Multiplicative renormalizability of quasi-parton distributions

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We extend our proof of the multiplicative renormalizability of quasi-quark PDFs in Ref. [1] to quasi-gluon PDFs, and demonstrated that quasi-gluon PDFs can be multiplicatively renormalized to all orders in perturbation theory, without mixing with other operators. We find that using a gauge-invariant UV regulator is essential for achieving this proof. With the multiplicative renormalizability of both quasi-quark and quasi-gluon PDFs, and QCD collinear factorization of quasi-PDFs into PDFs, we have now a solid theoretical foundation for extracting PDFs from lattice QCD calculated quasi-PDFs.

I. INTRODUCTION

The parton distribution functions (PDFs) encode important nonperturbative information of strong interactions. Based on QCD factorization [2], PDFs have been successfully extracted from high-energy-collision data with good precisions [3]. However, from both theoretical and practical points of view, extracting PDFs from first principle lattice QCD calculations must be done for testing non-perturbative sector of QCD, as well as needed for study partonic structure of hadrons that could be difficult to do scattering experiments with.

Calculating PDFs directly from Euclidean-space lattice QCD, if not impossible, is very difficult due to the time-dependence of the operators defining them [3]. Recently, two of us proposed a general approach to calculate PDFs in lattice QCD indirectly [4, 5], by extracting PDFs from lattice QCD calculations of good "lattice cross sections" (LCSs), which are defined as hadronic matrix elements satisfying 1) calculable in Euclidean-space lattice QCD, 2) renormalizable for ultraviolet (UV) divergences to ensure a reliable continue limit, and 3) factorizable to PDFs with infrared-safe matching coefficients. It is the factorization that relates lattice QCD calculable LCSs to the desired PDFs. We can then extract PDFs by a global analysis for lattice QCD data of LCSs, similar to the extraction of PDFs from experimental data of factorizable and measurable cross sections. All other proposals [6–8] could be special realizations of this general approach.

To extract the rich, precise and flavor separated information on PDFs, it is necessary to find as many good LCSs as possible, since different flavor PDFs are likely to be mixed to contribute to almost all lattice QCD calculated LCSs through QCD collinear factorization. For constructing good LCSs to extract PDFs, we have considered two types of operators in terms of correlation of (1) two gauge-dependent operators with proper gauge links [6] and (2) two gauge-invariant currents [4]. We refer the first type as quasi-parton operators in the following. Factorization properties of both types of operators have been studied in [4], in which we found that multiplicative renormalizability of these operators is a necessary condition for the collinear factorization to be valid. Renormalization of the second type of operators is almost trivial, for which one only needs to renormalize the gauge-invariant local currents, which is well-known to be multiplicative. On the other hand, the renormalization of the first type of operators is nontrivial due to the nonlocality of corresponding operators.

A lot of efforts have been devoted to understand the UV structure of quasi-quark operators [9–16]. The all-order multiplicative renormalizability of quasi-quark operators has been proved using two different methods: one relies on the auxiliary field technique [17, 18], and the other is based on diagrammatic expansion [1]. These proofs provide a firm theoretical basis for extracting PDFs from lattice QCD calculated quasi-quark PDFs [19–35].

The UV structure of quasi-gluon operators could be much more complicated, as we will explain. We define general bare quasi-gluon operators as

$$\mathcal{O}_{bg}^{\mu\nu\rho\sigma}(\xi) = F^{\mu\nu}(\xi)\,\Phi^{(a)}(\{\xi,0\})\,F^{\rho\sigma}(0)\,, \tag{1}$$

where $\Phi^{(a)}(\xi,0) = \mathcal{P}e^{-ig_s\int_0^1 \xi\cdot A^{(a)}(r\xi)\,dr}$ is a path ordered gauge link in adjoint representation. To be definite, we assume ξ_μ along z-direction and introduce a unit vector $n^\mu = (0,0,0,1)$, defining $v\cdot n \equiv v_z$ for any vector v_μ. Due to the dimensional derivative operator in $F^{\mu\nu}$, superficial power counting tells us that the vertex between gluon field strength and gauge link can be linearly UV divergent. By using a cutoff regularization, one-loop calculation in Refs. [36, 37] indeed shows uncanceled linear divergences for the vertex, which would make the multiplicative renormalization of quasi-gluon operators almost impossible. For the comparison, the corresponding vertex between quark and gauge link can be at most logarithmic UV divergent.

