattering. Here, it is our aim to study its impact onto the determination of M_W in hadronic collisions, taking inspiration from the phenomenological extraction of the unpolarized TMD PDF from low-energy data [30].

Formalism.

The impact of nonperturbative effects in Drell-Yan and Higgs production has been extensively investigated (see, e.g., [13, 18, 19, 31–34] for available calculations and fitting codes).

Different implementations of the nonperturbative contributions have been presented in the literature (see e.g. Refs. [13, 19] and references therein). In order to take into account possible differences between the valence and the sea quarks (and among different flavors in general), a flavor- and kinematic-dependent implementation of the nonperturbative part of the quark Sudakov exponent has been suggested in Refs. [30, 35]. In the present work, we choose a Gaussian functional form for the intrinsic transverse momentum distribution of the unpolarized TMD PDF. Its Fourier-conjugate expression reads

$$f_1^{aNP}(b_T^2) = \frac{1}{2\pi} e^{-g_{NP}^a b_T^2}, \quad (1)$$

where g_{NP}^a is related to the average intrinsic transverse momentum squared of a parton with flavor a . In general, the latter may also depend on kinematics, but here we will neglect this dependence.

We implemented the above ansatz in two publicly available tools for computing Drell-Yan differential cross sections: DYqT [36, 37] and DYRes [36, 38]. The DYqT program computes the q_T spectrum of an electroweak boson V ($V = \gamma^*, W^\pm, Z$) produced in hadronic collisions. The calculation combines the pure fixed-order QCD result up to $\mathcal{O}(\alpha_s^2)$ at high q_T ($q_T \sim M_V$) with the resummation of the logarithmically-enhanced contributions at small transverse-momenta ($q_T \ll M_V$) up to next-to-next-to-leading logarithmic (NNLL) accuracy. The rapidity of the vector boson and the leptonic kinematical variables are integrated over the entire kinematical range. At the same perturbative accuracy, the DYRes code also provides the full kinematics of the vector boson and of its decay products. It thus allows for the application of arbitrary cuts on the final-state kinematical variables and gives differential distributions in form of bin histograms, directly comparable to experimental measurements.

The original codes implement the nonperturbative TMD effects as a flavor- and kinematic-independent Gaussian exponential $e^{-g_{NP} b_T^2}$ whose strength is governed by a single parameter g_{NP} tuned at the electroweak scale. This factor incorporates the nonperturbative effects from both the TMD PDFs entering the cross section, including their evolution. In order to mimic a flavor dependence in each partonic intrinsic transverse momentum, we modify this simple implementation by decomposing g_{NP} into the sum $g_{NP}^a + g_{NP}^{a'}$, where the flavor indices span the range $a, a' = u_v, u_s, d_v, d_s, s, c, b, g$ (the subscripts referring to the valence and sea components, respectively). For each parton with flavor a , the nonperturbative contribution f_1^{aNP} of Eq. (1) is included in the corresponding term in the flavor sum of the TMD factorization formula [3]. In the following, we assume $g_{NP}^s = g_{NP}^c = g_{NP}^b = g_{NP}^g$, i.e., we assume that in total the intrinsic transverse-momentum depends on five flavors.

Analysis strategy.

The phenomenological extraction of Ref. [30] is based on about 1500 data points, however the nonperturbative parameters g_{NP}^a in Eq. (1) are not tightly constrained. A fit to Z/γ^* data from Tevatron produces the value $g_{NP} \sim 0.8 \text{ GeV}^2$ for the universal nonperturbative factor [32]. We recall that this value refers to the convolution of two TMD PDFs inside the cross section; hence, each parton should equally contribute with a nonperturbative width of $\approx 0.4 \text{ GeV}^2$. When we introduce the corresponding parameter g_{NP}^a for a single TMD PDF with flavor a , we split it as follows:

$$\exp(-g_{NP}^a b_T^2) \longrightarrow \exp[-[g_{evo} \ln(Q^2/Q_0^2) + g_a] b_T^2], \quad (2)$$

where the first term in the right hand side is the nonperturbative correction due the TMD PDF evolution, which is flavor independent (but, in principle, different

for quarks and gluons), and g_a is the genuine flavor-dependent contribution. Information on g_{evo} can be deduced from Ref. [13], where the TMD PDF was extracted from the global fit of SIDIS, Drell-Yan and Z -production data (g_{evo} corresponds to $g_2/4$ in Ref. [13]). At $Q = M_W$ and $Q_0 = 1$ GeV, we have $g_{evo} \ln(Q^2/Q_0^2) \approx 0.3$ GeV². In order to account for the uncertainties affecting the determination of g_{evo} , we choose to consider the interval $[0.2, 0.6]$ GeV² as a reasonable range and we vary g_a in Eq. (2) such that the g_{NP}^a values fall into this range.

