AN ANALYTIC APPROACH TO EMITTANCE GROWTH FROM THE BEAM-BEAM EFFECT WITH APPLICATIONS TO THE LHeC*

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Abstract

In colliders with asymmetric rigidity such as the proposed Large Hadron electron Collider, jitter in the weaker beam can cause emittance growth via coherent beam-beam interactions. The LHeC in this case would collide 7 TeV protons on 60 GeV electrons, which can be modeled using a weak-strong model. In this work we estimate the proton beam emittance growth by separating out the longitudinal angular kicks from an off-center bunch interaction and produce an analytic expression for the emittance growth per turn in systems like the LHeC.

INTRODUCTION

The beam-beam effect is the term given to the mutual lensing action that each beam in a collider causes on the other. In the proposed Large Hadron electron Collider (LHeC) the colliding beams would have an asymmetric collision between a 7 TeV proton beam, and a 60 GeV electron beam from a dedicated recirculating linac [1]. Due to the asymmetric rigidities in these beams, the beam-beam tune shift is 9.6×10^{-5} for the proton beam and 0.75 for the electron beam. Since the LHC proton-proton collisions run with an incoherent tune shift of 0.0037 regularly, it is the coherent effects that will drive emittance growth from these interactions. Furthermore, since this is a linac-ring system, the offset jitter that drives this increase will not reach an equilibrium, since the linac will continuously add a new beam with new jitter [2].

GROWTH MECHANISM

Due to the higher proton rigidity when compared to electrons, the proton beam can pull the electron beam in and through the proton beam. This action will add horizontal/vertical kicks (as the case may be for a given offset) in a manner that is coupled with the longitudinal position of the beam. An example of the kicks given are shown in Figure 1.

If we assume that a given kick causes only transverse momentum changes (i.e. Δp_x only), then we can estimate that the given normalized emittance growth is,

$$\Delta\varepsilon_n = \frac{1}{2}(\beta\gamma)\beta^* < \Delta p_x{}^2 > \frac{\sigma_{jitter}^2}{\sigma_x^2}\,. \tag{1}$$

Where $\Delta\varepsilon_n$ is the change per interaction of the normalized emittance for a given transverse dimension. Finding $<\Delta p_x^2>$ is our main challenge [3].

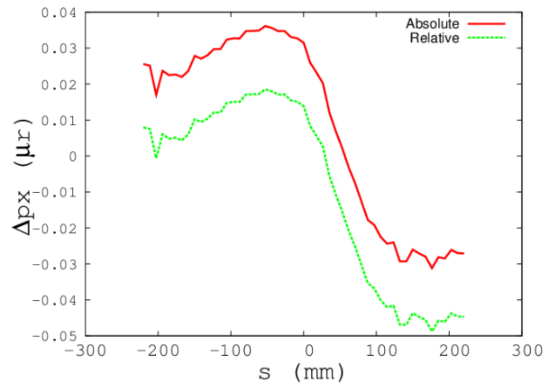

Figure 1: This figure shows the angular offset of the proton beam as a function of longitudinal position caused by an electron beam moving through the proton beam. Absolute is the total kick received, while relative is the kick with the average subtracted out.

DETERMINING <ΔPX²>

The simplest method of determining the $<\Delta p_x^2>$ would be to model the kicks received by the proton beam based on the relative position of the electron beam. The kicks are modeled using the Basetti-Erskine formula [4], and we have started out with three methods of determining the path of the electron beam through the proton beam. One simple way is to model the system in a beam-beam code such as GUINEA-PIG [5], another is to directly integrate using the equations of motion, and finally an attempt at a polynomial ansatz was made. The paths these methods make through the LHeC proton beam are shown in Figure 2.