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In principle, the general quasi-gluon operator in Eq. (1) could have 36 independent operators after taking into account the antisymmetry of gluon field strength, and all of them could be mixed under renormalization since they all have the same mass dimension. In practice, one usually constructs some linear combinations of Eq. (1) by contracting it with some "tensors". In Refs. [6] and [37] and the first lattice simulation [38], a reduced number of quasi-gluon operators were obtained by contracting $\mathcal{O}_{bg}^{\mu\nu\rho\sigma}$ by $n_\mu n_\rho g_{\nu i} g^i_\sigma$, $n_\mu n_\rho g_{\nu\sigma}$ and $g_{\mu 0} g_{\rho 0} g_{\nu\sigma}$, respectively. It is not clear how to renormalize these reduced quasi-gluon operators due to potential complications and uncertainties in mixing of these operators.

In this paper, we study the UV divergences of quasi-gluon operators to all order in perturbation theory. We first perform an explicit one-loop calculation of quasi-gluon PDFs of an asymptotic gluon of momentum p, defined as $\langle g(p)|\mathcal{O}_{bg}^{\mu\nu\rho\sigma}(\xi)|g(p)\rangle$. We use dimensional regularization (DR) to regularize both logarithmic and linear UV divergences, which respectively appear as poles around $d = 4$ and $d = 4 - 1/n$ at n-loop order. We find that linear UV divergences of the one-loop correction to gluon-gauge-link vertex are canceled under DR, which makes the multiplicative renormalizability of quasi-gluon operators a possibility. We then explore all possible UV divergent topologies of higher order diagrams. Using gauge invariance, we find that all linear UV divergences from the gluon-gauge-link vertex are canceled to all-orders in perturbation theory, leaving only linear UV divergences from the gauge link which can be easily exponentiated to all-order as an overall phase factor and then multiplicatively renormalized, just like the case of quasi-quark PDFs [1, 39, 40]. In addition, we find that all of the 36 independent quasi-gluon operators can be multiplicatively renormalized without mixing with any other operators. Combining with our proof for quasi-quark PDFs in Ref. [1], our work presented in this paper for quasi-gluon PDFs completes the proof of the multiplicative renormalization of quasi-parton operators.

II. UV DIVERGENCES AT ONE LOOP

We present the relevant one-loop Feynman diagrams for quasi-gluon PDFs of an asymptotic gluon of momentum p in Fig. 1, where the bubble in the diagram (e) includes all one-loop self-energy diagrams of the active gluon. For the complete one-loop contribution, additional Feynman diagrams are needed. Some of them are mirror of diagrams (b), (c), (d), (e) and (g), while the rest can be obtained by replacing external momentum p to $-p$ in all these Feynman amplitudes. For the following one-loop calculation, we take the linearly combined quasi-gluon operator in Ref. [6] as an example, but our conclusion is true for any of the 36 independent operators.