Thus, we generate random widths in the allowed range for the considered five flavors. We build 50 sets of flavor-dependent parameters together with a flavor-independent set where all the parameters are put equal to the central value of the variation range, $\overline{g_{NP}^a} = 0.4$ GeV². Our analysis is performed by first selecting "Z-equivalent" sets, and then making a template fit, as detailed here below.

Selection of "Z-equivalent" sets. For proton-proton collisions at $\sqrt{s} = 7$ TeV, we generate pseudodata for the q_T distribution of the Z boson (22 bins similar to the ATLAS ones [23]) using the flavor-independent set in the DYqT code at $\mathcal{O}(\alpha_s)$ and NLL accuracy. We do the same for proton-antiproton collisions at $\sqrt{s} = 1.96$ TeV (72 bins similar to the CDF ones [22]). We assign to each of the q_T bins an uncertainty equal to the experimental one. We compute the q_T distribution in the same conditions also for each of the 50 flavor-dependent sets. We calculate the χ^2 between each of these 50 distributions and the pseudodata generated by the flavor-independent set. We retain only those flavor-dependent sets that have a $\chi^2 < 80$ on the "CDF-like" bins ($\chi^2/\text{d.o.f.} < 1.1$) and a $\chi^2 < 44$ on the "ATLAS-like" bins ($\chi^2/\text{d.o.f.} < 2$). The first criterion selects 48 flavor-dependent sets out of 50; only 30 sets out of 50 match the second one, because the ATLAS data have smaller (experimental) uncertainties. We keep those flavor-dependent sets that fulfill both criteria. When considering all the bins, these sets have a total $\chi^2 < 124$ on the pseudodata ($\chi^2/\text{d.o.f.} < 1.3$). In practice, these selected flavor-dependent sets are equivalent to the flavor-independent one (with which the Z pseudodata are generated) at approximately 2σ level. Not surprisingly, this result implies that the Z boson data alone are not able to discriminate between flavor-independent and flavor-dependent sets of nonperturbative parameters. Data from flavor-sensitive processes are needed, in particular from SIDIS [39–42].

Template fit. Following the scheme introduced in [26], we perform a template fit to estimate the impact of our "Z-equivalent" flavor-dependent sets on the determination of M_W . We use the DYRes code at the same accuracy (NLL at small transverse momentum and $\mathcal{O}(\alpha_s)$ at large transverse momentum) and kinematics as before, using the MSTW2008 NLO PDF set [43], setting

central values for the renormalization, factorization and resummation scales $\mu_R = \mu_F = \mu_{res} = M_W$, and implementing ATLAS acceptance cuts on the final-state leptons [23]. In DYRes, the singularity of the resummed form factor at very large values of b_T ($b_T \gtrsim 1/\Lambda_{\text{QCD}}$) is avoided by the usual b_* prescription [2]. Similarly, the correct behavior at very low b_T is enforced by modifying the argument of the logarithmic terms as in Refs. [36, 38]. The form factor in Eq. (2) is usually interpreted as the nonperturbative contribution to TMD resummation for $b_T \gtrsim 1/\Lambda_{\text{QCD}}$. We generate templates with very high statistics (750 M events) for the m_T , $p_{T\ell}$ distributions¹ with different M_W masses in the range 80.370 GeV $\leq M_W \leq 80.400$ GeV, using the flavor-independent set for the nonperturbative parameters. Then, for each "Z-equivalent" flavor-dependent set we generate pseudodata with lower statistics (75 M events) for the same leptonic observables with the fixed value $M_W = 80.385$ GeV². Finally, for each pseudodata set we compute the χ^2 of the various templates and we identify the template with minimum χ^2 in order to establish how large is the shift in M_W induced by a particular choice of flavor-dependent nonperturbative parameters. The statistical uncertainty of the template-fit procedure has been estimated by considering statistically equivalent those templates for which $\Delta\chi^2 = (\chi^2 - \chi_{min}^2) \leq 1$. Consequently, we quote an uncertainty of 4 MeV for each of the obtained M_W shifts.

Impact on the M_W determination.

