Lowest order QED radiative effects in polarized SIDIS

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The explicit exact analytical expressions for the lowest-order radiative corrections to the semi-inclusive deep inelastic scattering of the polarized particles are obtained in the most compact, covariant and convenient for the numerical analysis form. The infrared divergence from the real photon emission is extracted and cancelled using the Bardin-Shumeiko approach. The contribution of the exclusive radiative tail is presented. The analytic results obtained within the ultrarelativistic approximation are also shown.

I. INTRODUCTION

Nowadays the polarized semi-inclusive deep-inelastic scattering (SIDIS) plays a crucial role in our understanding of the internal spin structure of the nucleons. Information on the three-dimensional structure of the polarized proton and neutron can be obtained by extracting the quark transverse momentum distributions from the various single spin asymmetries measured in SIDIS with polarized particles. Specifically, the Sivers and Collins contributions can be selected [1] from the present data on transversely polarized targets $\vec{p}(e,e'\pi)x$ in HERMES [2], $\vec{D}(\mu,\mu'\pi)x$ in COMPASS [3] and $^3\vec{\mathrm{He}}(e,e'\pi)x$ in JLab [4] which show a strong flavor dependence of transverse momentum distributions. Moreover in the near future, highly accurate experiments are planned at 12-GeV Jlab [5] that will provide unique opportunities for the breakthrough in the investigation of the nucleon structure by carrying out multi-dimensional precision studies of longitudinal and transverse spin and momentum degrees of freedom from SIDIS experiments with high luminosity in combination with large acceptance detectors.

It is well known that one of the important sources of the systematical uncertainties in SIDIS experiments with and without polarization of initial particles are the QED radiative corrections (RC). RC to the three-fold differential cross section ($d\sigma/dxdydz$, where x and y are the standard Bjorken variables and the z is the fraction of the virtual-photon energy transferred to the detected hadron) can be calculated using the patch SIRAD of FORTRAN code POLRAD [6] created based on the original calculations in refs. [7] and [8] for unpolarized and polarized particles. The calculation of RC to the five-fold differential cross section of unpolarized particles ($d\sigma/dxdydzdp_t^2d\phi_h$, where p_t is the detected hadron transverse momentum and ϕ_h is the azimuthal angle between the lepton scattering and hadron production planes) was performed in ref. [9]. These calculations did not contain the radiative tail from the exclusive reactions as a separate contribution involving the exclusive structure functions (SF). This limitation was addressed in ref. [10] in which the authors explicitly calculated the exclusive radiative tail and implemented the exclusive SF using the approach of MAID [11].

In the present paper we consider the general task of RC calculation when the initial nucleon can be arbitrary polarized. The analytical expressions for RC to SIDIS are obtained for the six-fold cross section with the longitudinally polarized lepton and arbitrary polarized target, $d\sigma/dxdydzdp_t^2d\phi_hd\phi$, where the azimuthal angle ϕ between the lepton scattering and ground planes is introduced to appropriately account the transverse target polarization. The contribution of the exclusive radiative tail to the total RC is also presented. Similarly to the previous analyses we calculated RC in the model-independent way. These corrections are induced by the unobservable real photon emission from the lepton leg, leptonic vertex correction, and vacuum polarization. The model independent correction is proportional to the leading logarithm $\log(Q^2/m^2)$, which is large because of high transferring momentum squared Q^2 (> 1 GeV^2) and small electron mass m. What is not accounted in this approach is the correction due to the real and additional virtual photon emission by hadrons. However, this correction should not be accounted in majority of cases, e.g., when the used model for SF was extracted from the experiment in which emission by hadrons had not been applied in RC procedure of experimental data.

The Bardin-Shumeiko approach [12] is used for extraction and cancellation of the infrared divergence coming from the real and virtual photon emission. In contrast to the widely used the Mo-Tsai approach [13, 14] the final expression for RC within the Bardin-Shumeiko approach does not depend on an artificial parameter that is introduced in [13, 14] for separation of the photon emission on the hard and soft parts.

In this paper we apply an approach for decomposition of the initial nucleon and virtual photon polarization as well as the real photon four-momentum over the respective bases (Appendix A). The polarization decom-

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position is used for the hadronic tensor representation in a covariant form. The momentum decomposition is used to simplify integration over the momentum of the unobserved photon. Specifically, this allows us to essentially reduce the number of pseudoscalars occurring after the convolution of the leptonic tensors of radiative effects with the hadronic tensor and present the final expressions for RC in a compact, covariant and convenient for numerical analysis form. All calculations have been performed in an exact way keeping the lepton mass at all stages of the calculation. The dependence of certain terms in the exact final expressions for RC on the electron mass is quite tricky and therefore we analyze respective contributions in the ultrarelativistic approximation allowing for extraction of the electron mass dependence explicitly and classifying all terms in RC as leading (i.e., containing the leading logarithms), next-to-leading (i.e., independent on the electron mass), and other potentially negligible (i.e., the terms vanishing in the approximation of $m \to 0$). Thus the results obtained in the paper contain both exact formulas for RC and expressions in ultrarelativistic approximations allowing us to explicitly control the dependence on the electron mass. Thus, the analytic expressions for RC are valid for experiments with muons (e.g., COMPASS [3]) in which the approximation of zero lepton mass could be not appropriate.

The rest of the article is organized as follows. The hadronic tensor, different sets for the SF used in the literature, as well as the lowest-order (Born) contribution to SIDIS process are discussed in Section II. The calculation of the lowest-order QED RC to the observables in SIDIS as well as the explicit results for both the semi-inclusive final hadronic state and exclusive radiative tail contributions are presented in Section III. The infrared divergence in these calculations are extracted from the real photon emission with the semi-inclusive final hadronic state by Bardin-Shumeiko approach [12] and then cancelled with the corresponding term from the leptonic vertex correction in such a way that the obtained results are free from an intermediate parameter $\bar{k}_0$. For the parameterization of the infrared and ultraviolet divergences the dimension regularization is used. The results of analyses of the exact expressions in ultrarelativistic approximation are given in Section IV. Particularly we show that the double leading logarithms coming from the terms with the soft photon emission and the leptonic vertex correction cancel in their sum. Brief discussion and conclusion are presented in Section V. Technical details and most cumbersome parts of the RC are presented in four Appendices. The bases for the decomposition of the initial target and virtual photon polarization as well as the real photon momentum are presented in Appendix A. The explicit expressions for the real photon emission quantities are presented in Appendix B. The details of the approach for the infrared divergence extraction and cancellation are given in Appendix C. The detail calculations of the additional virtual particle contributions are presented in Appendix D.

II. HADRONIC TENSOR AND BORN CONTRIBUTION

The six-fold differential cross section of SIDIS with polarized particles

$$e(k_1,\xi) + n(p,\eta) \longrightarrow e(k_2) + h(p_h) + x(p_x) \qquad (1)$$

($k_1^2 = k_2^2 = m^2$, $p^2 = M^2$, $p_h^2 = m_h^2$) where ξ (η) is the initial lepton (nucleon) polarized vector, can be described by the following set of variables

$$x = -\frac{q^2}{2qp},\ y = \frac{qp}{k_1 p},\ z = \frac{p_h p}{pq},$$
$$t = (q - p_h)^2,\ \phi_h,\ \phi. \qquad (2)$$

Here $q = k_1 - k_2$, ϕ_h is the angle between $(\mathbf{k_1}, \mathbf{k_2})$ and $(\mathbf{q}, \mathbf{p_h})$ planes and ϕ is the angle between $(\mathbf{k_1}, \mathbf{k_2})$ and the ground planes in the target rest frame reference system ($\mathbf{p} = 0$).

Also we use the following set of invariants:

$$\begin{aligned}
&S = 2pk_1,\ Q^2 = -q^2,\ Q_m^2 = Q^2 + 2m^2,\\
&X = 2pk_2,\ S_x = S - X,\ S_p = S + X,\\
&V_{1,2} = 2k_{1,2}p_h,\ V_+ = \frac{1}{2}(V_1 + V_2),\\
&V_- = \frac{1}{2}(V_1 - V_2) = \frac{1}{2}(m_h^2 - Q^2 - t),\\
&S' = 2k_1(p + q - p_h) = S - Q^2 - V_1,\\
&X' = 2k_2(p + q - p_h) = X + Q^2 - V_2,\\
&p_x^2 = (p + q - p_h)^2 = M^2 + t + (1 - z)S_x.\\
&\lambda_S = S^2 - 4M^2m^2,\ \lambda_Y = S_x^2 + 4M^2Q^2,\\
&\lambda_1 = Q^2(SX - M^2Q^2) - m^2\lambda_Y,\ \lambda_m = Q^2(Q^2 + 4m^2),\\
&\lambda'_S = S'^2 - 4m^2p_x^2,\ \lambda'_X = X'^2 - 4m^2p_x^2. \qquad (3)
\end{aligned}$$

Non-invariant variables including the energy p_{h0}, longitudinal p_l and transverse p_t (k_t) three-momenta of the detected hadron (the incoming or scattering lepton) with respect to the virtual photon direction in the target rest frame are expressed in terms of the above invariants:

$$\begin{aligned}
p_{h0} &= \frac{zS_x}{2M},\\
p_l &= \frac{zS_x^2 - 4M^2V_-}{2M\sqrt{\lambda_Y}} = \frac{zS_x^2 + 2M^2(t + Q^2 - m_h^2)}{2M\sqrt{\lambda_Y}},\\
p_t &= \sqrt{p_{h0}^2 - p_l^2 - m_h^2},\\
k_t &= \sqrt{\frac{\lambda_1}{\lambda_Y}}. \qquad (4)
\end{aligned}$$

As a result the quantities $V_{1,2}$ can be written through $\cos\phi_h$ and other variables defined in Eqs. (2-4) as:

$$\begin{aligned}
V_1 &= p_{h0}\frac{S}{M} - \frac{p_l(SS_x + 2M^2Q^2)}{M\sqrt{\lambda_Y}} - 2p_tk_t\cos\phi_h,\\
V_2 &= p_{h0}\frac{X}{M} - \frac{p_l(XS_x - 2M^2Q^2)}{M\sqrt{\lambda_Y}} - 2p_tk_t\cos\phi_h. \qquad (5)
\end{aligned}$$

The sine of ϕ_h is expressed as

$$\sin\phi_h = \frac{2\varepsilon_\perp p_h}{p_t\sqrt{\lambda_1}}, \tag{6}$$

where

$$\varepsilon_\perp^\mu = \varepsilon^{\mu\nu\rho\sigma} p_\nu k_{1\rho} q_\sigma \tag{7}$$

is the pseudovector with a normal direction to the scattering plane $(\mathbf{k_1}, \mathbf{k_2})$.

The lowest-order QED (Born) contribution to SIDIS is presented by Feynman graph on Fig. 1 (a). The cross section for this process reads:

$$d\sigma_B = \frac{(4\pi\alpha)^2}{2\sqrt{\lambda_S}Q^4} W_{\mu\nu} L_B^{\mu\nu} d\Gamma_B, \tag{8}$$

where the phase space is parameterized as:

$$\begin{aligned} d\Gamma_B &= (2\pi)^4 \frac{d^3k_2}{(2\pi)^3 2k_{20}} \frac{d^3p_h}{(2\pi)^3 2p_{h0}} \\ &= \frac{1}{4(2\pi)^2} \frac{SS_x dx dy d\phi}{2\sqrt{\lambda_S}} \frac{S_x dz dp_t^2 d\phi_h}{4Mp_l}. \end{aligned} \tag{9}$$

Since the initial lepton is considered to be longitudinally polarized, its polarization vector has the form [15]:

$$\xi = \frac{\lambda_e S}{m\sqrt{\lambda_S}} k_1 - \frac{2\lambda_e m}{\sqrt{\lambda_S}} p_1 = \xi_0 + \xi_1. \tag{10}$$

As a result the leptonic tensor is:

$$\begin{aligned} L_B^{\mu\nu} &= \frac{1}{2}\mathrm{Tr}[(\hat{k}_2 + m)\gamma_\mu(\hat{k}_1 + m)(1 + \gamma_5\hat{\xi})\gamma_\nu] \\ &= 2[k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - \frac{Q^2}{2} g^{\mu\nu} \\ &+ \frac{i\lambda_e}{\sqrt{\lambda_S}} \varepsilon^{\mu\nu\rho\sigma}(Sk_{2\rho}k_{1\sigma} + 2m^2 q_\rho p_\sigma)]. \end{aligned} \tag{11}$$

According to [16] the hadronic tensor for SIDIS process $\gamma^* + n \to h + X$ can be decomposed in the terms of the scalar spin-independent $H_{ab}^{(0)}$ and spin-dependent $H_{abi}^{(S)}$ structures functions

$$W_{\mu\nu} = \sum_{a,b=0}^{3} e_\mu^{\gamma(a)} e_\nu^{\gamma(b)} (H_{ab}^{(0)} + \sum_{\rho,i=0}^{3} \eta^\rho e_\rho^{h(i)} H_{abi}^{(S)}). \tag{12}$$

where $e_\mu^{\gamma(a)}$ (or $e_\nu^{\gamma(b)}$) and $e_\rho^{h(i)}$ are the complete set of the basis vectors for the polarization four-vectors of the virtual photon and nucleon in the target rest frame. These vectors can be represented in a covariant form [17] using (A1) and (A2).