We choose Feynman gauge, and assume ξ_z to be positive for definiteness. The diagram (a) in Fig. 1 gives

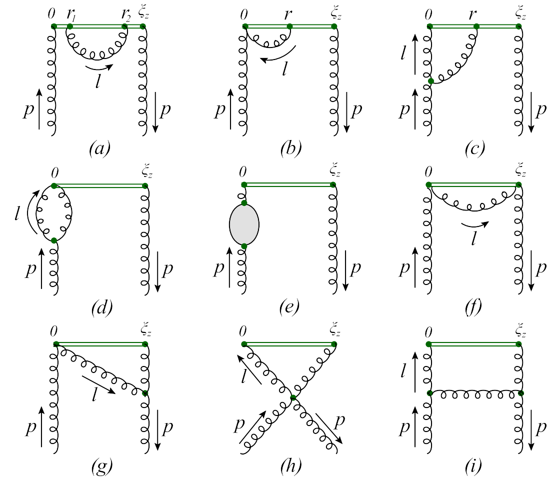

FIG. 1. Some typical Feynman diagrams for quasi-gluon PDFs of an asymptotic gluon of momentum p at one-loop order.

$$M_{1a} = \frac{g_s^2 \mu_r^{4-d} C_A}{i}\, e^{-ip_z\xi_z} \int_0^{\xi_z} dr_1 \int_{r_1}^{\xi_z} dr_2 \frac{d^d l}{(2\pi)^d} \frac{e^{il_z(r_2-r_1)}}{l^2}$$
$$\overset{\text{UV}}{=\!=} \frac{\alpha_s C_A}{\pi} e^{-ip_z\xi_z} \left(-\frac{\pi\mu_r\xi_z}{3-d} + \frac{1}{4-d} \right), \tag{2}$$

where μ_r is renormalization scale to compensate the mass dimension in DR. This diagram contributes to both linear and logarithmic UV divergences, as expected.

To understand where in this one-loop phase space the UV divergences in Eq. (2) come from, it is instructive to distinguish l_z - the z-component of the loop momentum l from $\bar{l}_\mu$ - the other components of l, as $l_\mu = \bar{l}_\mu - l_z n_\mu$ with $l^2 = \bar{l}^2 - l_z^2$ [1]. If $\bar{l}^2$ is constrained in a finite region in Eq. (2), integrating l_z, r_1 and r_2 cannot generate any UV divergence. Furthermore, there is no UV divergence if we do not include the region where $|r_2 - r_1|$ is very small, which can be demonstrated by introducing the following decomposition,

$$\frac{1}{(l+q)^2} = \frac{1}{(\bar{l}+\bar{q})^2} + \frac{(l_z+q_z)^2}{(l+q)^2(\bar{l}+\bar{q})^2}, \tag{3}$$

where q can be any unintegrated momentum. By applying this decomposition to $\frac{1}{l^2}$ in Eq. (2), the first term is free of l_z, and thus the integration of l_z gives $\delta(r_2 - r_1)$, while the second term is UV finite under the integration of $\bar{l}$. That is, the UV divergence in Eq. (2) can only come from the region of phase space where $\bar{l}$ are in UV region while $|r_2 - r_1|$ is very small. Therefore, we conclude that, with DR, all UV divergences of diagram (a) in Fig. 1 come from a region localized in spacetime.

By decomposing both $\frac{1}{l^2}$ and $\frac{1}{(p-l)^2}$ using Eq. (3), we obtain many terms for diagrams (b) and (c) and found that these terms are either free of l_z in denominator, which result in $\delta(r)$ or its derivatives, or UV finite under the integration of $\bar{l}$. Thus the UV divergences of these two diagrams are also localized in spacetime,

$$M_{1b} \overset{\text{UV}}{=\!=} \frac{\alpha_s C_A}{\pi} e^{-ip_z\xi_z} \left(\frac{-i}{p_z\xi_z} \frac{\pi\mu_r\xi_z}{3-d} \right), \tag{4}$$

$$M_{1c} \overset{\text{UV}}{=\!=} \frac{\alpha_s C_A}{\pi} e^{-ip_z\xi_z} \left(\frac{i}{p_z\xi_z} \frac{\pi\mu_r\xi_z}{3-d} + \frac{3}{4}\frac{1}{4-d} \right), \tag{5}$$

where diagram (b) has only linear UV divergence, while (c) has both linear and logarithmic UV divergence.

