

Quark fragmentation as a probe of dynamical mass generation

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We address the propagation and hadronization of a struck quark by studying the gauge invariance of the color-averaged cut quark propagator, and by relating this to the single inclusive quark fragmentation correlator by means of new sum rules. Using suitable Wilson lines, we provide a gauge-invariant definition for the mass of the color-averaged dressed quark and decompose this into the sum of a current and an interaction-dependent component. The latter, which we argue is an order parameter for dynamical chiral symmetry breaking, also appears in the sum rule for the twist-3 \tilde{E} fragmentation function, providing a specific experimental way to probe the dynamical generation of mass in Quantum Chromo Dynamics.

Introduction. One of the crucial properties of the strong force is confinement, namely the fact that partons cannot exist as free particles outside of hadrons. As a consequence, any individual parton struck in a high-energy scattering process and freed from its parent hadron must transform into at least one hadron. During this hadronization process, the colored and typically nearly massless parton interacts with the surrounding matter and the vacuum to produce massive and colorless hadrons. Hadronization is thus tightly connected to the dynamical generation of the mass, the spin, and the size of hadrons, but the details of the quark-to-hadron transition are still unknown. Unraveling hadronization dynamics is not only of fundamental importance to understand the nature of visible matter, but also to tackle hadron tomography studies at current and future facilities, such as the 12 GeV upgrade at Jefferson Lab [1] and a future US-based Electron-Ion Collider [2], where measuring the transverse momentum of one final state hadron is crucial to provide a handle into the transverse motion of quarks and gluons in the hadron target [3–10].

In this letter, we address the propagation and hadronization of a struck quark by studying the gauge invariance properties of the “inclusive jet correlator” defined in Ref. [11–14], *i.e.*, the color-averaged cut quark propagator supplemented by suitable Wilson lines, and by relating this by means of sum rules to the fragmentation correlator [15] utilized in perturbative Quantum Chromo Dynamics (QCD) to describe the semi-inclusive transition of a quark into a single hadron.

In particular, through the Dirac decomposition of the jet correlator, we provide a gauge-invariant definition of the jet mass M_j that was previously introduced in Ref. [14]. In this letter, we elucidate for the first time its nature and properties, recognizing that M_j can be decomposed into the sum of the current quark mass and of an interaction-dependent “correlation mass”. Thus the

jet correlator can also be interpreted as a color-averaged propagator for a *dressed quark*.

We find that the jet mass and the correlation mass are experimentally accessible through sum rules for the unpolarized collinear twist-3 E and \tilde{E} fragmentation functions (FFs). A similar sum rule for the \tilde{D}^\perp FF also supplies information on the average transverse momentum of the produced hadrons. These sum rules encode in a quantitative and rigorous way the intimate connection between hadronization and the dynamical generation of mass and momentum, providing one with a novel way to quantify the quark dressing process, the dynamical breaking of the chiral symmetry, and the nature of the QCD vacuum.

The inclusive jet correlator. Let us consider the unintegrated color-averaged gauge-invariant cut propagator for a quark of momentum k :

$$\Xi_{ij}(k; w) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{ik \cdot \xi} \times \frac{\text{Tr}_c}{N_c} \langle \Omega | \mathcal{T} W_1 \psi_i(\xi) \bar{\psi}_j(0) W_2 | \Omega \rangle, \quad (1)$$

where $|\Omega\rangle$ is the interacting QCD vacuum, and, for simplicity, we omitted the flavor index of the quark field ψ .

Gauge invariance is implemented via the Wilson lines $W_1 \equiv W_1(\infty, \xi; w)$ and $W_2 \equiv W_2(0, \infty; w)$, and the time-ordering operator \mathcal{T} exchanges, when needed, the ψ and $\bar{\psi}$ fields as well as the ending point ξ of the Wilson line W_1 and the starting point 0 of W_2 . We consider Wilson lines with displacements in the $w = n_+$ longitudinal direction defined by a light-like vector n_+ , followed by a displacement in the plane transverse to this and to another light-like vector n_- with $n_+ \cdot n_- = 1$ [15]. For sake of brevity, we will henceforth omit any explicit dependence on w .