Due to the parity and current conservation, hermiticity as well as $p\eta \equiv 0$ only the following set of independent SF $H_{ab}^{(0)}$ and $H_{abi}^{(S)}$ in (12) survives [16]: five spin-independent $H_{00}^{(0)}$, $H_{11}^{(0)}$, $H_{22}^{(0)}$, $\mathrm{Re}H_{01}^{(0)}$, $\mathrm{Im}H_{01}^{(0)}$ and thirteen spin-dependent $H_{002}^{(S)}$, $\mathrm{Re}H_{012}^{(S)}$, $\mathrm{Im}H_{012}^{(S)}$, $\mathrm{Re}H_{021}^{(S)}$, $\mathrm{Im}H_{021}^{(S)}$, $\mathrm{Re}H_{023}^{(S)}$, $\mathrm{Im}H_{023}^{(S)}$, $H_{112}^{(S)}$, $\mathrm{Re}H_{121}^{(S)}$, $\mathrm{Im}H_{121}^{(S)}$, $\mathrm{Re}H_{123}^{(S)}$, $\mathrm{Im}H_{123}^{(S)}$, $H_{222}^{(S)}$. All the rest SF have to be set zero [16].

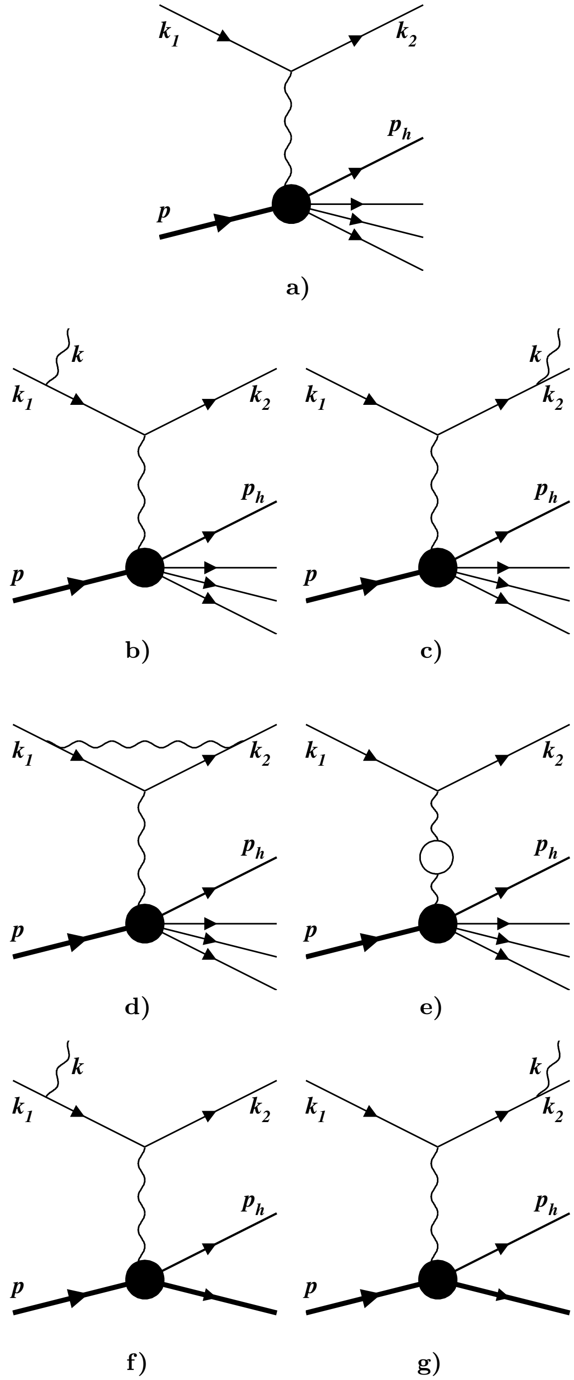

FIG. 1. Feynman graphs for the lowest order (a), SIDIS (b-e) and exclusive radiative tails (f,g) contributions to the lowest order RC for SIDIS scattering

The hadronic tensor in terms of these SF can be obtained by substitution (A1) and (A2) into (12) resulting in:

$$\begin{aligned} W_{\mu\nu} = \sum_{i=1}^{9} w_{\mu\nu}^i \mathcal{H}_i &= -g_{\mu\nu}^\perp \mathcal{H}_1 + p_\mu^\perp p_\nu^\perp \mathcal{H}_2 + p_{h\mu}^\perp p_{h\nu}^\perp \mathcal{H}_3 \\ &+ (p_\mu^\perp p_{h\nu}^\perp + p_{h\mu}^\perp p_\nu^\perp)\mathcal{H}_4 + i(p_\mu^\perp p_{h\nu}^\perp - p_{h\mu}^\perp p_\nu^\perp)\mathcal{H}_5 \end{aligned}$$

$$+(p^{\perp}_{\mu}n_{\nu}+n_{\mu}p^{\perp}_{\nu})\mathcal{H}_6+i(p^{\perp}_{\mu}n_{\nu}-n_{\mu}p^{\perp}_{\nu})\mathcal{H}_7$$
$$+(p^{\perp}_{h\mu}n_{\nu}+n_{\mu}p^{\perp}_{h\nu})\mathcal{H}_8$$
$$+i(p^{\perp}_{h\mu}n_{\nu}-n_{\mu}p^{\perp}_{h\nu})\mathcal{H}_9. \qquad (13)$$

Here: $g^{\perp}_{\mu\nu}=g_{\mu\nu}-q_{\mu}q_{\nu}/q^2$ and $n^{\mu}=\varepsilon^{\mu\nu\rho\sigma}q_{\nu}p_{\rho}p_{h\sigma}$.

The generalized SF $\mathcal{H}_i$ can be expressed via $H^{(0)}_{ab}$, $H^{(S)}_{abi}$ using the decomposition of the nucleon polarized three-vector $\boldsymbol{\eta}=(\eta_1,\eta_2,\eta_3)$ over the basis (A2) in the following way

$$\begin{aligned}
\mathcal{H}_1&=H^{(0)}_{22}-\eta_2H^{(S)}_{222},\\
\mathcal{H}_2&=\frac{4}{\lambda_Y^2p_t^2}[\lambda_Yp_t^2Q^2(H^{(0)}_{00}-\eta_2H^{(S)}_{002})+\lambda_3^2S_x^2(H^{(0)}_{11}\\
&\qquad-\eta_2H^{(S)}_{112})-\lambda_2\lambda_Y(H^{(0)}_{22}-\eta_2H^{(S)}_{222})\\
&\qquad-2S_x\lambda_3p_tQ\sqrt{\lambda_Y}(\mathrm{Re}H^{(0)}_{01}-\eta_2\mathrm{Re}H^{(S)}_{012})],\\
\mathcal{H}_3&=\frac{1}{p_t^2}(H^{(0)}_{11}-H^{(0)}_{22}+\eta_2(H^{(S)}_{222}-H^{(S)}_{112})),\\
\mathcal{H}_4&=\frac{2}{\lambda_Yp_t^2}[\lambda_3S_x(H^{(0)}_{22}-H^{(0)}_{11}+\eta_2(H^{(0)}_{112}-H^{(S)}_{222}))\\
&\qquad+p_tQ\sqrt{\lambda_Y}(\mathrm{Re}H^{(0)}_{01}-\eta_2\mathrm{Re}H^{(S)}_{012})],\\
\mathcal{H}_5&=\frac{2Q}{p_t\sqrt{\lambda_Y}}(\mathrm{Im}H^{(0)}_{01}-\eta_2\mathrm{Im}H^{(S)}_{012}),\\
\mathcal{H}_6&=\frac{4M}{\lambda_Y^{3/2}p_t^2}[Qp_t\sqrt{\lambda_Y}(\eta_1\mathrm{Re}H^{(S)}_{021}+\eta_3\mathrm{Re}H^{(S)}_{023})\\
&\qquad-\lambda_3S_x(\eta_1\mathrm{Re}H^{(S)}_{121}+\eta_3\mathrm{Re}H^{(S)}_{123})],\\
\mathcal{H}_7&=\frac{4M}{\lambda_Y^{3/2}p_t^2}[Qp_t\sqrt{\lambda_Y}(\eta_1\mathrm{Im}H^{(S)}_{021}+\eta_3\mathrm{Im}H^{(S)}_{023})\\
&\qquad-\lambda_3S_x(\eta_1\mathrm{Im}H^{(S)}_{121}+\eta_3\mathrm{Im}H^{(S)}_{123})],\\
\mathcal{H}_8&=\frac{2M}{\sqrt{\lambda_Y}p_t^2}(\eta_1\mathrm{Re}H^{(S)}_{121}+\eta_3\mathrm{Re}H^{(S)}_{123}),\\
\mathcal{H}_9&=\frac{2M}{\sqrt{\lambda_Y}p_t^2}(\eta_1\mathrm{Im}H^{(S)}_{121}+\eta_3\mathrm{Im}H^{(S)}_{123}). \qquad (14)
\end{aligned}$$

Here $\lambda_2=V_-^2+m_h^2Q^2$, $\lambda_3=V_-+zQ^2$ and V_- is defined in Eqs.(3).

Finally we find the Born contribution in the form:

$$\sigma^B\equiv\frac{d\sigma^B}{dxdydzdp_t^2d\phi_hd\phi}=\frac{\alpha^2SS_x^2}{8MQ^4p_l\lambda_S}\sum_{i=1}^{9}\theta_i^B\mathcal{H}_i, \qquad (15)$$

where $\theta_i^B=L^{\mu\nu}w^i_{\mu\nu}/2$:

$$\begin{aligned}
\theta_1^B&=Q^2-2m^2,\\
\theta_2^B&=(SX-M^2Q^2)/2,\\
\theta_3^B&=(V_1V_2-m_h^2Q^2)/2,\\
\theta_4^B&=(SV_2+XV_1-zQ^2S_x)/2,\\
\theta_5^B&=\frac{2\lambda_eS\varepsilon_{\perp}p_h}{\sqrt{\lambda_S}},\\
\theta_6^B&=-S_p\varepsilon_{\perp}p_h,\\
\theta_7^B&=\frac{\lambda_eS}{4\sqrt{\lambda_S}}[\lambda_YV_+-S_pS_x(zQ^2+V_-)],\\
\theta_8^B&=-2V_+\varepsilon_{\perp}p_h,\\
\theta_9^B&=\frac{\lambda_e}{2\sqrt{\lambda_S}}[S(Q^2(zS_xV_+-m_h^2S_p)+V_-(SV_2\\
&\qquad-XV_1))+2m^2(4M^2V_-^2+\lambda_Ym_h^2\\
&\qquad-zS_x^2(zQ^2+2V_-))]. \qquad (16)
\end{aligned}$$