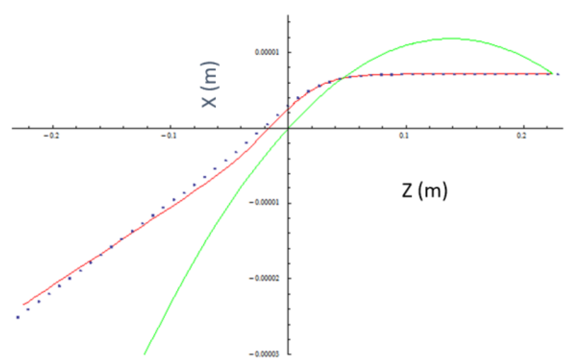

Figure 2: This figure shows the comparison of the three methods used so far to calculate the path of the electron beam through the proton beam. The dotted line shows the path as calculated using GUINEA-PIG, the red line is the path directly integrated from the equations of motion, and the green path represents our polynomial ansatz.

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Using GUINEA-PIG and a simplified map of the LHC lattice we can simulate the beam-beam interactions and how they affect the circulating proton beam. The data from multiple random number seeds, as well as the growth rates predicted using the three methods already described are shown in Figure 3.

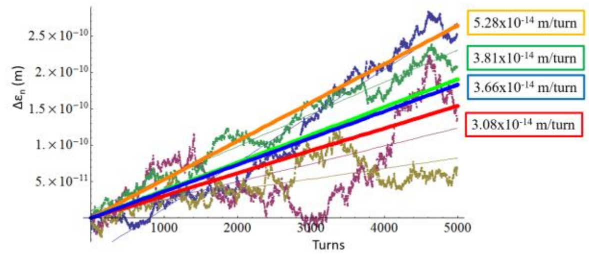

Figure 3: The solid orange line is the rate predicted by GUINEA-PIG, The solid green line is the rate predicted by the ansatz, the solid blue line is the average of the four simulation rates, and the solid red line is the directly integrated path. The point data is the simulated emittances of four different test distributions sent through the same set of kicks in our model system.

TOWARDS AN ANALYTIC MODEL

While it is possible to estimate the emittance growth rate in a system like the LHeC by either using a beam-beam code like GUINEA-PIG or to directly integrate the equations of motion, it would be far simpler if we could use a formula to get a quick guess. The attempt at an ansatz for the path of the electron beam through the proton beam was an early attempt at this.

We can begin to model these systems analytically by making a series of assumptions; we assume the following about the system.

- One beam's motion can be considered constant (Reference) and one beam can be considered as moving (Colliding). (i.e. a weak strong system)
- Both beams are round at impact, and have a gaussian profile so that the round beam Basetti-Erskine approximation can be used.
- The colliding beam can be considered a gaussian disk of charge moving through the reference beam.

These assumptions prove to be very good for the LHeC system since the electron bunch is much less rigid than the proton bunch, and much shorter. In the following notation we will use subscripts to denote the quantity a beam "sees" so for instance D_C would be the disruption parameter "seen" by the colliding beam. If we start with the equations of motion for the colliding beam,

$$r''(z) + \frac{2Nr_{particle}}{\gamma\sqrt{2}\sigma_z} f(r(z), \sigma_r) e^{-\left(\frac{z}{\sigma_z}\right)^2} = 0, \quad (2)$$

where,

$$f(r(z), \sigma_r) = \frac{e^{\frac{-r(z)^2}{2\sigma_r^2}} - 1}{r(z)}. \quad (3)$$

We can cast this into dimensionless coordinates (z_0=z/σ_z), and make use of the beam-beam disruption parameter,

$$D_{x,y} = \frac{2Nr_{particle}\sigma_z}{\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}, \quad (4)$$

to reduce the equations of motion to,

$$r''(z_0) + \frac{2D_C}{\sqrt{2}} f(r(z_0), \sigma_r) e^{-z_0^2} = 0. \quad (5)$$