In this letter, we will study the cut propagator Ξ as an object of intrinsic theoretical interest in a formal twist expansion in powers of $1/k^-$. This can be justified in scattering processes with a hard scale, such as the 4-momentum transfer Q in deep inelastic scattering. In such processes, the n_\pm vectors are determined by the initial or final state momenta, and $k^- \propto Q \gg k_T \gg k^+$. Our considerations will be, however, process independent.

By taking the discontinuity (Disc) of the propagator – or its cut, in diagrammatic terms – Eq.(1) can be interpreted as a gauge-invariant inclusive quark-to-jet amplitude squared, or “inclusive jet correlator” in short, in which all the quark hadronization products cross the cut and are on-shell [11–14, 16]. The adjective “inclusive” is to stress that none of the jet’s constituents is actually measured, hence the absence of an axis and radius in Eq. (1), contrary to semi-inclusive definitions of jets [17–22].

The color averaging of the initial-state quark, implemented as $\text{Tr}_c[\dots]/N_c$ in Eq. (1), arises naturally in QCD factorization theorems and in the definition of the fragmentation correlator, to which the cut quark propagator Ξ is connected through the sum rules we will prove later. In our analysis, color-averaging plays a crucial role, technically, as it enables a spectral decomposition of the cut propagator Ξ . At the conceptual level, it implements the color neutralization that has to take place in order for all the states propagating through the cut to be on-shell hadrons, as confinement dictates for a physical process. Therefore, Eq. (1) can also be interpreted as a *color-averaged* (or *color-screened*) version of the gauge-invariant Feynman propagator for a dressed quark with four-momentum k .

Spectral decomposition. The cut propagator Ξ can be given a spectral representation by rewriting Eq. (1) as a convolution of a quark bilinear \tilde{S} and the Fourier transform \tilde{W} of a Wilson line:

$$\Xi_{ij}(k) = \text{Disc} \int d^4p \frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}_{ij}(p) \tilde{W}(k-p) | \Omega \rangle, \quad (2)$$

where

$$\tilde{S}_{ij}(p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot p} \mathcal{T} \psi_i(\xi) \bar{\psi}_j(0), \quad (3)$$

$$\tilde{W}(k-p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot (k-p)} \mathcal{T} W(0, \xi). \quad (4)$$

In Eq. (3), the operator \mathcal{T} acts as an ordinary time-ordering operator, while in Eq. (4) it acts only on the endpoints of the path for the Wilson line, orienting this from 0 to ξ or vice versa.

The quark operator \tilde{S} can furthermore be decomposed in Dirac space assuming invariance under Lorentz

and parity transformations:

$$\tilde{S}_{ij}(p) = \hat{s}_3(p^2) \not{p}_{ij} + \sqrt{p^2} \hat{s}_1(p^2) \mathbb{I}_{ij}, \quad (5)$$

where we refer to $\hat{s}_{1,3}$ as “spectral operators”. In principle, when working in an axial $v \cdot A = 0$ gauge, we should also add a structure proportional to \not{v} to the right hand side of Eq. (5) [25]. However, in our explicit calculations we adopt the light-cone gauge, $v = n_+$, and this additional gauge-fixing term would only contribute at twist-4 level, which is not relevant to the matter discussed in this letter.

We can obtain a connection with the usual Källén-Lehman spectral representation of the (gauge-variant) quark propagator [13, 23, 24] by noticing that the quark’s Feynman propagator in momentum space is given by the expectation value of \tilde{S} on the interacting vacuum. It is then possible to write

$$\begin{aligned} \frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}(p) | \Omega \rangle &= \int_{-\infty}^{+\infty} \frac{d\mu^2}{(2\pi)^4} \frac{\mathbf{i}}{p^2 - \mu^2 + i\epsilon} \\ &\times \left\{ \not{p} \rho_3(\mu^2) + \sqrt{p^2} \rho_1(\mu^2) \right\} \theta(\mu^2). \end{aligned} \quad (6)$$