The quantities $H^{(0)}_{ab}$ and $H^{(S)}_{abi}$ can be expressed through another set of the SF presented in [18]. Taking into account that $\eta_1=\cos(\phi_s-\phi_h)S_{\perp}$, $\eta_2=\sin(\phi_s-\phi_h)S_{\perp}$ and $\eta_3=S_{||}$ we find that:

$$\begin{aligned}
H^{(0)}_{00}&=C_1F_{UU,L},\\
H^{(0)}_{01}&=C_1(-F_{UU}^{\cos\phi_h}+iF_{LU}^{\sin\phi_h}),\\
H^{(0)}_{11}&=C_1(F_{UU}^{\cos2\phi_h}+F_{UU,T}),\\
H^{(0)}_{22}&=C_1(F_{UU,T}-F_{UU}^{\cos2\phi_h}),\\
H^{(S)}_{002}&=C_1F_{UT,L}^{\sin(\phi_h-\phi_s)},\\
H^{(S)}_{012}&=C_1(F_{UT}^{\sin\phi_s}-F_{UT}^{\sin(2\phi_h-\phi_s)}\\
&\qquad+i(F_{LT}^{\cos\phi_s}-F_{LT}^{\cos(2\phi_h-\phi_s)})),\\
H^{(S)}_{021}&=-C_1(F_{UT}^{\sin(2\phi_h-\phi_s)}+F_{UT}^{\sin\phi_s}\\
&\qquad+i(F_{LT}^{\cos(2\phi_h-\phi_s)}+F_{LT}^{\cos\phi_s})),\\
H^{(S)}_{023}&=-C_1(F_{UL}^{\sin\phi_h}+iF_{LL}^{\cos\phi_h}),\\
H^{(S)}_{121}&=C_1(F_{UT}^{\sin(3\phi_h-\phi_s)}+F_{UT}^{\sin(\phi_h+\phi_s)}\\
&\qquad+iF_{LT}^{\cos(\phi_h-\phi_s)}),\\
H^{(S)}_{123}&=C_1(F_{UL}^{\sin2\phi_h}+iF_{LL}),\\
H^{(S)}_{112}&=C_1(F_{UT}^{\sin(3\phi_h-\phi_s)}+F_{UT,T}^{\sin(\phi_h-\phi_s)}\\
&\qquad-F_{UT}^{\sin(\phi_h+\phi_s)}),\\
H^{(S)}_{222}&=C_1(F_{UT}^{\sin(\phi_h+\phi_s)}+F_{UT,T}^{\sin(\phi_h-\phi_s)}\\
&\qquad-F_{UT}^{\sin(3\phi_h-\phi_s)}), \qquad (17)
\end{aligned}$$

where

$$C_1=\frac{4Mp_l(Q^2+2xM^2)}{Q^4}. \qquad (18)$$

III. LOWEST-ORDER RADIATIVE CORRECTIONS

The six matrix elements shown in Fig. 1b-g contribute to the lowest-order QED RC to the cross section of the base SIDIS process (Fig. 1a). A critical difference in the graphs a)-e) comparing to the graphs f) and g) is the distinct final unobserved hadronic state: continuum of particles in the former and a single hadron in the latter case. The underlying processes are semi-inclusive and exclusive hadron leptoproduction, respectively. At the level of RC, both of them include the unobservable real photon emission from the lepton leg as presented in Fig. 1 (b,c) and (f,g). The contribution to RC from the semi-inclusive process contains also the leptonic vertex correction and vacuum polarization (Fig. 1d,e). Thus these two separate contributions to the total RC to the SIDIS cross section are considered in two separate subsections below.

A. Semi-Inclusive Contribution

The real photon emission in semi-inclusive process,

$$e(k_1,\xi)+n(p,\eta)\to e(k_2)+h(p_h)+x(\tilde{p}_x)+\gamma(k), \tag{19}$$

where k is a real photon four-momentum depicted on Fig. 1 (b,c) is described by the set variables presented in (2) and three additional quantities:

$$R=2kp,\ \tau=\frac{kq}{kp},\ \phi_k, \tag{20}$$

where ϕ_k is an angle between $(\mathbf{k}_1,\mathbf{k}_2)$ and $(\mathbf{k},\mathbf{q})$ planes. Its sine in the covariant form is:

$$\sin\phi_k=\frac{2\varepsilon_\perp k\sqrt{\lambda_Y}}{R\sqrt{\lambda_1(Q^2+\tau(S_x-\tau M^2))}}. \tag{21}$$

The contribution of real photon emission from the leptonic leg is:

$$d\sigma_R=\frac{(4\pi\alpha)^3}{2\sqrt{\lambda_S}\tilde{Q}^4}\widetilde{W}_{\mu\nu}L^{\mu\nu}_R d\Gamma_R. \tag{22}$$

Here the symbol of "tilde" denotes that the arguments of the hadronic tensor such as Q^2, W^2, z, t and ϕ_h are defined through the shifted $q\to q-k$, *i.e.* $\tilde{Q}^2=-(q-k)^2=Q^2+R\tau$. The phase space of the considered process has a form:

$$d\Gamma_R=(2\pi)^4\frac{d^3k}{(2\pi)^3 2k_0}\frac{d^3k_2}{(2\pi)^3 2k_{20}}\frac{d^3p_h}{(2\pi)^3 2p_{h0}}, \tag{23}$$

where

$$\frac{d^3k}{k_0}=\frac{RdRd\tau d\phi_k}{2\sqrt{\lambda_Y}}. \tag{24}$$

For the representation of explicit results in the simplest way the leptonic tensor $L^{\mu\nu}_R$ in (22) is separated into two parts:

$$L^{\mu\nu}_R=L^{\mu\nu}_{R0}+L^{\mu\nu}_{R1}. \tag{25}$$

The first term includes the part of the leptonic tensor that contains spin independent terms and terms containing ξ_0, i.e., the part of the polarization vector (10):

$$L^{\mu\nu}_{R0}=-\frac{1}{2}\mathrm{Tr}[(\hat{k}_2+m)\Gamma^{\mu\alpha}_R(\hat{k}_1+m)(1+\gamma_5\hat{\xi}_0)\bar{\Gamma}^\nu_{R\alpha}], \tag{26}$$

where

$$\begin{aligned}\Gamma^{\mu\alpha}_R&=\left(\frac{k_1^\alpha}{kk_1}-\frac{k_2^\alpha}{kk_2}\right)\gamma^\mu-\frac{\gamma^\mu\hat{k}\gamma^\alpha}{2kk_1}-\frac{\gamma^\alpha\hat{k}\gamma^\mu}{2kk_2},\\ \bar{\Gamma}^\nu_{R\alpha}&=\gamma_0\Gamma^{\nu\dagger}_{R\alpha}\gamma_0\\ &=\left(\frac{k_{1\alpha}}{kk_1}-\frac{k_{2\alpha}}{kk_2}\right)\gamma^\nu-\frac{\gamma^\nu\hat{k}\gamma_\alpha}{2kk_2}-\frac{\gamma_\alpha\hat{k}\gamma^\nu}{2kk_1}.\end{aligned} \tag{27}$$

The second term in (25) is proportional only to the residual part ξ_1 of the polarization vector ξ

$$L^{\mu\nu}_{R1}=-\frac{1}{2}\mathrm{Tr}[(\hat{k}_2+m)\Gamma^{\mu\alpha}_R(\hat{k}_1+m)\gamma_5\hat{\xi}_1\bar{\Gamma}^\nu_{R\alpha}]. \tag{28}$$

As it is shown below this part of the leptonic tensor gives non-vanishing contribution to RC in the ultrarelativistic approximation both for the semi-inclusive (71) and exclusive (73) final hadronic states.

The convolution of the leptonic tensors $L^{\mu\nu}_{R0}$ and $L^{\mu\nu}_{R1}$ with the shifted hadronic tensor can be presented as

$$\begin{aligned}\widetilde{W}_{\mu\nu}L^{\mu\nu}_{R0}&=\sum_{i=1}^{9}\tilde{w}^i_{\mu\nu}\tilde{\mathcal{H}}_iL^{\mu\nu}_{R0}=-2\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta^0_{ij}R^{j-3},\\ \widetilde{W}_{\mu\nu}L^{\mu\nu}_{R1}&=\sum_{i=5,7,9}\tilde{w}^i_{\mu\nu}\tilde{\mathcal{H}}_iL^{\mu\nu}_{R1}=-2\sum_{i=5,7,9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta^1_{ij}R^{j-3},\end{aligned} \tag{29}$$

where i enumerates the contributions of respective SF in (13). The sum over j represents the decomposition of the leptonic (26, 28) and hadronic tensor convolutions over R. In this decomposition quantities $\theta^{0,1}_{ij}$ do not depend on R. Their explicit expressions are presented in Appendix B. The number of terms is different for different SF: $k_i=\{3,3,3,3,3,4,4,4,4\}$.

The lowest order SIDIS process (1) is described by the four independent four-momenta such us p, k_1, q and p_h. Therefore, the Born cross section contains only one pseudoscalar $\varepsilon^{\mu\nu\rho\sigma}p_{h\,\mu}p_\nu k_{1\rho}q_\sigma=\varepsilon^\mu_\perp p_h$. This pseudoscalar contributes to $\theta^B_{5,6,8}$ as it was shown in Eqs. (16) and, according to Eqs. (6,7), can be expressed in terms of the variables (2-4) as: $\varepsilon^{\mu\nu\rho\sigma}p_{h\,\mu}p_\nu k_{1\rho}q_\sigma=\varepsilon^\mu_\perp p_h=p_t\sqrt{\lambda_1}\sin\phi_h/2$. When we deal with real photon emission the additional independent four-momentum k appears. As a result the number of pseudoscalar quantities that can exist in the expressions for the cross section grows up to five. They are not independent and their number can be reduced to two, namely $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$, using the decomposition of the photonic four-momentum over the basis introduced in Appendix A by Eqs. (A5). As shown in Eqs. (A9) the rest three pseudoscalars are expressed through the linear combination of $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$. The explicit expression for $\varepsilon_\perp k$ follows from (21)

$$\varepsilon_\perp k=\frac{\sin\phi_k R\sqrt{\lambda_1(Q^2+\tau(S_x-\tau M^2))}}{2\sqrt{\lambda_Y}}. \tag{30}$$

After substitution (29) into (22)

$$d\sigma_R = -\frac{\alpha^3}{4\pi^2\tilde{Q}^4\sqrt{\lambda_S}}\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}R^{j-3}\frac{d^3k}{k_0}\frac{d^3k_2}{k_{20}}\frac{d^3p_h}{p_{h0}}$$
$$= -\frac{\alpha^3 SS_x^2 dxdydzdp_t d\phi_h d\phi d\tau d\phi_k dR}{64\pi^2 Mp_l\lambda_S\sqrt{\lambda_Y}\tilde{Q}^4}$$
$$\times\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}R^{j-2}, \quad (31)$$

where $\theta_{ij}=\theta^0_{ij}$ for $i=1-4,6,8$ and $\theta_{ij}=\theta^0_{ij}+\theta^1_{ij}$ for $i=5,7,9$, we found that the term with $j=1$ in (31) contains the infrared divergence at $R\to 0$ that does not allow to perform the straightforward integration of $d\sigma_R$ over the photonic variable R. For the correct extraction and cancellation of the infrared divergence the Bardin-Shumeiko approach [12] is used. Following to this method the identical transformation:

$$d\sigma_R = d\sigma_R - d\sigma_R^{IR} + d\sigma_R^{IR} = d\sigma_R^F + d\sigma_R^{IR}, \quad (32)$$

is performed. Here $d\sigma_R^F$ is the infrared free contribution and $d\sigma_R^{IR}$ contains only the $j=1$-term in which arguments of SF are taken for $k=0$,

$$d\sigma_R^{IR} = -\frac{\alpha^3}{4\pi^2 Q^4\sqrt{\lambda_S}}\sum_{i=1}^{9}\frac{\mathcal{H}_i\theta_{i1}}{R^2}\frac{d^3k}{k_0}\frac{d^3k_2}{k_{20}}\frac{d^3p_h}{p_{h0}}. \quad (33)$$

This decomposition allows us to perform the treatment of the infrared divergence analytically since the arguments of the SF in (33) do not depend on photonic variables. Due to $\theta_{i1}=4F_{IR}\theta_i^B$ one can find that this contributions can be factorized in front of the Born cross section

$$d\sigma_R^{IR} = -\frac{\alpha}{\pi^2}d\sigma^B\frac{F_{IR}}{R^2}\frac{d^3k}{k_0}, \quad (34)$$

where

$$F_{IR} = \left(\frac{k_1}{z_1}-\frac{k_2}{z_2}\right)^2, \quad (35)$$

$z_{1,2}=kk_{1,2}/kp$, and the explicit expressions of these quantities are given in Appendix B (see (B4)).

The term (34) is then separated into the soft δ_S and hard δ_H parts:

$$\sigma_R^{IR} = \frac{\alpha}{\pi}(\delta_S+\delta_H)\sigma_0 \quad (36)$$

by introduction of the infinitesimal photonic energy $\bar{k}_0\to 0$ that is defined in the system $\mathbf{p_1}+\mathbf{q}-\mathbf{p_h}=0$:

$$\delta_S = -\frac{1}{\pi}\int\frac{d^3k}{k_0}\frac{F_{IR}}{R^2}\theta(\bar{k}_0-k_0),$$
$$\delta_H = -\frac{1}{\pi}\int\frac{d^3k}{k_0}\frac{F_{IR}}{R^2}\theta(k_0-\bar{k}_0). \quad (37)$$

The explicit integration, detail of which are described in Appendix C, results in the final explicit expressions for these two contributions in the form:

$$\delta_S = 2(Q_m^2L_m-1)\left(P_{IR}+\log\frac{2\bar{k}_0}{\nu}\right)+\frac{1}{2}S'L_{S'}$$
$$+\frac{1}{2}X'L_{X'}+S_\phi,$$
$$\delta_H = 2(Q_m^2L_m-1)\log\frac{p_x^2-(M+m_\pi)^2}{2\bar{k}_0\sqrt{p_x^2}}. \quad (38)$$

The sum of δ_S and δ_H does not depend on the separated photonic energy $\bar{k}_0$ but includes the term representing the infrared divergence

$$P_{IR} = \frac{1}{n-4}+\frac{1}{2}\gamma_E+\log\frac{1}{2\sqrt{\pi}} \quad (39)$$

as well as the arbitrary parameter ν. These two quantities should be cancelled by summing the infrared divergent part with the contribution from the leptonic vertex correction that is considered below.