The diagram (d) in Fig. 1 gives

$$M_{1d} \overset{\text{UV}}{=\!=} -\frac{3\alpha_s C_A}{4\pi} \frac{e^{-ip_z\xi_z}}{4-d}, \tag{6}$$

where the logarithmic UV divergence comes from the region where all components of l^μ go to infinity, and thus, is localized in spacetime. The logarithmic UV divergence of diagram (e) is from the one-loop self-energy correction for the gluon, which is well-known and localized in spacetime, and can be removed by the renormalization of gluon field.

All other diagrams in Fig. 1 are free of UV divergence, simply because the loop cannot be localized in spacetime due to finite ξ_z. For example, the diagram (f) gives

$$M_{1f} = \frac{ig_s^2 \mu_r^{4-d} C_A}{p_z^2} \int \frac{d^d l}{(2\pi)^d} \frac{e^{il_z\xi_z - ip_z\xi_z}}{l^2}, \tag{7}$$

which is UV divergent only if $\xi_z = 0$. In summary, we conclude that the UV divergences of quasi-gluon PDFs at one-loop can only be emerged from a region localized in coordinate spacetime.

The linear UV divergence in Eq. (2) from the diagram (a) is harmless because it can be easily exponentiated to all-order as an overall phase factor and then multiplicatively renormalized, just like the case of quasi-quark PDFs [1, 39, 40]. However, the presence of linear UV divergence in Eqs. (4) and (5) from the diagrams (b) and (c), respectively, could challenge the multiplicative renormalizability. Fortunately, we find that with DR, the linear UV divergences from these two diagrams are canceled. On the contrary, the linear divergences from the diagrams (b) and (c) do not cancel if one uses a cutoff regularization that breaks the gauge symmetry [36, 37]. This implies that gauge invariance plays an important role to remove the linear UV divergences that may challenge the multiplicative renormalizability. In the following, we will use gauge invariance to show that all linear divergences, except that from self-energy of gauge links, are canceled by summing over all contributions, and quasi-gluon PDFs could be multiplicatively renormalized.

III. UV DIVERGENCES AT HIGH ORDERS

From the one-loop diagrams in Fig. 1, we can generate all high order loop diagrams by adding gluons (or a quark-antiquark pair) to them. Because of the isolation of z-component in the definition of quasi-parton operators, both 3-dimensional (3-D) and 4-D integration of loop momentum $\bar{l}$ and l could lead to UV divergence. In Ref. [1], we introduced the change of divergence index $\Delta\omega_3$ and $\Delta\omega_4$ for the 3-D and 4-D integration of loop momenta of higher order diagrams, respectively, and showed that it is sufficient, although it is not necessary, that quasi-parton operators are renormalizable if $\Delta\omega_3 \leq 0$ and $\Delta\omega_4 \leq 0$ are satisfied for all corresponding higher order diagrams. Based on the power counting rules derived in Ref. [1], we find that the only case that may increase superficial UV divergence of quasi-gluon operators at high orders is when we add a gluon with both ends of it attached to the gauge link, where the 3-D integration gets $\Delta\omega_3 = 1$. By applying the decomposition in Eq. (3) to the added gluon's momentum, it is straightforward to show that dimensional regularized UV divergences at any loop level are localized in spacetime, in the same way as quasi-quark operators shown in [1]. As a result, we find that $\Delta\omega_3$ is effectively irrelevant for studying UV divergences. Because $\Delta\omega_4 \leq 0$ for all cases, there are only finite topologies of high order diagrams in Fig. 2 that have UV divergences.