Using the operator decomposition for \tilde{S} given in Eq. (5) and the Cutkosky rule [26, 27], we can then connect the spectral operators $\hat{s}_{1,3}$ to the chiral-odd and -even Källén-Lehman spectral functions $\rho_{1,3}$:

$$(2\pi)^3 \text{Disc} \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_{1,3}(p^2) | \Omega \rangle = \rho_{1,3}(p^2) \theta(p^2) \theta(p^-). \quad (7)$$

The TMD inclusive jet correlator. It is useful to consider the cut quark propagator integrated over the plus component of the partonic momentum [14],

$$J_{ij}(k^-, \mathbf{k}_T) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k), \quad (8)$$

where the time ordering is now trivial since $\xi^- = 0$ [28, 29]. As explained later, this integrated correlator is also of interest for the derivation of sum rules for fragmentation functions.

The Transverse Momentum Dependent (TMD) correlator J can be decomposed in Dirac structures, with coefficients determined by Dirac projections of Ξ defined as

$$J^{[\Gamma]} \equiv \frac{1}{2} \int dk^+ \text{Tr} \left[\Xi \frac{\Gamma}{2} \right], \quad (9)$$

where Γ is a generic Dirac matrix and Tr represent the trace on the Dirac indexes. The structures of interest for the present discussion are:

$$\alpha(k^-) \equiv J^{[\gamma^-]} = \frac{k^-}{\mathbf{k}_T^2} J^{[\gamma^i]}, \quad (10)$$

$$\zeta(k^-) \equiv \frac{k^-}{\Lambda} J^{[\mathbb{I}]}, \quad (11)$$

where Λ is a scale with the dimension of a mass introduced for power counting purposes. The kinematic dependence of the coefficients and all the remaining Dirac structures will be discussed in detail in Ref. [30]. The integrated cut quark propagator can then be decomposed up to terms of $\mathcal{O}(\Lambda^2/(k^-)^2)$ as:

$$J(k^-, \mathbf{k}_T) = \frac{1}{2}\alpha(k^-)\left[\gamma^+ + \frac{\mathbf{k}_T}{k^-}\right] + \frac{\Lambda}{2k^-}\zeta(k^-)\mathbb{I} \quad (12)$$

Note that one can explicitly factor a $\theta(k^-)$ function out of α and ζ because of four-momentum conservation, if one assumes that the particles passing the cut all have physical four-momenta. We can thus rewrite the coefficient of the chiral-odd Dirac structure as

$$\zeta(k^-) = \frac{\theta(k^-)}{2(2\pi)^3\Lambda}M_j(k^-), \quad (13)$$

where M_j is a *gauge-invariant* mass term, potentially function of k^- , and the chosen normalization will become clear after performing the explicit calculation in the light-cone gauge.

We calculate ζ in the light-cone gauge $A \cdot n_+ = 0$ starting from the definition (11). Using the convolution representation (2) and the spectral decomposition (7), we obtain:

$$\zeta(k^-) = \frac{\theta(k^-)}{2(2\pi)^3\Lambda} \left\{ \int_0^{+\infty} dp^2 \sqrt{p^2} \rho_1^{lsg}(p^2) \right\}, \quad (14)$$

where the superscript *lsg* stresses the use of the light-cone gauge. The gauge-invariant M_j mass has a particularly simple and k^- -independent form in this gauge, and is completely determined by the first moment of the spectral function ρ_1^{lsg} :

$$M_j = \int_0^{+\infty} d\mu^2 \sqrt{\mu^2} \rho_1^{lsg}(\mu^2). \quad (15)$$

The integral at the right hand side is summing over all the discontinuities of the quark propagator. In the light-cone gauge, therefore, M_j can be interpreted as the average mass generated by the chirality flipping component of the quark-to-jet amplitude squared (however, one has to be careful with a probabilistic interpretation because, in a confined theory, the spectral functions are not guaranteed to be positive definite.) Choosing another gauge, the right hand side of this equation would also receive contributions from the Wilson line, and its physical interpretation would be less immediate. Interpretation aside, we stress that M_j is gauge-invariant, thanks to the gauge invariance of the operator Ξ in Eq. (1), and distinct from the average invariant mass of the fragmented hadrons. We also note that under renormalization M_j would acquire an additional scale dependence.