This arrangement has the advantage that a given disruption parameter will define a unique path for the colliding beam through the reference beam. For uniformity we assume a 1 σ_r initial offset and assume a linear scaling in the momentum regime. That means that using our dimensionless coordinates,

$$\langle\Delta p_r^2\rangle = 4D_r^2 \left(\frac{\sigma_{r_C}}{\sigma_{z_C}}\right)^2 N(D_C) \left(\frac{\sigma_{jitter}}{\sigma_{r_R}}\right)^2, \quad (6)$$

where,

$$N(D_C) = \frac{\int \frac{\left(e^{-\frac{r(z_0;D_C)^2}{2}} - 1\right)^2}{r(z_0;D_C)^2} e^{-2z_0^2} dz_0}{\int e^{-2z_0^2} dz_0}. \quad (7)$$

N(D_C) is the momentum change squared, however if we assume that the bunches are damped well in our circulator ring, then the overall kick that the whole reference bunch receives will need to be removed. This is accomplished with,

$$I(D_C) = \frac{\left(\int \frac{e^{-\frac{r(z_0;D_C)^2}{2}} - 1}{r(z_0;D_C)} e^{-2z_0^2} dz_0\right)^2}{\int e^{-2z_0^2} dz_0}. \quad (8)$$

Eq. 7, and Eq. 8 can be best understood when viewed graphically, as is shown in Figure 4.

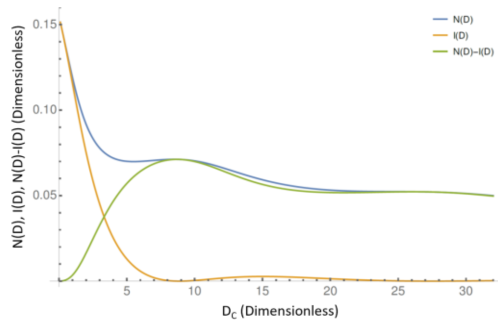

Figure 4: This plot shows N(D_C), I(D_C), and N(D_C)-I(D_C) in blue, orange, and green respectively.

In Figure 4 we see that the green line, which can be interpreted as a system with perfect damping of offsets in the recirculating portion of the machine will have a maximum growth rate at a disruption parameter seen by the colliding beam of 8.89. This is the point where the colliding beam has equal paths on both sides of the reference beam. This can be seen in Figure 5.

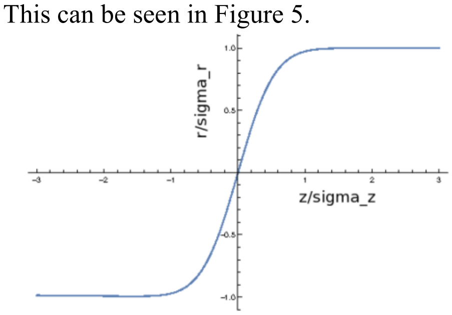

Figure 5: This is the path of the colliding beam through the reference beam at the point of maximum emittance growth.

Thus, half of the beam receives a kick in one direction, and half receives a kick in the other direction. This is also why the $N(D_C)$ and $N(D_C)$-$I(D_C)$ lines are equal at that point. Above this value both lines are very close, this is because the reference beam has such a strong pull that it will "suck" the colliding beam into it and keep it there for the remainder of the interaction.

One other issue that could confound this system, is the fact that the beams experience an hourglass effect as they go through the Interaction Point (IP). These can be added to the curves shown in Figure 4, extending them to accommodate an hourglass effect. In order to keep this method useful we need to keep the parameters dimensionless. Thus we create the dimensionless quantity,

$$e_C = \frac{\sigma_{z_R}}{\beta^*_R}. \quad (9)$$

A similar term e_R can be made for the reference beam, but due to the assumptions made that will have a negligible effect on our system since the bunch length of the colliding beam is so short. This is included in the dimensionless equations by changing the σ_z term which has been normalized to one, and recast it as $(1+(z_0e_C)^2)$. Examples of $N(D_C,e_C)$ and $N(D_C,e_C)$-$I(D_C,e_C)$ are shown in Figures 6 and 7 respectively.