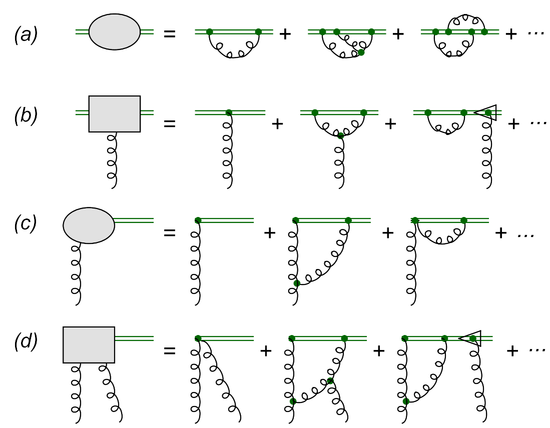

FIG. 2. Four topologies of diagrams which could give UV divergences to the quasi-gluon operators.

The blobs with topologies (a) and (c) in Fig. 2 denote one-particle-irreducible diagrams, and they both have linear superficial UV divergences. Because of the potential linear UV divergences, diagrams with one more gluon attached to the blobs can generate logarithmic UV divergences. Another possibility to produce logarithmic divergences is when a gluon is attached to the gauge link outside of, but very close to, the blobs, as shown in Fig. 2 (b) and (d), with the attachment denoted by a triangle. The blobs of topologies (b) and (d) include both kinds of logarithmic divergent diagrams mentioned here.

The topologies (a) and (b) in Fig. 2 are the same as that for quasi-quark operators, and their divergences can be renormalized similarly. Linear divergences from the diagrams of the topology (a) can be removed by an overall factor as the mass renormalization of a test particle moving along the gauge link [39], and its logarithmic divergences caused by end points of the gauge link can be removed by multiplying $Z_{wg}^{-1/2}$ - the "wave function" renormalization of the test particle [40]. The diagrams of the topology (b) has only logarithmic UV divergence, which can be taken care of by QCD renormalization [40].

The UV divergences from diagrams of topologies (c) and (d) in Fig. 2 are different from that of quasi-quark operators, and are studied in next two sections, respectively.

IV. RENORMALIZATION OF GLUON-GAUGE-LINK VERTEX

For the definiteness of following discussion, we assume that the gauge link in diagrams of the topology (c) in Fig. 2 starts at an arbitrary coordinate ξ_{1z} with the operator $F^{\mu\nu}(\xi_{1z})$, and ends at another arbitrary coordinate ξ_{2z} with no additional operators. With the "bare" coupling constant g_s, and "bare" field operators for the gluons, the Faddeev-Popov ghost and the antighost given by the symbols A, c and $\bar{c}$ respectively, a generalized Ward identity of the non-Abelian field relevant to this topology can be derived [41],

$$\begin{aligned} &\langle -i\partial_\lambda^y A_d^\lambda(y)[\Phi(\{\xi_{2z},\xi_{1z}\})]_{ab}\, F_b^{\mu\nu}(\xi_{1z})\rangle \\ &= \langle g_s\bar{c}_d(y)c_e(\xi_{2z})[t_e\Phi(\{\xi_{2z},\xi_{1z}\})]_{ab}\, F_b^{\mu\nu}(\xi_{1z})\rangle, \end{aligned} \tag{8}$$

where the t represents SU(3) generators of the adjoint representation. A pictorial representation of Eq. (8) is given in Fig. 3, where "1PR" denotes one-particle-reducible diagrams. The topology of the left-hand side of Fig. 3 is the same as that of the diagram (c) in Fig. 2, but is contracted with external gluon momentum and expressed in coordinate space. The topology of the first term on the right-hand side of Fig. 3 is nonlocal in spacetime, and thus has no UV divergence. Furthermore, after the renormalization of QCD Lagrangian and gaugelink-related topologies (a) and (b) in Fig. 2, UV divergences of 1PR diagrams will be canceled. That is, the generalized Ward identity in Eq. (8) ensures that the topology (c) in Fig. 2 is free of UV divergence if it is contracted by external gluon momentum.

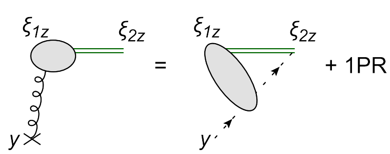

FIG. 3. Pictorial representation of the generalized Ward identity in Eq. (8) with dashed line represents the ghost field.