The calculation of the α coefficient follows closely the strategy applied to ζ . We obtain:

$$\alpha(k^-) = \frac{\theta(k^-)}{2(2\pi)^3} \left\{ \int_0^{+\infty} dp^2 \rho_3^{lsg}(p^2) \right\} = \frac{\theta(k^-)}{2(2\pi)^3}, \quad (16)$$

where we have furthermore used the gauge-independent normalization property $\int_0^{+\infty} ds \rho_3(s) = 1$ [24]. Putting all elements together, up to twist 3 we find

$$4(2\pi)^3 J(k^-, \mathbf{k}_T) = \left\{ \gamma^+ + \frac{M_j}{k^-} \mathbb{I} + \frac{\mathbf{k}_T}{k^-} \right\} \theta(k^-). \quad (17)$$

The jet correlator can thus be directly compared to the cut quark propagator with current quark mass m_q , but now depends on the non-perturbative dressed quark mass M_j .

Being related to the trace of the cut propagator, the mass M_j is intrinsically different from the mass function which appears in non-perturbative treatments of the uncut quark propagator [31, 32]. M_j is gauge invariant and scale dependent, whereas the mass function is gauge dependent, but renormalization group invariant. Nonetheless, being a scale that characterizes the physics of a color-screened dressed quark, it also provides a window on color confinement.

Remarkably, the gauge-invariant mass M_j is also experimentally accessible because it contributes explicitly to the mass sum rules for the twist-3 fragmentation functions E and \tilde{E} that we shall prove in the following.

Sum rules for single-hadron fragmentation functions. The unintegrated correlator describing the fragmentation of a quark into a single unpolarized hadron is [15, 28, 33–36]:

$$\Delta_{ij}^h(k, P_h) = \sum_{S_h} \int \frac{d^4\xi}{(2\pi)^4} e^{ik \cdot \xi} \quad (18)$$

$$\times \frac{\text{Tr}_c}{N_c} \langle \Omega | \mathcal{T} W_1 \psi_i(\xi) a_h^\dagger(P_h S_h) a_h(P_h S_h) \bar{\psi}_j(0) W_2 | \Omega \rangle,$$

where a_h^\dagger and a_h create and destroy a hadron h with momentum P_h and spin S_h .

Integrating over the suppressed quark momentum component k^+ [28, 37, 38], one defines the TMD quark-to-single-hadron fragmentation correlator:

$$\Delta_{ij}^h(z, \mathbf{P}_{h\perp}) = \int \frac{dk^+}{2z} \text{Disc} [\Delta_{ij}^h(k, P_h)]_{k^+ = P_h^+ / z}, \quad (19)$$

where, in the parton frame, $\mathbf{k}_T = 0$ and $\mathbf{P}_{h\perp}$ is the hadronic transverse momentum relative to the quark [36, 39]. The parametrization of this correlator in terms of TMD FFs is known up to twist 3 [15, 33]. Here we are only interested in

$$\Delta_{ij}^h(z, \mathbf{P}_{h\perp}) = \frac{\gamma^+}{2} D_1^h + \frac{M_h}{2P_h^-} E^h + \frac{\mathbf{P}_{h\perp}}{2zP_h^-} D^{\perp h}, \quad (20)$$

where D_1^h is the unpolarized twist-2 TMD FF and E^h and $D^{\perp h}$ are the unpolarized twist-3 TMD FFs.

We can now derive a master four-momentum sum rule for unintegrated correlators, from which we can later derive sum rules for specific fragmentation functions. Relying on the commutation properties of the momentum operator associated to the vector P_h^μ [40],

$$\hat{P}^\mu = \sum_{h,S_h} \int \frac{dP_h^- d^2 \mathbf{P}_{h\perp}}{2P_h^- (2\pi)^3} P_h^\mu \hat{a}_h^\dagger(P_h, S_h) \hat{a}_h(P_h, S_h), \quad (21)$$

and integrating Eq. (18) by parts with vanishing boundary conditions for the fields, we obtain:

$$\sum_h \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2) P_h^\mu \Delta^h(k, P_h) = k^\mu \Xi^{uncut}(k). \quad (22)$$

A detailed proof will be presented in Ref. [30].