The term S_ϕ has a form:

$$S_\phi = -\frac{Q_m^2}{2\sqrt{\lambda_m}}\Bigg\{\log\frac{X'-\sqrt{\lambda'_X}}{X'+\sqrt{\lambda'_X}}\log\frac{(z-z_1)(z-z_3)}{(z-z_2)(z-z_4)}$$
$$+\sum_{i,j}^{4}S_j(-1)^{i+1}\Bigg(\frac{1}{2}\delta_{ij}\log^2(z-z_i)$$
$$+(1-\delta_{ij})\Bigg[\log(z-z_i)\log(z_i-z_j)$$
$$-\mathrm{Li}_2\left(\frac{z-z_i}{z_j-z_i}\right)\Bigg]\Bigg)\Bigg\}\Bigg|_{z=z_d}^{z=z_u}, \quad (40)$$

where

$$\mathrm{Li}_2(x) = -\int\limits_0^x\frac{\log|1-y|}{y}dy \quad (41)$$

is Spence's dilogarithm and

$$z_{1,2} = \frac{1}{\sqrt{\lambda'_X}}\left(X'-S'+\frac{2p_x^2(Q^2\mp\sqrt{\lambda_m})}{X'-\sqrt{\lambda'_X}}\right),$$
$$z_{3,4} = \frac{1}{\sqrt{\lambda'_X}}\left(S'-X'-\frac{2p_x^2(Q^2\pm\sqrt{\lambda_m})}{X'+\sqrt{\lambda'_X}}\right),$$
$$z_u = \sqrt{\frac{\lambda'_S}{\lambda'_X}}-1,\ z_d = \frac{X'(S'-X')-2p_x^2Q^2}{\lambda'_X},$$
$$S_j = \{1,1,-1,-1\}. \quad (42)$$

Note, the absolute value of the argument in the logarithmic function is used when computing the expression (40).

The infrared free contribution $d\sigma_R^F$ from (32) integrated over the three photonic variables reads:

$$\sigma_R^F = -\frac{\alpha^3 SS_x^2}{64\pi^2 Mp_l\lambda_S\sqrt{\lambda_Y}}\int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}d\phi_k\int\limits_0^{R_{max}}dR$$

$$\times \sum_{i=1}^{9}\left[\frac{\theta_{i1}}{R}\left(\frac{\tilde{\mathcal{H}}_i}{\tilde{Q}^4}-\frac{\mathcal{H}_i}{Q^4}\right)+\sum_{j=2}^{k_i}\tilde{\mathcal{H}}_i\theta_{ij}\frac{R^{j-2}}{\tilde{Q}^4}\right], \tag{43}$$

where the limits of integration are:

$$R_{max} = \frac{p_x^2-(M+m_\pi)^2}{1+\tau-\mu},$$
$$\tau_{max/min} = \frac{S_x \pm \sqrt{\lambda_Y}}{2M^2} \tag{44}$$

and the quantity μ is defined in Eq. (B3).

The additional virtual particle contributions consist of the leptonic vertex correction (Fig.1 (d)) and vacuum polarization by leptons and hadrons (Fig.1 (e)). These contributions are given by Eq. (8) with the replacement of the leptonic tensor $L_B^{\mu\nu}$ by

$$L_V^{\mu\nu} = \frac{1}{2}\mathrm{Tr}[(\hat{k}_2+m)\Gamma_V^\mu(\hat{k}_1+m)(1+\gamma_5\hat{\xi})\gamma^\nu] + \frac{1}{2}\mathrm{Tr}[(\hat{k}_2+m)\gamma^\mu(\hat{k}_1+m)(1+\gamma_5\hat{\xi})\bar{\Gamma}_V^\nu], \tag{45}$$

where

$$\Gamma_V^\mu = \Lambda^\mu + \Pi_\alpha^{l\mu}\gamma^\alpha + \frac{\alpha}{2\pi}\delta_{vac}^h\gamma^\mu, \tag{46}$$

and $\bar{\Gamma}_V^\nu = \gamma_0\Gamma_V^{\nu\,\dagger}\gamma_0$.

The first two terms corresponding to the leptonic vertex correction Λ_μ and vacuum polarization by leptons $\Pi_\alpha^{l\mu}$ are calculated analytically using Feynman rules while the fit for the vacuum polarization by hadrons δ_{vac}^h can be taken from the experimental data [19].

Since Λ_μ and $\Pi_\alpha^{l\mu}$ contain the ultraviolet divergence while Λ_μ also includes the infrared divergent term the dimensional regularization is used for the calculation of the loop integrals:

$$\Lambda_\mu = -ie^2\int\frac{d^nl}{(2\pi)^n\nu^{n-4}} \times\frac{\gamma_\alpha(\hat{k}_2-\hat{l}+m)\gamma_\mu(\hat{k}_1-\hat{l}+m)\gamma^\alpha}{l^2(l^2-2lk_2)(l^2-2lk_1)},$$
$$\Pi_{\alpha\mu}^l = -\frac{ie^2}{Q^2}\int\frac{d^nl}{(2\pi)^n\nu^{n-4}} \times\left\{\sum_{i=e,\mu,\tau}\frac{\mathrm{Tr}[(\hat{l}+m_i)\gamma_\alpha(\hat{l}-\hat{q}+m_i)\gamma_\mu]}{(l^2-m_i^2)((l-q)^2-m_i^2)}\right\}. \tag{47}$$

Details of the calculations are presented in Appendix D; Λ_μ and $\Pi_{\alpha\mu}^i$ have the following structure:

$$\Lambda_\mu = \frac{\alpha}{2\pi}\left(\delta_{vert}^{UV}(Q^2)\gamma_\mu - \frac{1}{2}mL_m[\hat{q},\gamma_\mu]\right),$$
$$\Pi_{\alpha\mu}^l = \sum_{i=e,\mu,\tau}\frac{\alpha}{2\pi}\delta_{vac}^{i\,UV}(Q^2)g_{\alpha\mu}^\perp, \tag{48}$$

where the second term in Λ_μ is the anomalous magnetic moment. To remove the ultraviolet divergence the standard on mass-shell renormalization procedure is used: $\delta_{vert}^{UV}(Q^2)$ and $\delta_{vac}^{i\,UV}(Q^2)$ are substituted by the difference of these quantities and their values at $Q^2=0$:

$$\delta_{vert} = \delta_{vert}^{UV}(Q^2)-\delta_{vert}^{UV}(0),$$
$$\delta_{vac}^i = \delta_{vac}^{i\,UV}(Q^2)-\delta_{vac}^{i\,UV}(0). \tag{49}$$

Here $\delta_{vert}^{UV}(0) = 2-P_{UV}-2P_{IR}-3\log(m/\nu)$, $\delta_{vac}^{i\,UV}(0) = 4(P_{UV}+\log(m_i/\nu))/3$ and the ultraviolet free terms have a form:

$$\delta_{vert} = -2(Q_m^2L_m-1)\left(P_{IR}+\log\frac{m}{\nu}\right)-2 +\left(\frac{3}{2}Q^2+4m^2\right)L_m-\frac{Q_m^2}{\sqrt{\lambda_m}}\left(\frac{1}{2}\lambda_mL_m^2 +2\mathrm{Li}_2\left(\frac{2\sqrt{\lambda_m}}{Q^2+\sqrt{\lambda_m}}\right)-\frac{\pi^2}{2}\right),$$
$$\delta_{vac}^l = \sum_{i=e,\mu,\tau}\delta_{vac}^i = \sum_{i=e,\mu,\tau}\left[\frac{2}{3}\frac{Q^2+2m_i^2}{\sqrt{\lambda_m^i}}L_m^i -\frac{10}{9}+\frac{8m_i^2}{3Q^2}\left(1-\frac{2m_i^2}{\sqrt{\lambda_m^i}}L_m^i\right)\right]. \tag{50}$$

The quantity L_m is defined in (C10) while the expressions for λ_m^i and L_m^i is defined by Eqs. (D3).

Finally the contribution of the inelastic tail to the six-fold SIDIS cross section reads:

$$\sigma^{in} = \frac{\alpha}{\pi}(\delta_{VR}+\delta_{vac}^l+\delta_{vac}^h)\sigma^B+\sigma_R^F+\sigma^{AMM}, \tag{51}$$

where the sum of the infrared divergent terms

$$\delta_{VR} = \delta_S+\delta_H+\delta_{vert} = 2(Q_m^2L_m-1)\log\frac{p_x^2-(M+m_\pi)^2}{m\sqrt{p_x^2}}+\frac{1}{2}S'L_{S'} +\frac{1}{2}X'L_{X'}+S_\phi-2+\left(\frac{3}{2}Q^2+4m^2\right)L_m -\frac{Q_m^2}{\sqrt{\lambda_m}}\left(\frac{1}{2}\lambda_mL_m^2+2\mathrm{Li}_2\left(\frac{2\sqrt{\lambda_m}}{Q^2+\sqrt{\lambda_m}}\right) -\frac{\pi^2}{2}\right) \tag{52}$$

is free both from the infrared divergent term P_{IR} appearing in δ_S and δ_{vert} that are defined by Eqs. (38) and (50), and the arbitrary parameter ν.

At last the contribution of the anomalous magnetic moment coming from the second term in Λ_μ given by Eqs. (48) has a form:

$$\sigma^{AMM} = \frac{\alpha^3m^2SS_x^2}{16\pi MQ^2p_l\lambda_S}L_m\sum_{i=1}^{9}\theta_i^{AMM}\mathcal{H}_i, \tag{53}$$

with:

$$\theta_1^{AMM} = 6,$$
$$\theta_2^{AMM} = -\frac{\lambda_Y}{2Q^2},$$
$$\theta_3^{AMM} = -2m_h^2-2\frac{V_-^2}{Q^2},$$

$$\theta_4^{AMM} = -2S_x\left(z + \frac{V_-}{Q^2}\right),$$
$$\theta_5^{AMM} = \frac{2\lambda_e(2S+S_x)\varepsilon_\perp p_h}{\sqrt{\lambda_S}Q^2},$$
$$\theta_7^{AMM} = \frac{\lambda_e(2S+S_x)}{4\sqrt{\lambda_S}Q^2}(S_x(SV_2 - XV_1 - zS_pQ^2) +4M^2Q^2V_+),$$
$$\theta_9^{AMM} = \frac{\lambda_e}{2\sqrt{\lambda_S}Q^2}(S_x^2(4m^2(m_h^2 - z(zQ^2+2V_-)) +V_1V_-) - 4(M^2(Q^2-4m^2)+S^2)(m_h^2Q^2 +V_-^2) + zQ^2S_x(S_x(zQ^2+V_1+V_-) +2SV_+) + 2SS_xV_-V_+),$$
$$\theta_6^{AMM} = \theta_8^{AMM} = 0. \qquad (54)$$

B. Exclusive Radiative Tail

The exclusive radiative tail is the process

$$e(k_1,\xi) + n(p,\eta) \to e(k_2) + h(p_h) + u(p_u) + \gamma(k), \qquad (55)$$

where p_u is the four-momentum of undetected hadron ($p_u^2 = m_u^2$) shown in Fig. 1 (f,g). The final unobserved state contains the photon radiated from the lepton line and a hadron produced in an exclusive reaction of γ^* and p. The process (55) gives a contribution to the RC in SIDIS because two observed particles in the final state can have the same momenta as the unobserved particles in SIDIS process (1). The square of the invariant mass of the unobserved state $p_x^2 = (p+q-p_h)^2 = 2k(p+q-p_h)+m_u^2$ depends on the photonic variables. Emission of the soft photons would result in $p_x^2 = m_u^2$. This is beyond the kinematic region of SIDIS. Therefore the process (55) being the contribution to RC to the SIDIS cross section does not contain the infrared divergence [10].

Description of the exclusive process without the radiated photon requires the only five of the six presented in Eqs. (2) variables of SIDIS: x, y, t, ϕ_h and ϕ. The process with the radiated photon is additionally described by the three photonic variables R, τ and ϕ_k introduced above by Eq. (20). In this case the sixth SIDIS variable z is expressed through other SIDIS and photonic variables:

$$z = \frac{M^2 - m_u^2 + t - R(1+\tau-\mu)}{S_x} + 1, \qquad (56)$$

where μ is defined by Eq. (B3). Since we calculate RC to SIDIS we need to keep z and use this equation in order to express R in terms of z and two remaining photonic variables:

$$R_{ex} = \frac{p_x^2 - m_u^2}{1+\tau-\mu}, \qquad (57)$$

and therefore to reduce the integration over the photon momentum to the two-dimensional integral in respect of variables τ and ϕ_k.