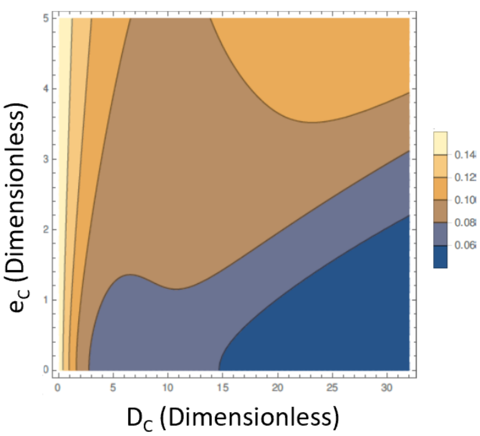

Figure 6: This is a plot of $N(D_C,e_C)$ over a range of both numbers.

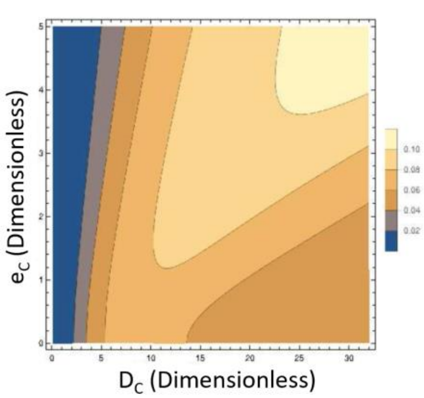

Figure 7: This is a plot of $N(D_C,e_C)$-$I(D_C,e_C)$ over values of both D_C and e_C.

If we pull together all of the information so far, then we can show that for systems that follow our basic assumptions, we can calculate the per-turn growth rate as,

$$\Delta\epsilon_n = 2\beta^*(\beta\gamma)D_R^2\left(\frac{\sigma_{rC}}{\sigma_{zC}}\right)^2\left(\frac{\sigma_{jitter}}{\sigma_{rR}}\right)^2 E(D_c,e_C,e_R), \quad (10)$$

Where,

$$E(D_c,e_C,e_R) = \mathrm{N}(D_c,e_C,e_R)\text{-}\mathrm{kI}(D_c,e_C,e_R). \quad (11)$$

Where k is 0 if we assume no offset corrections in the recirculating beam, and 1 if we assume there are. e_C and e_R can be used as appropriate, but are 0 if not needed. Eq 10 and 11 when combined with either graphs such as Figures 4 6 and 7, or when interpolating from lookup tables can provide a very rapid method of estimating per-turn emittance growth rates in asymmetric systems like those found in the LHeC. Examples of these new methods, are shown in Figure 8.

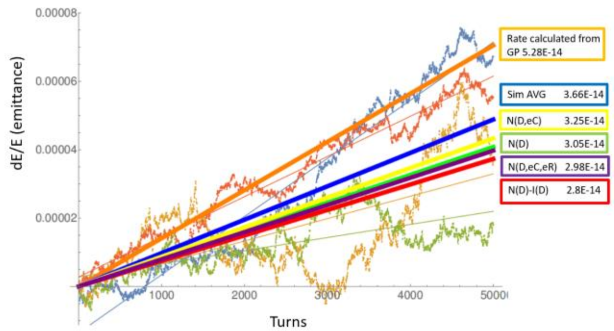

Figure 8: In this figure we see a variety of analytic methods shown in this work as compared to the same growth data shown in Figure 3.

The analytic method that most closely matches the average of the simulations is $N(D_C,e_C)$. Since these simulations include hourglass, but do not correct for offset errors in the recirculation, this was to be expected.

CONCLUSIONS AND FURTHER WORK

This work has made strides in creating an analytic formula to estimate the emittance growth rates for the beam-beam effect. There is however more work to be done, both in expanding from the current focusing interactions (ep collisions) to defocusing interactions (pp collisions), and also in moving away from looking up numbers on a curve to a true formula. Though one interesting insight gained from this study is the fact that for high D_C the growth rate doesn't increase anymore.

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