We can obtain sum rules for the fragmentation functions by integrating in the parton frame over the suppressed momentum components on both sides of Eq. (22), which implies calculating a discontinuity [28, 37, 38], and using suitable Dirac projections of the result. Since the sum over the hadron spin S_h is fundamental in the manipulations involving the momentum operator (21) [40], we can only obtain sum rules for FFs involving unpolarized hadrons. In particular, for $\Gamma = \{\gamma^-, \mathbb{I}, \gamma^i\}$, and suitably choosing $\mu = -$ or $\mu = i$, we obtain:

$$[\Gamma = \gamma^-] \quad \sum_{h,S_h} \int dz z D_1^h(z) = 1, \quad (23)$$

$$[\Gamma = \mathbb{I}] \quad \sum_{h,S_h} \int dz M_h E^h(z) = M_j, \quad (24)$$

$$[\Gamma = \gamma^i] \quad \sum_{h,S_h} \int dz M_h^2 D^{\perp(1)h}(z) = 0. \quad (25)$$

The full set of sum rules for FFs up to twist 3 will be presented in Ref. [30], with preliminary results already discussed at various conferences [41]. Note that we identified the integrated TMD FFs $f = D_1^h, E^h$ with their collinear counterpart, *i.e.*, $f(z) \equiv \int d^2 \mathbf{P}_{h\perp} f(z, P_{h\perp}^2)$. While this is correct only when considering bare distributions, the sum rules hold true also in the renormalized case [39, 42]. Furthermore, we define the first moment of any FF f as $f^{(1)}(z) = \int d^2 \mathbf{P}_{h\perp} [\mathbf{P}_{h\perp}^2 / (2z^2 M_h^2)] f(z, P_{h\perp}^2)$ [36].

Eq. (23) is the well-known momentum sum rule for the unpolarized twist-2 FF [42]. The sum rule (24) was first introduced in Ref. [14], but it is proven here for the first time in field theory. It generalizes the sum rule proposed in Ref. [43], which however neglected the non-perturbative component of the dressed quark mass.

We refer to Eq. (24) as the “mass sum rule” because of its physical interpretation: the non-perturbative color-screened dressed quark mass M_j corresponds to the average of all the possible masses of the particles produced in the hadronization of the quark, weighted by the chiral-odd collinear twist-3 fragmentation function $E^h(z)$. The sum rule (25) is new and its implications will be discussed in the following.

Twist-three fragmentation and the dynamical component of the jet mass. Let us now consider the equations of motion relations discussed in Ref. [15], that relate the twist-2 and the twist-3 fragmentation functions. In particular, we need

$$E^h = \tilde{E}^h + z \frac{m_q}{M_h} D_1^h, \quad D^{\perp h} = \tilde{D}^{\perp h} + z D_1^h, \quad (26)$$

where the functions with a tilde are related to the parametrization of the dynamical twist-3 quark-gluon-quark correlator of type $\tilde{\Delta}_A^\alpha$ [15] and m_q is the current quark mass.

Within the “Wandzura-Wilczek (WW) approximation” [44], namely assuming that the \tilde{E} , \tilde{D}^\perp FFs are negligible compared to the quark-quark FFs without the tilde, M_j reduces to the current quark mass, as can be seen by setting $\tilde{E} = 0$ in Eq. (26) and using the sum rules (23) and (24). This suggests a decomposition of M_j into the sum of the current quark mass and of an interaction-dependent (or dynamical) mass m_q^{corr} generated by quark-gluon-quark correlations, which we name “correlation mass”:

$$M_j = m_q + m_q^{corr}. \quad (27)$$

In analogy with the generation of the quark mass induced by dynamical chiral symmetry breaking [31, 32], we expect $m_q^{corr} = O(\Lambda_{CQD})$. For light quarks, therefore, M_j might be substantially larger than the current quark mass m_q . Combining the sum rules (23) and (24) with Eq. (26) and (27) we obtain:

$$\sum_{h,S_h} \int dz M_h \tilde{E}^h(z) = m_q^{corr}, \quad (28)$$

and one clearly sees that m_q^{corr} is generated by the interaction-dependent quark-gluon-quark correlations encoded in the \tilde{E} function. Thus, Eq. (28) generalizes the sum rule $\int dz \tilde{E} = 0$ discussed in Ref. [15] to the non-perturbative case.