The contribution of the exclusive radiative tail in the form similar to (22) reads:

$$d\sigma_R^{ex} = \frac{(4\pi\alpha)^3}{2\sqrt{\lambda_S}\tilde{Q}^4}\widetilde{W}_{ex}^{\mu\nu}L_{\mu\nu}^R d\Gamma_R^{ex}, \qquad (58)$$

where the hadronic tensor $W_{ex}^{\mu\nu}$ describes the exclusive process $\gamma^* + n \to h + u$ and has the same structure as the hadronic tensor in Eq. (13) but with the SF dependent only on Q^2, W^2 and t variables. The leptonic tensor $L_{\mu\nu}^R$ as well as its convolution with the hadronic structures $\tilde{w}_i^{\mu\nu}$ are the same as in Eqs. (26-29).

The phase space of this process is:

$$d\Gamma_R^{ex} = \frac{1}{(2\pi)^8}\frac{d^3k_2}{2k_{20}}\frac{d^3k}{2k_0}\frac{d^3p_h}{2p_{h0}}\frac{d^3p_u}{2p_{u0}} \times\delta^4(k_1+p-k_2-p_h-p_u-k) = \frac{2R_{ex}SS_x^2dxdyd\phi dzd\phi_h dp_t^2 d\tau d\phi_k}{(4\pi)^8(1+\tau-\mu)Mp_l\sqrt{\lambda_S\lambda_Y}}. \qquad (59)$$

The use of the phase space (59) and convolution of leptonic and hadronic tensors (29) with replacement $\tilde{\mathcal{H}}_i \to \tilde{\mathcal{H}}_i^{ex}$ in (58) and subsequent integration of the obtained expression over two photonic variables results in the contribution of the exclusive radiative tail to SIDIS process in the form:

$$\sigma_R^{ex} = -\frac{\alpha^3SS_x^2}{2^9\pi^5Mp_l\lambda_S\sqrt{\lambda_Y}}\int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}d\phi_k \times\sum_{i=1}^{9}\sum_{j=1}^{k_i}\frac{\tilde{\mathcal{H}}_i^{ex}\theta_{ij}R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}. \qquad (60)$$

IV. ULTRARELATIVISTIC APPROXIMATION

In Section III all contributions to the lowest-order RC are presented by exact formulas. Some of them have a rather complicate analytical structure. However, due to smallness of the leptonic mass compared to other quantities that describe kinematics of the process it is rather useful to obtain RC in the ultrarelativistic approximation keeping the leptonic mass m only as an argument of the logarithmic function. This allows us to simplify the analytical expressions essentially as well as clarify the leading log behavior of the obtained results. In the other words, the lowest-order QED RC in this approximation has the form

$$\sigma_{RC} = \frac{\alpha}{\pi}\left[Al_m + B + \mathcal{O}\left(\frac{m^2}{Q^2}\right)\right], \qquad (61)$$

where $l_m = \log Q^2/m^2$, and the terms A and B are independent of the leptonic mass and represent the lowest order leading and next-to-leading contributions to the RC to the cross section, respectively.

The terms in (52) that are factorized in front of the Born contribution, are essentially simplified resulting in more transparent structure after applying the ultrarelativistic approximation, e.g., the terms (36)

$$\sigma_R^{IR} = \frac{\alpha}{\pi}\biggl[(l_m-1)\biggl(2P_{IR}+2\log\frac{m}{\nu} + \log\frac{(p_x^2-(M+m_\pi)^2)^2}{S'X'}\biggr)+\frac{1}{2}l_m^2 - \frac{1}{2}\log^2\frac{S'}{X'}+\mathrm{Li}_2\biggl\{1-\frac{Q^2p_x^2}{S'X'}\biggr\}-\frac{\pi^2}{3}\biggr]\sigma_0 \quad (62)$$

contains both l_m and l_m^2. The latter comes from the soft photon emission whose contribution cancels in the sum with the leptonic vertex correction:

$$\delta_{VR} = (l_m-1)\log\frac{(p_x^2-(M+m_\pi)^2)^2}{S'X'}+\frac{3}{2}l_m - \frac{1}{2}\log^2\frac{S'}{X'}+\mathrm{Li}_2\biggl\{1-\frac{Q^2p_x^2}{S'X'}\biggr\}-\frac{\pi^2}{6}-2. \quad (63)$$

The ultrarelativistic approximation for the hard photon emission contribution (43,60) requires additional care because of integration over photonic variables and nontrivial dependence of the integrand on the leptonic mass. Specifically, the integrand contains the terms $1/z_1$ and $1/z_1^2$:

$$\int\limits_0^{2\pi}\frac{d\phi_k}{z_1} = \frac{2\pi\sqrt{\lambda_Y}}{\sqrt{(Q^2+\tau S)^2+4m^2(\tau(S_x-\tau M^2)+Q^2)}},$$
$$\int\limits_0^{2\pi}\frac{d\phi_k}{z_1^2} = \frac{2\pi(Q^2S_p+\tau(SS_x+2M^2Q^2))\sqrt{\lambda_Y}}{((Q^2+\tau S)^2+4m^2(\tau(S_x-\tau M^2)+Q^2))^{3/2}} \quad (64)$$

These have a sharp peaking behavior in the region $\tau\to\tau_s\equiv -Q^2/S$ due to smallness of the lepton mass. The integration of the expressions (64) over ϕ_k and τ gives:

$$\int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}\frac{d\phi_k}{z_1} = 2\pi\sqrt{\frac{\lambda_Y}{\lambda_S}}\log\frac{S+\sqrt{\lambda_S}}{S-\sqrt{\lambda_S}},$$
$$\int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}\frac{d\phi_k}{z_1^2} = \frac{2\pi\sqrt{\lambda_Y}}{m^2}. \quad (65)$$

Since

$$\lim_{m\to 0}\log\frac{S+\sqrt{\lambda_S}}{S-\sqrt{\lambda_S}} = l_m+\log\frac{S^2}{Q^2M^2} \quad (66)$$

the terms containing $1/z_1$ contribute to the leading and next-to-leading RC. The terms containing $1/z_1^2$ also contain m^2 in numerators and therefore contribute to the next-to-leading RC only (the only exception is $\hat\theta^0_{53}$ that is discussed below). The similar conclusions are true for the terms containing $1/z_2$ and $1/z_2^2$ terms.

Actually the integrand in (65) contains SF. Therefore we make the identical transformation for extraction of the leading and next-to-leading terms:

$$\int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}d\phi_k\frac{\mathcal{G}(\tau,\phi_k)}{z_1} = 2\pi\sqrt{\frac{\lambda_Y}{\lambda_S}}\log\frac{\sqrt{\lambda_S}+S}{\sqrt{\lambda_S}-S}\mathcal{G}(\tau_s,0) + \int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}d\phi_k\frac{\mathcal{G}(\tau,\phi_k)-\mathcal{G}(\tau_s,0)}{z_1},$$
$$m^2\int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}d\phi_k\frac{\mathcal{G}(\tau,\phi_k)}{z_1^2} = 2\pi\sqrt{\lambda_Y}\mathcal{G}(\tau_s,0) + \int\limits_{\tau_{min}}^{\tau_{max}}d\tau\int\limits_0^{2\pi}d\phi_k m^2\frac{\mathcal{G}(\tau,\phi_k)-\mathcal{G}(\tau_s,0)}{z_1^2}, \quad (67)$$

where $\mathcal{G}(\tau,\phi_k)$ is a regular function of τ and ϕ_k. The second term in the r. h. s. of the first transformation does not include the leading terms and the second term in the second equality is proportional to m^2 and vanishes in the ultrarelativistic approximation.

The approach of extraction of the leading and next-to-leading contributions can be illustrated by consideration of the terms originated from the convolution of the leptonic tensor (28) with the hadronic structures $\widetilde{w}^i_{\mu\nu}$. Summing up the terms $\theta^1_{ij}R^{j-3}$ in the last expression of Eq. (29) and keeping the leptonic mass only in the term m^2/z_1^2 (in θ^1_{ij} the term $1/z_2^2$ is proportional to m^4) results in

$$\widetilde{w}^i_{\mu\nu}L^{\mu\nu}_{R1} = -2\sum_{j=1}^{k_i}\theta^1_{ij}R^{j-3} = \frac{m^2}{z_1^2}\theta^1_i(R,\tau,\phi_k) \quad (68)$$

with the quantities $\theta^1_i(R,\tau_s,0)$ expressed through (16) as:

$$\theta^1_i(R,\tau_s,0) = \frac{4R}{S(S-R)}\theta^B_i\biggl(k_1\to\biggl(1-\frac{R}{S}\biggr)k_1\biggr). \quad (69)$$

The replacement in the brackets is applied for any kinematic variable defined through k_1, e.g., $S\to S-R$, $Q^2\to(1-S/R)Q^2$, and $\varepsilon_\perp p_h\to(1-S/R)\varepsilon_\perp p_h$. Note that $R=R_{ex}$ has to be used for the exclusive radiative tail.

The resulting equation for the $\sigma_R^{\xi_1}$ is obtained using the second equation of (67) with the regular function $\mathcal{G}(\tau,\phi_k)$,

$$\mathcal{G}(\tau,\phi_k) = \int\limits_0^{R_{max}}\frac{RdR}{(Q^2+\tau R)^2}\sum_{i=5,7,9}\theta^1_i(R,\tau,\phi_k)\tilde{\mathcal{H}}_i. \quad (70)$$

Therefore the contribution from the second part ξ_1 of the lepton polarized vector (10) reads:

$$\sigma_R^{\xi_1} = -\frac{\alpha S_x^2}{\pi M p_l S^2}\int\limits_0^{R^s_{max}}\frac{p^s_l RdR}{(S_x-R)^2}\tilde\sigma^B_{pl}, \quad (71)$$

where

$$p_l^s = \frac{zSS_x(S_x - R) + 2M^2(RV_1 - 2SV_-)}{2M\sqrt{S(4M^2Q^2(S-R) + S(S_x - R)^2)}},$$
$$R^s_{max} = S(p_x^2 - (M+m_\pi)^2)/S', \quad (72)$$

and $\tilde{\sigma}^B_{pl}$ is a proportional to λ_e part of the Born contribution with the following replacement: $m \to 0$, $S \to S - R$, $Q^2 \to Q^2(1 - R/S)$, $V_1 \to V_1(1 - R/S)$ and $z \to zS_x/(S_x - R)$.

Similar calculation of the exclusive radiative tail results in:

$$\sigma_R^{ex\ \xi_1} = -\frac{\alpha S_x^2 R^s_{ex} p_l^{s\ ex}}{\pi M p_l S S'(S_x - R^s_{ex})} \frac{d\tilde{\sigma}^{ex\ B}_{pl}}{d\tilde{x} d\tilde{y} d\tilde{p}_t d\phi_h d\phi}, \quad (73)$$

where

$$p_l^{s\ ex} = \frac{1}{2M\sqrt{S(4M^2Q^2(S - R^s_{ex}) + S(S_x - R^s_{ex})^2)}}$$
$$[(S_x - R^s_{ex})(S(S_x - 2V_- + m_h^2 - m_u^2)$$
$$-R^s_{ex}(S - V_1)) - Q^2(S - R^s_{ex})(S_x - R^s_{ex})$$
$$+M^2(S(S_x - 4V_-) - R^s_{ex}(S - 2V_1))], \quad (74)$$

$R^s_{ex} = S(p_x^2 - m_u^2)/S'$ and the exclusive Born cross section reads:

$$\frac{d\sigma^{ex\ B}_{pl}}{dxdydp_t d\phi_h d\phi} = \frac{\alpha^2 S S_x}{64\pi^3 Q^4 M p_l \lambda_S}$$
$$\times \sum_{i=5,7,9} \mathcal{H}_i^{ex} \theta_i^B (z \to \frac{t + M^2 - m_u^2}{S_x} + 1). \quad (75)$$

Finally, we consider the extraction of the leading and next-to-leading terms in the quantity $\hat{\theta}^0_{53}$ given in Appendix B. In contrast to other $\hat{\theta}^0_{ij}$, the quantity $\hat{\theta}^0_{53}$ includes terms $1/z_1^2$ without factors proportional to m^2 and therefore can potentially result in electron mass singularity $\sim m^{-2}$ after integration (65). This is, however, is not the case because $\hat{\theta}^0_{53} = 0$ at the peak point, i. e., for $\tau = \tau_s = -Q^2/S$ (and $\mu = V_1/S$). Explicit integration in the limit $m^2 \to 0$,

$$\int\limits_{\tau_{min}}^{\tau_{max}} d\tau \int\limits_0^{2\pi} d\phi_k \hat{\theta}^0_{53} = -\frac{2\lambda_e \pi p_t \sin\phi_h \sqrt{\lambda_Y}}{M^2 S^2 \sqrt{Q^2(SX - M^2Q^2)}} \left[4M^2Q^2(SX - M^2Q^2)\left(l_m + \log\frac{S^2}{Q^2M^2} - 3\right) + S^2\lambda_Y\right] \quad (76)$$

shows that $\hat{\theta}^0_{53}$ has a standard form $A\log(Q^2/m^2) + B$.