The outcome of our template fit is summarized in Tabs. I and II for 5 representative sets out of the 30 "Z-equivalent" sets. The former table lists the values of the g_{NP}^a parameter in Eq. (2) for each of the 5 considered flavors $a = u_v, d_v, u_s, d_s, s = c = b = g$. The latter table shows the corresponding shifts induced in M_W when applying our analysis to the m_T , $p_{T\ell}$ distributions for the W^+ and the W^- production at the LHC ($\sqrt{s} = 7$ TeV).

| Set | u_v | d_v | u_s | d_s | s |
|-----|-------|-------|-------|-------|------|
| 1 | 0.34 | 0.26 | 0.46 | 0.59 | 0.32 |
| 2 | 0.34 | 0.46 | 0.56 | 0.32 | 0.51 |
| 3 | 0.55 | 0.34 | 0.33 | 0.55 | 0.30 |
| 4 | 0.53 | 0.49 | 0.37 | 0.22 | 0.52 |
| 5 | 0.42 | 0.38 | 0.29 | 0.57 | 0.27 |

TABLE I: Values of the g_{NP}^a parameter in Eq. (2) for the flavors $a = u_v, d_v, u_s, d_s, s = c = b = g$. Units are GeV².

¹ Our analysis is performed on 30 bins in the interval $[60, 90]$ GeV for m_T and on 20 bins in the interval $[30, 50]$ GeV for $p_{T\ell}$.

² The factor-of-10 reduction in statistics is justified by a sanity check performed analyzing the χ^2 profile of different samples with the same inputs but different statistics [26].

| | ΔM_{W^+} | | ΔM_{W^-} | |
|-----|------------------|-------------|------------------|-------------|
| Set | m_T | $p_{T\ell}$ | m_T | $p_{T\ell}$ |
| 1 | 2 | -4 | -2 | 2 |
| 2 | 1 | -11 | -1 | -3 |
| 3 | 0 | 4 | -3 | -6 |
| 4 | 1 | 4 | -2 | -5 |
| 5 | 2 | -5 | 0 | -5 |

TABLE II: Shifts in M_{W^\pm} (in MeV) induced by the corresponding sets of flavor-dependent intrinsic transverse momenta outlined in Tab. I (Statistical uncertainty: 4 MeV).

As expected, the shifts induced by the analysis performed on $p_{T\ell}$ are generally larger than for the m_T case, since the latter is less sensitive to q_T^W -modelling effects.

For set 2, the shift induced on M_{W^+} by the $p_{T\ell}$ analysis is 11 MeV, a particularly large value compared to the corresponding uncertainty quoted by ATLAS (3 MeV). In general, taking also into account the statistical uncertainty of our analysis, the absolute value of the shifts induced when considering the $p_{T\ell}$ observable can exceed 10 MeV.

Different flavor-dependent sets may induce artificial asymmetric shifts for M_{W^+} and M_{W^-} in the flavor-independent template fits. For instance, if $M_{W^-} > M_{W^+}$ (which corresponds to the ATLAS findings [23]) a template fit to the $p_{T\ell}$ observable based on sets 1 and 2 would lead to different shifts $\Delta M_{W^-} > \Delta M_{W^+}$ such that the difference between the two masses is enhanced. In this case, a fit with the corresponding flavor-dependent nonperturbative contributions would lead to a reduction of the mass gap. On the contrary, using sets 3 and 4 one would obtain the opposite result. Finally, set 5 would induce the same shift $\Delta M_{W^+} = \Delta M_{W^-}$, leading to a higher value for the average M_W and thus challenging the consistency with the prediction of the electroweak global fit.

Outlook and future developments.

In this work, we investigated the uncertainties on the determination of M_W at the LHC induced by a possible flavor dependence of the partonic intrinsic transverse momentum. From these outcomes, we point out that a “flavor-blind” data analysis may not be a sufficiently accurate option, especially when a total uncertainty lower than 10 MeV is expected for M_W at the LHC [44].

Future data from flavor-sensitive processes such as SIDIS (from the 12 GeV upgrade at Jefferson Lab [45], from the COMPASS collaboration [46], and from a future Electron-Ion Collider with both proton and deuteron beams [41, 42]) will shed new light on the flavor decomposition of the unpolarized TMD PDF. These low-energy SIDIS data involve also the study of the flavor dependence in the fragmentation function (the unpolarized TMD FF). Therefore, new data from semi-inclusive e^+e^- annihilations will also be needed for the flavor de-

composition of the TMD FF [35].

All these data will improve our knowledge of the partonic structure of hadrons, and may help in reducing the uncertainties in precision measurements at high energies.

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