To understand the renormalization of the UV divergence of the topology (c) in Fig. 2, we represent the diagrams of this topology as $\Gamma^{\lambda\mu\nu}(p,n)$, which could be referred as the gluon-gauge-link vertex, where p and λ are the momentum and Lorentz index of the external gluon, respectively. Lorentz symmetry combined with antisymmetry between μ and ν enable us to do the general decomposition $\Gamma^{\lambda\mu\nu}(p,n)=\sum_{i=1}^4 c_i\Pi_i^{\lambda\mu\nu}$ with

$$\begin{aligned} &\Pi_1^{\lambda\mu\nu}=g^{\mu\lambda}p^\nu-g^{\nu\lambda}p^\mu,\, \Pi_2^{\lambda\mu\nu}=(p^\mu n^\nu-p^\nu n^\mu)n^\lambda, \\ &\Pi_3^{\lambda\mu\nu}=(p^\mu n^\nu-p^\nu n^\mu)p^\lambda,\, \Pi_4^{\lambda\mu\nu}=g^{\mu\lambda}n^\nu-g^{\nu\lambda}n^\mu. \end{aligned} \tag{9}$$

Since $p_\lambda\Gamma^{\lambda\mu\nu}=0$ from our discussion above, we obtain $c_2\, p\cdot n+c_3\, p^2+c_4=0$, and consequently,

$$\begin{aligned} \Gamma^{\lambda\mu\nu}(p,n)=&c_1\Pi_1^{\lambda\mu\nu}+c_2(\Pi_2^{\lambda\mu\nu}-p\cdot n\Pi_4^{\lambda\mu\nu}) \\ &+c_3(\Pi_3^{\lambda\mu\nu}-p^2\Pi_4^{\lambda\mu\nu}). \end{aligned} \tag{10}$$

Since c_1 and c_2 have mass dimension 0, locality of UV divergences ensures that they can be at most logarithmic divergent, while c_3 is UV finite due to its mass dimension at -1. The only potential linearly UV divergent coefficient c_4 is removed by gauge invariance. We have therefore demonstrated that the cancellation of linear UV divergences of diagrams (b) and (c) in Fig. 1 at one-loop order can be generalized to all orders, which makes the multiplicative renormalizability of quasi-gluon PDFs a possibility.

At the lowest order in α_s, we have $\Gamma^{\lambda\mu\nu}(p,n)\propto\Pi_1^{\lambda\mu\nu}$. If we want $\Gamma^{\lambda\mu\nu}(p,n)$ not to mix with other operators under renormalization, we need its UV divergence to be proportional to $\Pi_1^{\lambda\mu\nu}$ to all orders. Fortunately, it is always true. For the case with μ (or ν) along z-direction or the case with both μ and ν not along z-direction, the coefficients of c_2 are proportional to $\Pi_1^{\lambda\mu\nu}$ or equal to zero, respectively. Therefore, the components of $\Gamma^{\lambda\mu\nu}(p,n)$ do not mix with each others at all, although two different renormalization constants are needed for the two different choices.

In summary, the choice of operators to define quasi-gluon PDFs is not unique. If we choose either $F^{z\bar{\nu}}$ or $F^{\bar{\mu}\bar{\nu}}$ for gluon-gauge-link vertex, $\Gamma^{\lambda\mu\nu}$, we can remove the UV divergences of the vertex by multiplying a corresponding renormalization factor $Z_{vg1}^{-1/2}$ or $Z_{vg2}^{-1/2}$, respectively.