V. CONCLUSION

Newly achieved accuracies in modern SIDIS experiments in TJNAF and CERN require renewed attention to RC calculations and their implementation in data analysis software. In this paper we obtained the exact analytical expressions for the lowest-order model-independent part of QED RC to the SIDIS cross section with the longitudinally polarized initial lepton and arbitrary polarized target and demonstrated how the leading and next-to-leading contributions can be extracted. The model-independent RC includes i) the contributions of radiated SIDIS processes and loop diagrams (51) and ii) the contribution of the exclusive radiative tail (60). The methodology developed in this paper is the extension of the covariant approach for the RC calculations developed earlier: i) the method of covariant extraction and cancellation of the infrared divergence suggested by Bardin and Shumeiko [12]; ii) the set of integration variables used in RC calculation to DIS [15], iii) RC to unpolarized and polarized SIDIS in the quark-parton model [6–8], iv) RC for SIDIS of unpolarized particles [9], and v) the calculation of the exclusive radiative tail for unpolarized SIDIS [10]. The calculations of RC in SIDIS measurements were performed by the model independent way that involves constructing and using the SIDIS (and exclusive) hadronic tensor containing the eighteen SIDIS and exclusive SF. We obtained the explicit form of the hadronic tensor using approaches of [16] and [17] and demonstrated that the Born cross section exactly coincides with that given by [18]. The next step in the RC calculation includes coding of the formulae and numeric evaluation of the effects of the RC. However, this requires models of the SIDIS/exclusive SF that are not known now. Therefore, a broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SF as well as SF in resonance region and exclusive SF. Further development will include development of i) iteration procedure with fitting of measured SF and joining with models beyond SIDIS measurements at each iteration step, and ii) tools for generation of the radiated photon. Such generator can be constructed based on a code for RC in SIDIS in the same way of how RADGEN [20] is constructed based on POLRAD 2.0. Generation of semi-inclusive processes based on DIS Monte Carlo generators can provide only approximate cross sections, because a part of the SIDIS cross section involving pure semi-inclusive SF and respective convolutions of the leptonic and hadronic tensors are not presented in such DIS Monte Carlo generators.

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Appendix A: Bases in the four-dimensional space

In this Appendix three bases in the four-dimensional space that are used in our analyses, are presented. The first two are used for the decomposition of the initial target and virtual photon polarization in the hadronic tensor defined by (12). The latter allows us to decompose the real photon momentum in such a way that all five pseudoscalar quantities appearing in processes (19) and (55) reduce down to two: $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$.

For the decomposition of the hadronic tensor over the SF it is convenient to introduce the reference system $(\mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ in the target rest frame where the two polar axises are defined as: $\mathbf{z}_h$ is chosen in the virtual photon three-momentum direction $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ and the $\mathbf{x}_h$ along the part of the registrated hadronic momentum that is transverse to the $\mathbf{z}_h$-axis. The direction of the rest axial $\mathbf{y}_h$-axis is defined as $\mathbf{y}_h = \mathbf{z}_h \times \mathbf{x}_h$. In this system the complete basis for polarization vectors can be presented into covariant form [17] both for the virtual photon

$$
\begin{aligned}
e_\mu^{\gamma(0)} &= \frac{2Q}{\sqrt{\lambda_Y}} p_\mu^\perp, \\
e_\mu^{\gamma(1)} &= \frac{1}{p_t}\left[p_{h\mu}^\perp - \frac{S_x(m_h^2 + (2z-1)Q^2 - t)}{\lambda_Y} p_\mu^\perp\right], \\
e_\mu^{\gamma(2)} &= 2\frac{\varepsilon^{\mu\nu\rho\sigma} p_\nu q_\rho p_{h\sigma}}{p_t\sqrt{\lambda_Y}}, \\
e_\mu^{\gamma(3)} &= \frac{q_\mu}{Q},
\end{aligned} \tag{A1}
$$

and nucleon:

$$
\begin{aligned}
e_\mu^{h(0)} &= \frac{p_\mu}{M}, \\
e_\mu^{h(1)} &= \frac{1}{p_t}\left[p_{h\mu}^\perp - \frac{S_x(m_h^2 + (2z-1)Q^2 - t)}{\lambda_Y} p_\mu^\perp\right], \\
e_\mu^{h(2)} &= 2\frac{\varepsilon^{\mu\nu\rho\sigma} p_\nu q_\rho p_{h\sigma}}{p_t\sqrt{\lambda_Y}}, \\
e_\mu^{h(3)} &= \frac{2M^2 q_\mu - S_x p_\mu}{M\sqrt{\lambda_Y}},
\end{aligned} \tag{A2}
$$

where $Q = \sqrt{Q^2}$ and for any four-vector $a_\mu^\perp = a_\mu + aq\ q_\mu/Q^2$. Note, that direction of $e^{h(2)}$ (and $e^{\gamma(2)}$ as well) is chosen in such a way that the projection of $\mathbf{k}_{1,2}$ on $\mathbf{y}_h$ reads $\mathbf{y}_h \cdot \mathbf{k}_1 = \mathbf{y}_h \cdot \mathbf{k}_2 = -e^{h(2)}k_1 = -e^{h(2)}k_2 = -k_t \sin(\phi_h)$.

The components of these two bases in the reference system $(\mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ read:

$$
\begin{aligned}
e_\mu^{\gamma(0)} &= \frac{1}{2MQ}(\sqrt{\lambda_Y}, 0, 0, S_x), & e_\mu^{h(0)} &= (1,0,0,0), \\
e_\mu^{\gamma(1)} &= (0,1,0,0), & e_\mu^{h(1)} &= (0,1,0,0), \\
e_\mu^{\gamma(2)} &= (0,0,1,0), & e_\mu^{h(2)} &= (0,0,1,0), \\
e_\mu^{\gamma(3)} &= \frac{1}{2MQ}(S_x, 0, 0, \sqrt{\lambda_Y}), & e_\mu^{h(3)} &= (0,0,0,1).
\end{aligned} \tag{A3}
$$

In the rest frame system the virtual photon longitudinal and transverse polarizations correspond to $e^{\gamma(0)}$ and $e^{\gamma(1,2)}$, respectively, and the left and right circular polarizations are defined as

$$
e^{\gamma(\pm)} = \mp\frac{1}{\sqrt{2}}(e^{\gamma(1)} \pm ie^{\gamma(2)}). \tag{A4}
$$

To decompose the photonic four-momentum the other the reference system $(\mathbf{x}_l, \mathbf{y}_l, \mathbf{z}_l)$ in the rest target frame has to be introduced. In this system the polar $\mathbf{z}_l$-axis has the same direction as three-vector $\mathbf{q}$, other polar $\mathbf{x}_l$-axis is chosen along the incoming or outgoing lepton part that is transverse to $\mathbf{q}$, at last the axial $\mathbf{y}_l$-axis is defined as $\mathbf{y}_l = \mathbf{z}_l \times \mathbf{x}_l$. As a result $(\mathbf{x}_l, \mathbf{y}_l)$ is the scattering plane. In the covariant form this basis reads as:

$$
\begin{aligned}
e_\mu^{l(0)} &= \frac{p_\mu}{M}, \\
e_\mu^{l(1)} &= \sqrt{\frac{\lambda_Y}{\lambda_1}}\left[\frac{1}{2}(k_{1\mu} + k_{2\mu}) - \frac{S_p Q^2}{\lambda_Y} p_\mu^\perp\right], \\
e_\mu^{l(2)} &= -\frac{2\varepsilon_{\perp\mu}}{\sqrt{\lambda_1}}, \\
e_\mu^{l(3)} &= \frac{2M^2 q_\mu - S_x p_\mu}{M\sqrt{\lambda_Y}}.
\end{aligned} \tag{A5}
$$

Note, that direction of $\mathbf{y}_l$ is chosen in such a way that the projection of $\mathbf{p}_h$ on $\mathbf{y}_l$ is $\mathbf{y}_l \cdot \mathbf{p}_h = -e^{l(2)}p_h = p_t \sin(\phi_h)$. The two reference systems $(\mathbf{x}_h, \mathbf{y}_h, \mathbf{z}_h)$ and $(\mathbf{x}_l, \mathbf{y}_l, \mathbf{z}_l)$ can be expressed through each other in a following way:

$$
\begin{aligned}
\mathbf{x}_h &= \mathbf{x}_l \cos(\phi_h) + \mathbf{y}_l \sin(\phi_h), \\
\mathbf{y}_h &= -\mathbf{x}_l \sin(\phi_h) + \mathbf{y}_l \cos(\phi_h), \\
\mathbf{z}_h &= \mathbf{z}_l
\end{aligned} \tag{A6}
$$

where $\cos(\phi_h)$ and $\sin(\phi_h)$ are defined by Eqs. (5) and (6) respectively.

It should be also noted that for $i = \gamma, h, l$

$$
\begin{aligned}
e_\mu^{i(a)} e_\nu^{i(b)} g^{\mu\nu} &= g^{ab}, \\
e_\mu^{i(a)} e_\nu^{i(b)} g_{ab} &= g_{\mu\nu}.
\end{aligned} \tag{A7}
$$

The photonic four-momentum can be decomposed into the following way $k = k_{(a)} e^{(a)}$ where

$$
k_{(0)} = k e^{l(0)} = \frac{R}{2M},
$$

$$k_{(1)} = -ke^{l(1)} = \frac{R(Q^2 S_p + \tau(SS_x + 2M^2 Q^2) - z_1 \lambda_Y)}{2\sqrt{\lambda_1 \lambda_Y}},$$
$$k_{(2)} = -ke^{l(2)} = \frac{2\varepsilon_\perp k}{\sqrt{\lambda_1}},$$
$$k_{(3)} = -ke^{l(3)} = \frac{R(S_x - 2\tau M)}{2M\sqrt{\lambda_1}}. \quad \text{(A8)}$$

This decomposition for the four-momentum of the real unobservable photon allows us to express all pseudoscalars through the linear combinations two of them $\varepsilon_\perp p_h$ and $\varepsilon_\perp k$:

$$\begin{aligned}
\varepsilon^{\mu\nu\rho\sigma} k_\mu p_{h\ \nu} k_{1\rho} q_\sigma = \frac{1}{2\lambda_1}(&R\varepsilon_\perp p_h(\tau(Q^2 S + 2m^2 S_x)\\
&+Q^2(4m^2 + Q^2 - z_1 S_p))\\
&+\varepsilon_\perp k(Q^2(SV_2 + XV_1 - zQ^2 S_x)\\
&-4m^2 S_x(zQ^2 + V_-))),
\end{aligned}$$

$$\begin{aligned}
\varepsilon^{\mu\nu\rho\sigma} k_\mu p_\nu p_{h\ \rho} q_\sigma = \frac{1}{2\lambda_1}(&R\varepsilon_\perp p_h(z_1\lambda_Y - Q^2 S_p\\
&-\tau(SS_x + 2M^2 Q^2))\\
&+\varepsilon_\perp k(S_x(zQ^2 S_p - SV_2 + XV_1)\\
&-4V_+ M^2 Q^2)),
\end{aligned}$$

$$\begin{aligned}
\varepsilon^{\mu\nu\rho\sigma} k_\mu p_\nu k_{1\rho} p_{h\ \sigma} = \frac{1}{2\lambda_1}(&R\varepsilon_\perp p_h(\tau\lambda_S + 2m^2 S_x\\
&+Q^2 S - z_1(SS_x + 2M^2 Q^2))\\
&+\varepsilon_\perp k(2m^2(4V_- M^2 - zS_x^2)\\
&+S(SV_2 - XV_1 - zQ^2 S_x)\\
&+2V_1 M^2 Q^2)). \quad \text{(A9)}
\end{aligned}$$