The correlation mass vanishes when neglecting quark-gluon-quark correlations, as in the already discussed WW approximation. However, when interactions are considered, the spectral representation (15) shows that m_q^{corr} and M_j receive contributions from all possible propagating hadronic states, including baryons. As a

consequence, contrary to m_q , we expect that m_q^{corr} and M_j do not vanish even in the chiral limit. Therefore, they can also be considered order parameters for dynamical chiral symmetry breaking and are moreover experimentally accessible, at least in principle, via the mass sum rule for the E and \tilde{E} FFs.

Finally, combining the sum rules (23) and (25) with the D -type equation of motion in Eq. (26), we obtain:

$$\begin{aligned} \sum_{h S_h} \int dz M_h^2 \tilde{D}^{\perp(1)h}(z) &= - \sum_{h S_h} \int dz z M_h^2 D_1^{(1)h}(z) \\ &\equiv -\frac{1}{2} \langle \mathbf{P}_{h\perp}^2 / z \rangle. \end{aligned} \quad (29)$$

This new sum rule provides an experimental way of accessing the average squared transverse momentum acquired by unpolarized hadrons fragmenting off an unpolarized quark, and probes the dynamical nature of the hadronization process in analogy with the way the sum rule for \tilde{E} probes dynamical chiral symmetry breaking. Indeed, the transverse momentum acquired during the hadronization is generated by the quark-gluon-quark correlations and would vanish in the absence of these, as it can be directly checked assuming the WW approximation. Similar sum rules involving transversely polarized quarks or hadrons have been discussed in Refs. [36, 45].

Summary and outlook. With the jet mass M_j defined in Eqs. (11) and (13), we have for the first time proposed a *gauge-invariant* definition of a color-screened dressed quark mass, which is furthermore experimentally accessible by using the sum rules (24) and (28) for twist-3 collinear FFs. In perturbation theory, M_j is proportional to the current quark mass, but in the full theory it remains non-zero also in the chiral limit. Therefore we also recognize M_j as an order parameter for dynamical chiral symmetry breaking. This provides a novel, rigorous connection between hadronization and the dynamical generation of mass in QCD.

The dynamical component of the jet mass, quantified by the correlation mass m_q^{corr} in Eq. (27), is generated by the quark-gluon-quark correlations encoded in the \tilde{E} fragmentation function. The \tilde{E} FF, and thus the sum rule (28) connecting this to m_q^{corr} , can be experimentally accessed by looking at chiral odd observables at twist-3 level and higher in semi-inclusive hadron production in polarized electron-hadron scattering, electron-positron annihilation, and hadronic collisions. As proposed in Ref. [14, 41], the correlation mass itself may also directly contribute to the inclusive DIS g_2 structure function at large Bjorken x , and to an analogous asymmetry in electron-positron induced dihadron production. More generally, in scattering processes, the jet mass M_j (rigorously defined as the trace of the gauge-invariant cut quark propagator) appears to play a sim-

ilar phenomenological role to that of the constituent mass in quark models.

With the sum rule (29) we have also proven that the transverse momentum acquired during the hadronization process is a fully dynamical quantity, namely it is generated by the quark-gluon-quark correlations and it vanishes in the absence of these. This sum rule can be, in principle, experimentally accessed at twist-2 level through the TMD FF D_1^h , about which limited information is so far available [8, 46–48].

Finally, we note that the master sum rule (22) and the calculational techniques we have introduced have a general applicability. In particular, we expect the convolution and the spectral representation of the quark correlator to be also applicable to the study of the gauge invariance of other correlators, for example the recently introduced virtuality-dependent parton distributions [49] that play an important role in the direct lattice QCD calculation of PDFs in momentum space [50, 51]. Moreover, our methods are not limited to staple-like Wilson lines, making their domain of applicability potentially wide.

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