V. RENORMALIZATION OF GLUON-GLUON-GAUGE-LINK VERTEX

Finally, we exam the renormalization of UV divergence of the topology (d) in Fig. 2, whose diagrams involve two gluons and a gaugelink, and could be referred as the gluon-gluon-gauge-link vertex. Similar to the Ward identity in Eq. (8), we construct the following Ward identity for the "bare" fields and operators,

$$\begin{aligned} &\langle\partial_\lambda^x A_d^\lambda(x)\partial_\rho^y A_e^\rho(y)[\Phi(\{\xi_{2z},\xi_{1z}\})]_{ab}\, F_b^{\mu\nu}(\xi_{1z})\rangle \\ &+i\delta^{(d)}(x-y)\delta_{de}\langle[\Phi(\{\xi_{2z},\xi_{1z}\})]_{ab}\, F_b^{\mu\nu}(\xi_{1z})\rangle \\ &=g_s\langle f^{afg}\bar{c}_e(y)c_f(\xi_{2z})\partial_\lambda^x A_d^\lambda(x)[\Phi(\{\xi_{2z},\xi_{1z}\})]_{gb}\, F_b^{\mu\nu}(\xi_{1z})\rangle. \end{aligned} \tag{11}$$

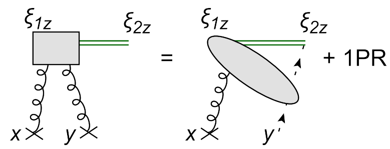

FIG. 4. Pictorial interpretation of the generalized Ward identity in Eq. (11) with dashed line represents the ghost field.

A pictorial interpretation of Eq. (11) is given in Fig. 4. Similar to Fig. 3, the topology of the left-hand side of Fig. 4 is the same as that of the diagram (d) in Fig. 2, except the external gluons of the diagrams are contracted with their respective momenta and expressed in coordinate space. The right-hand side of the equation in Fig. 4 is UV finite after all previously discussed renormalizations performed, including QCD Lagrangian, gauge links, and gluon-gauge-link vertex. That is, we find that the topology (d) in Fig. 2 is free of UV divergence if both external gluons are contracted by their respective momenta.

Similar to the discussion of the gluon-gauge-link vertex, the Ward identity helps reduce the superficial UV divergence of the topology (d) in Fig. 2. Since the diagrams of the topology (d) have only superficial logarithmic divergence, the additional reduction of the superficial UV divergence from the Ward identity makes the topology (d) in Fig. 2 UV finite. Therefore, after the renormalization of topology (c), the topology (d) requires no any additional renormalization. This is similar to the case of renormalizing gluon vertexes of QCD Lagrangian, where gauge invariance guarantees that four-gluon vertex will be free of UV divergence once three-gluon vertex is renormalized.

VI. SUMMARY

We demonstrated that the UV divergences of quasi-gluon operators are actually similar to the UV divergences of quasi-quark operators if all divergences are regularized in a gauge invariant scheme. We proved that UV divergences of all 36 pure quasi-gluon operators are localized in spacetime, and could be multiplicatively renormalized without mixing with each others,

$$\mathcal{O}_g^{\mu\nu\rho\sigma}(\xi) = e^{-C_g|\xi_z|} Z_{wg}^{-1} Z_{vg1}^{-s/2} Z_{vg2}^{-(2-s)/2} \mathcal{O}_{bg}^{\mu\nu\rho\sigma}(\xi), \quad (12)$$

where s is the number of z components chosen for Lorentz indices $\{\mu, \nu, \rho, \sigma\}$, and C_g, Z_{wg}, Z_{vg1} and Z_{vg2} are renormalization constants. Like the quasi-quark PDFs, quasi-gluon PDFs defined by quasi-gluon operators could be examples of "good lattice cross sections", as defined in Refs. [4, 5], which could be calculated in lattice QCD and factorized into normal PDFs, from which PDFs could be extracted by QCD global analysis of the data of these quasi-PDFs generated by the first principle lattice QCD calculations.

Note added: While our paper is being finalized, a preprint by Zhang *et al.* [42] appeared, in which these authors reached the similar conclusion although the approach is very different.

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