Appendix B: Explicit expression for θ_{ij}

For all $i = 1-8$, the quantities $\theta^0_{i1} = 4F_{IR}\theta^B_i$ and F_{IR} are defined by (B5). The other θ^0_{ij} read:

$$\begin{aligned}
\theta^0_{12} &= 4\tau F_{IR},\\
\theta^0_{13} &= -4 - 2F_d\tau^2,\\
2\theta^0_{22} &= S_x S_p F_{1+} + 2m^2 S_p F_{2-} + 2(S_x - 2\tau M^2)F_{IR} - \tau S_p^2 F_d,\\
2\theta^0_{23} &= (4m^2 + \tau(2\tau M^2 - S_x))F_d - S_p F_{1+} + 4M^2,\\
\theta^0_{32} &= 2((\mu V_- - \tau m_h^2)F_{IR} + V_+(\mu m^2 F_{2-} + V_- F_{1+} - \tau V_+ F_d)),\\
\theta^0_{33} &= (2\mu^2 m^2 + \tau(\tau m_h^2 - \mu V_-))F_d - \mu V_+ F_{1+} + 2m_h^2,\\
\theta^0_{42} &= (SV_1 - XV_2)F_{1+} + m^2(\mu S_p + 2V_+)F_{2-} - 2\tau S_p V_+ F_d + ((\mu - 2\tau z)S_x + 2V_-)F_{IR},\\
2\theta^0_{43} &= (8\mu m^2 + \tau((2\tau z - \mu)S_x - 2V_-))F_d - (\mu S_p + 2V_+)F_{1+} + 4zS_x,\\
\theta^0_{52} &= \frac{\lambda_e S}{\lambda_1\sqrt{\lambda_S}}[\varepsilon_\perp p_h(2(S_x(Q^2 + 4m^2) + 2\tau(SX - 2M^2(Q^2 + 2m^2)))F_{IR} + Q^2(S_p(S_x F_{1+} + 2m^2 F_{2-})\\
&\quad -\tau(4SX + S_x^2)F_d)) + 2\frac{\varepsilon_\perp k}{R}(m^2(S_x(SV_2 - XV_1 - zQ^2 S_p) + 4M^2 Q^2 V_+)F_{2-}\\
&\quad +((Q^2 + 4m^2)(4M^2 V_- - zS_x^2) + S_p(SV_2 - XV_1))F_{IR})],\\
\theta^0_{53} &= \hat\theta^0_{53} + \frac{\lambda_e S}{\lambda_1\sqrt{\lambda_S}}[\varepsilon_\perp p_h(8m^2(\tau(\tau M^2 - S_x) - Q^2)F_{21} + (Q^2(4\tau M^2 + S_p) + 2\tau SS_x)F_{1+} + \tau(4m^2(2\tau M^2 - S_x)\\
&\quad +Q^2(S_x - 4S) - 2\tau S^2)F_d) + 2\frac{\varepsilon_\perp k}{R}(2m^2(S_x(2zQ^2 + 2V_- + (\tau z - \mu)S_x) - 4M^2(\mu Q^2 + \tau V_-))F_{21}\\
&\quad +\tau(2m^2(zS_x^2 - 4M^2 V_-) - 2M^2 Q^2 V_1 + S(zS_x Q^2 - SV_2 + XV_1))F_d)],\\
\hat\theta^0_{53} &= \frac{2\lambda_e S}{\lambda_1\sqrt{\lambda_S}}F_{21}[\frac{\varepsilon_\perp k}{R}(2(\mu Q^2 + \tau V_1)(SX - M^2 Q^2) + (Q^2 + \tau S)(zQ^2 S_x - SV_2 - XV_1)) - \varepsilon_\perp p_h(Q^2 + \tau S)^2],\\
\theta^0_{62} &= \frac{1}{2\lambda_1}[\varepsilon_\perp p_h((4M^2 Q^2(Q^2 + 4m^2) - S_x^2(Q^2 - 4m^2) - 8Q^2 SX)(S_x F_{1+} + 2m^2 F_{2-} - \tau S_p F_d)\\
&\quad +2S_p(2\tau(2M^2(Q^2 + 2m^2) - SX) - S_x(Q^2 + 4m^2))F_{IR}) + 2S_p\frac{\varepsilon_\perp k}{R}(m^2(S_x(zS_p Q^2 - SV_2 + V_1 X)\\
&\quad -4M^2 Q^2 V_+)F_{2-} + ((Q^2 + 4m^2)(zS_x^2 - 4M^2 V_-) + S_p(XV_1 - SV_2))F_{IR})],
\end{aligned}$$

$$\theta^0_{63}=\frac{1}{2\lambda_1}[2\varepsilon_\perp p_h((2Q^2(SX-2M^2Q^2)-\tau S_x(S_x^2+3SX-4m^2M^2)-(Q^2+2m^2)S_x^2)F_{1+}$$
$$+m^2(2\tau(2M^2(Q^2+2m^2)-SX)-S_x(Q^2+4m^2))F_{2-}-Q^2S_pF_{IR}+S_p(\tau^2(S_x^2+2SX$$
$$-2M^2(Q^2+4m^2))+2\tau S_x(Q^2+2m^2)-4m^2Q^2)F_d+S_pS_x^2)+\frac{\varepsilon_\perp k}{R}((((Q^2+4m^2)(zS_x^2-4M^2V_-)$$
$$+S_p(XV_1-SV_2))(S_xF_{1+}+2m^2F_{2-})+2m^2(S_x(zQ^2S_p-SV_2+XV_1)-4M^2Q^2V_+)F_{2+}$$
$$+(4\tau(M^2Q^2(4SV_-+S_x(V_2-V_-))+2SX(SV_2-XV_1)+2m^2S_p(4M^2V_--zSx^2))$$
$$+(3\tau S_x+2(Q^2-2m^2))(SV_2-XV_1-zQ^2S_p)S_x+8(Q^2-2m^2)M^2Q^2V_+)F_d)],$$

$$\theta^0_{64}=\frac{1}{2\lambda_1}[\varepsilon_\perp p_h(((Q^2+4m^2)S_x+2\tau(SX-2M^2(Q^2+2m^2)))F_{1+}+S_p(\tau Q^2F_d-2S_x))$$
$$+\frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(4M^2V_--zS_x^2)+S_p(SV_2-XV_1))F_{1+}+\tau(4M^2Q^2V_++S_x(SV_2-XV_1-zQ^2S_p))F_d)],$$

$$\theta^0_{72}=\frac{\lambda_eS}{2\sqrt{\lambda_S}}[Q^2(4M^2V_--zS_x^2)F_{1+}+m^2(\mu\lambda_Y-2S_x(zQ^2+V_-))F_{2-}+(2(4\tau M^2-S_x)V_+$$
$$+(\mu-2\tau z)S_pS_x-2SV_2+2XV_1)F_{IR}+\tau(Q^2(zS_xS_p-4M^2V_+)+S_x(XV_1-SV_2))F_d],$$

$$\theta^0_{73}=\frac{\lambda_eS}{4\sqrt{\lambda_S}}[(S_x(4zQ^2+2V_--\mu S_x)-8\mu M^2Q^2)F_{1+}+2m^2(4\mu\tau M^2+2V_--(\mu+2\tau z)S_x)F_{2-}$$
$$+2(2V_+-\mu S_p)F_{IR}+\tau(4(S_x-2\tau M^2)V_++S_p((2\tau z-\mu)S_x-2V_-))F_d],$$

$$\theta^0_{74}=\frac{\lambda_eS}{4\sqrt{\lambda_S}}[((\mu+2\tau z)S_x-2V_--4\mu\tau M^2)F_{1+}+\tau(\mu S_p-2V_+)F_d],$$

$$\theta^0_{82}=\frac{1}{\lambda_1}[\varepsilon_\perp p_h((Q^2S_x(S_xV_+-2SV_2)-2V_-(2\lambda_1+Q^2SS_x))F_{1+}-2m^2(2\mu\lambda_1+Q^2S_pV_+)F_{2-}$$
$$+V_+(2m^2(2\tau(2(Q^2+2m^2)M^2-SX)-(Q^2+4m^2)S_x)F_{2+}+(4m^2((3Q^2+4m^2)S_x$$
$$+\tau(2SX-4(3Q^2+2m^2)M^2-S_x^2))+Q^2(\tau(12SX+S_x^2)+2Q^2(S_x-6\tau M^2)))F_d))$$
$$+2V_+\frac{\varepsilon_\perp k}{R}((((Q^2+4m^2)(zS_x^2-4M^2V_-)+S_p(XV_1-SV_2))F_{IR}+m^2(S_x(XV_1-SV_2+zS_pQ^2)$$
$$-4Q^2V_+M^2)F_{2-})],$$

$$\theta^0_{83}=\frac{1}{2\lambda_1}[\varepsilon_\perp p_h((2\mu(Q^2-2m^2)Q^2S_p+\tau(2(Q^2+8m^2)S_xV_++Q^2(\mu S_xS_p+2SV_1-2XV_2))$$
$$-2\tau^2(4(Q^2+4m^2)V_+M^2-S_p(SV_1+XV_2)))F_d+2\mu m^2(2\tau(2(Q^2+2m^2)M^2-SX)-(Q^2+4m^2)S_x)F_{2-}$$
$$+4S_pS_xV_--2\mu m^2Q^2S_pF_{2+}+(2\tau(X^2V_2-S^2V_1)+8m^2V_-(2\tau M^2-S_x)+Q^2(4\mu(SX-2Q^2M^2)$$
$$-S_x(2V_-+\mu S_x)))F_{1+})+2\frac{\varepsilon_\perp k}{R}((((Q^2+4m^2)(zS_x^2-4M^2V_-)+S_p(XV_1-SV_2))(V_-F_{1+}+\mu m^2F_{2-})$$
$$+\mu m^2(S_x(zS_pQ^2+XV_1-SV_2)-4Q^2V_+M^2)F_{2+}+(\mu(Q^2-2m^2)(4Q^2V_+M^2+S_x(SV_2-XV_1$$
$$-zS_pQ^2))+\tau(S_xS_pV_2(V_1+V_+)+2V_-V_+((S_x-4S)X+2(3Q^2+8m^2)M^2)+zS_x(Q^2(XV_2-SV_1)$$
$$-(Q^2+8m^2)S_xV_+)))F_d)],$$

$$\theta^0_{84}=\frac{\mu}{2\lambda_1}[\varepsilon_\perp p_h(((Q^2+4m^2)S_x+2\tau(SX-2(Q^2+2m^2)M^2))F_{1+}+S_p(\tau Q^2F_d-2S_x))$$
$$+\frac{\varepsilon_\perp k}{R}(((Q^2+4m^2)(4M^2V_--zS_x^2)+S_p(SV_2-XV_1))F_{1+}+\tau(4Q^2V_+M^2+S_x(SV_2-XV_1$$
$$-zS_pQ^2))F_d)],$$

$$\theta^0_{91}=\frac{2\lambda_eS}{\sqrt{\lambda_S}}(Q^2(zS_xV_+-m_h^2S_p)+V_-(SV_2-XV_1))F_{IR},$$

$$\theta^0_{92} = \frac{\lambda_e S}{\sqrt{\lambda_S}}(Q^2 S_x(zV_- - m_h^2)F_{1+} + m^2(Q^2(\mu z S_x - 2m_h^2) + V_-(\mu S_x - 2V_-))F_{2-}$$
$$+\tau(Q^2(m_h^2 S_p - zS_xV_+) + V_-(XV_1 - SV_2))F_d + (2V_-(2\mu S - V_+) + 2\tau(zS_xV_+ - m_h^2 S_p)$$
$$-\mu(V_1 + V_-)S_x)F_{IR}),$$
$$\theta^0_{93} = \frac{\lambda_e S}{2\sqrt{\lambda_S}}((2(2m_h^2 Q^2 + V_-^2) - \mu S_x(2zQ^2 + V_-))F_{1+} + m^2(\mu((2\tau z - \mu)S_x + 2V_-) - 4\tau m_h^2)F_{2-}$$
$$+\mu(2V_+ - \mu S_p)F_{IR} + \tau(2\tau(m_h^2 S_p - zS_xV_+) + \mu S_x(V_- + V_1) + 2V_-(V_+ - 2\mu S))F_d),$$
$$\theta^0_{94} = \frac{\lambda_e S}{4\sqrt{\lambda_S}}((2\tau(2m_h^2 - \mu z S_x) + \mu(\mu S_x - 2V_-))F_{1+} + \mu\tau(\mu S_p - 2V_+)F_d). \quad \text{(B1)}$$

The quantities θ^1_{ij} have a form:

$$\theta^1_{51} = 0,$$
$$\theta^1_{52} = \frac{2m^2\lambda_e}{\lambda_1\sqrt{\lambda_S}}[\varepsilon_\perp p_h(2(2m^2\lambda_Y + (Q^2 + \tau S)(2M^2Q^2 + SS_x))F_{21} - S_x\lambda_Y F_{1+} + (2Q^2XS_x + \tau S_x(2S^2 - S_p^2)$$
$$+4M^2Q^2(\tau S - Q^2) - 4m^2\lambda_Y)F_d) + 2\frac{\varepsilon_\perp k}{R}(S_x(zS_pQ^2 + XV_1 - SV_2) - 4M^2Q^2V_p)(XF_d - SF_{21})],$$
$$\theta^1_{53} = \frac{2m^2\lambda_e}{\lambda_1\sqrt{\lambda_S}}[\varepsilon_\perp p_h(2((Q^2 + 2m^2)(2\tau M^2 + X) - (\tau X + 2m^2)S)F_{21} - \lambda_Y F_{1+} + (4m^2(S_x - 2\tau M^2) + 2Q^2 S$$
$$+\tau(S^2 + X^2))F_d) + 2\frac{\varepsilon_\perp k}{R}((2m^2(zS_x^2 - 4M^2V_-) + 2M^2Q^2V_2 + X(XV_1 - SV_2 - zS_xQ^2))F_{21}$$
$$+(2m^2(4M^2V_- - zS_x^2) + 2M^2Q^2V_1 + S(SV_2 - XV_1 - zQ^2S_x))F_d)],$$
$$\theta^1_{71} = 0,$$
$$\theta^1_{72} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}((4M^2(\tau SV_- - Q^2V_+) - S_x^2(\tau zS + zQ^2 + V_1) + \mu\lambda_Y S)F_{21} + (4M^2(Q^2V_+ - \tau XV_-)$$
$$+S_x^2(\tau zX + V_2 - zQ^2) - \mu\lambda_Y X)F_d),$$
$$\theta^1_{73} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[(2M^2(\mu(Q^2 + \tau S) - 2\tau V_+) + S_x((\tau z - 2\mu)S + 2V_+ - zQ^2) + (\mu - \tau z)S_x^2)F_{21}$$
$$+(2M^2(\mu(Q^2 - \tau X) + 2\tau V_+) + S_x((2\mu - \tau z)S - zQ^2 - 2V_+) - \mu S_x^2)F_d],$$
$$\theta^1_{74} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[(2\mu\tau M^2 + \mu X - \tau zS_x - V_2)F_{21} + (2\mu\tau M^2 + V_1 - \mu S - \tau zS_x)F_d],$$
$$\theta^1_{91} = \frac{4\lambda_e m^2(m_h^2\lambda_Y + 4M^2V_-^2 - zS_x^2(zQ^2 + 2V_-))}{\sqrt{\lambda_S}}F_{IR},$$
$$\theta^1_{92} = \frac{2m^2\lambda_e}{\sqrt{\lambda_S}}[2m^2(2m_h^2(2\tau M^2 - S_x) + 2(zS_x - 2\mu M^2)V_- + z(\mu - \tau z)S_x^2)F_{2+}$$
$$+S_x(zQ^2(\mu S - V_+) + V_-((\mu + \tau z)S - V_1) - m_h^2(Q^2 + \tau S))F_{21}$$
$$+(S_x((m_h^2 - zV_-)(\tau X + 3Q^2 + 8m^2) + (V_- + zQ^2)(V_2 - \mu X))$$
$$+2(4M^2(\mu V_- - \tau m_h^2) + (z\tau - \mu)zS_x^2)(Q^2 + 2m^2))F_d],$$
$$\theta^1_{93} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[4m^2(m_h^2 + \mu^2M^2 - \mu zS_x)F_{2+} + (2m_h^2(\tau X - Q^2) + S_x(\mu(\tau z - \mu)S - 2\tau zV_+ + \mu zQ^2$$
$$+\mu V_1) + 2V_-(V_2 - \mu X))F_{21} + (\mu((Q^2 + 2m^2)(5zS_x - 4\mu M^2) + (\mu - \tau z)XS_x)$$
$$-2m_h^2(\tau S + 3Q^2 + 4m^2) + S_x(2\tau zV_+ - \mu V_2 - 2\mu zm^2) + 2V_-(\mu S - V_1))F_d],$$
$$\theta^1_{94} = \frac{m^2\lambda_e}{\sqrt{\lambda_S}}[(\mu(\tau zS_x + \mu X - V_2) - 2\tau m_h^2)F_{21} + (\mu(\tau zS_x + V_1 - \mu S) - 2\tau m_h^2))F_d]. \quad \text{(B2)}$$

The variable μ is defined as

$$\mu = \frac{kp_h}{kp} = \frac{p_{h0}}{M} + \frac{p_l(2\tau M^2 - S_x)}{M\sqrt{\lambda_Y}} -2Mp_t\cos(\phi_h - \phi_k)\sqrt{\frac{(\tau_{max}-\tau)(\tau-\tau_{min})}{\lambda_Y}} \quad \text{(B3)}$$

The quantities F_i ($i = d, 1+, 2+, 2-, IR$) are expressed through:

$$\begin{aligned} z_1 &= \frac{k_1 k}{pk} \\ &= \frac{Q^2 S_p + \tau(SS_x + 2M^2Q^2) - 2M\sqrt{\lambda_z}\cos\phi_k}{\lambda_Y}, \\ z_2 &= \frac{k_1 k}{pk} \\ &= \frac{Q^2 S_p + \tau(XS_x - 2M^2Q^2) - 2M\sqrt{\lambda_z}\cos\phi_k}{\lambda_Y}, \\ \lambda_z &= (\tau_{max}-\tau)(\tau-\tau_{min})\lambda_1 \end{aligned} \quad \text{(B4)}$$

in the following way:

$$\begin{aligned} F_{2\pm} &= F_{22} \pm F_{21} = \frac{1}{z_2^2} \pm \frac{1}{z_1^2}, \\ F_d &= \frac{1}{z_1 z_2}, \\ F_{1+} &= \frac{1}{z_1} + \frac{1}{z_2}, \\ F_{IR} &= m^2 F_{2+} - (Q^2 + 2m^2)F_d. \end{aligned} \quad \text{(B5)}$$

Appendix C: Calculation of δ_S and δ_H

The dimensional regularization is used for calculation of δ_S in (37),

$$\begin{aligned} \frac{d^3k'}{k'_0} &\to \frac{d^{n-1}k'}{(2\pi\nu)^{n-4}k'_0} \\ &= \frac{2\pi^{n/2-1}k_0'^{n-3}dk'_0(1-x^2)^{n/2-2}dx}{(2\pi\nu)^{n-4}\Gamma(n/2-1)}, \end{aligned} \quad \text{(C1)}$$

where $x = \cos\theta$ (θ is defined as the spatial angle between the photon three-momentum and $\mathbf{k}'_i$ ($i = 1-3$) that are introduced below) and ν is an arbitrary parameter of the dimension of a mass. The Feynman parameterization of propagators in F_{IR}:

$$F_{IR} = \frac{R^2}{4k_0'^2}\int_0^1 dy\mathcal{F}(x,y), \quad \text{(C2)}$$

where y is the Feynman parameter and

$$\mathcal{F}(x,y) = \frac{m^2}{k_{10}'^2(1-x\beta_1)^2} + \frac{m^2}{k_{20}'^2(1-x\beta_2)^2} - \frac{Q_m^2}{k_{30}'^2(1-x\beta_3)^2}. \quad \text{(C3)}$$

The energies of the real photon (k'_0), initial (k'_{10}) and final (k'_{20}) leptons are defined in the system $\mathbf{p} + \mathbf{q} - \mathbf{p}_h = 0$ while $k'_{30} = yk'_{10} + (1-y)k'_{20}$ and $\beta_i = |\mathbf{k}'_i|/k'_{i0}$.

Then, the substitution Eqs. (C1) and (C3) into the definitions of δ_S by Eq. (37), integration over k'_0, and expanding the obtained result into Laurent series around $n = 4$ result in

$$\delta_S = \delta_S^{IR} + \delta_S^1, \quad \text{(C4)}$$

where

$$\delta_S^{IR} = -\frac{1}{2}\left[P_{IR} + \log\frac{\bar{k}_0}{\nu}\right]\int_0^1 dy \int_{-1}^1 dx\mathcal{F}(x,y) \quad \text{(C5)}$$

and

$$\delta_S^1 = -\frac{1}{4}\int_0^1 dy\int_{-1}^1 dx\log(1-x^2)\mathcal{F}(x,y). \quad \text{(C6)}$$

Here P_{IR} is the infrared divergent term defined by Eq. (39). Since $k_{30}'^2 - |\mathbf{k}'_3|^2 = m^2 + y(1-y)Q^2$ the integration over x and y variables in δ_S^{IR} is performed explicitly:

$$\delta_S^{IR} = 2(Q_m^2 L_m - 1)\left[P_{IR} + \log\frac{\bar{k}_0}{\nu}\right], \quad \text{(C7)}$$

For the covariant analytical integration in δ_S^1 we express the initial and final lepton energies through the invariants:

$$k'_{10} = \frac{S'}{2\sqrt{p_x^2}}, \qquad k'_{20} = \frac{X'}{2\sqrt{p_x^2}}, \quad \text{(C8)}$$

As a result,

$$\delta_S^1 = 2(Q_m^2 L_m - 1)\log(2) + \frac{1}{2}S'L_{S'} + \frac{1}{2}X'L_{X'} + S_\phi, \quad \text{(C9)}$$

where the quantities L_m, $L_{S'}$ and $L_{X'}$ are

$$\begin{aligned} L_m &= \frac{1}{\sqrt{\lambda_m}}\log\frac{\sqrt{\lambda_m}+Q^2}{\sqrt{\lambda_m}-Q^2}, \\ L_{S'} &= \frac{1}{\sqrt{\lambda'_S}}\log\frac{S'+\sqrt{\lambda'_S}}{S'-\sqrt{\lambda'_S}}, \\ L_{X'} &= \frac{1}{\sqrt{\lambda'_X}}\log\frac{X'+\sqrt{\lambda'_X}}{X'-\sqrt{\lambda'_X}} \end{aligned} \quad \text{(C10)}$$

and

$$S_\phi = \frac{1}{2}Q_m^2\int_0^1 \frac{dy}{\beta_3(m^2+y(1-y)Q^2)}\log\frac{1-\beta_3}{1+\beta_3}. \quad \text{(C11)}$$

The explicit expression for S_ϕ after integration over y is given in Eq. (40).

For the calculation of δ_H we carry out integration in the same reference system $\mathbf{p}+\mathbf{q}-\mathbf{p}_h=0$

$$\delta_H=-\frac{1}{\pi}\int\limits_{\bar{k}_0}^{k_0^{max\prime}}k_0'dk_0'\int\limits_0^\pi\sin(\theta_k')d\theta_k'\int\limits_0^{2\pi}d\phi_k'\frac{F_{IR}}{R^2}, \tag{C12}$$

where θ_k' is the angle between $\mathbf{k}$ and $\mathbf{q}$ three momenta, and ϕ_k' is the angle between $(\mathbf{k}_1,\mathbf{k}_2)$ and $(\mathbf{k},\mathbf{q})$ planes.

In this system

$$\begin{aligned}z_1&=\frac{2k_0'}{R}(k_{10}'-k_t'\cos\phi_k'\sin\theta_k'-k_{13}'\cos\theta_k'),\\ z_2&=\frac{2k_0'}{R}(k_{20}'-k_t'\cos\phi_k'\sin\theta_k'-k_{23}'\cos\theta_k'),\end{aligned} \tag{C13}$$

that allows us to take the first integration in respect to ϕ_k:

$$\delta_H=\int\limits_{\bar{k}_0}^{k_0^{max\prime}}\frac{dk_0'}{2k_0'}\int\limits_0^\pi\sin(\theta_k')d\theta_k'\left[\frac{Q_m^2}{B_1-B_2}\left(\frac{1}{\sqrt{C_2}}-\frac{1}{\sqrt{C_1}}\right)-\frac{m^2B_1}{C_1^{3/2}}-\frac{m^2B_2}{C_2^{3/2}}\right]. \tag{C14}$$

Here

$$B_i=k_{i0}'-\cos(\theta_k')k_{i3}',\ C_i=B_i^2-\sin^2(\theta_k')k_t'^2 \tag{C15}$$

for $i=1,2$.

After the integration in respect to θ_k' and the use of the following replacements:

$$k_t'=\sqrt{k_{10}'^2-k_{13}'^2-m^2}=\sqrt{k_{20}'^2-k_{23}'^2-m^2},$$

$$k_{13}'=\frac{2k_{10}'q_0'+Q^2}{2\sqrt{Q^2+q_0'^2}},\ k_{23}'=\frac{2k_{20}'q_0'-Q^2}{2\sqrt{Q^2+q_0'^2}} \tag{C16}$$

with $q_0'=k_{10}'-k_{20}'$, the hard contribution δ_H are expressed in the form

$$\delta_H=2\int\limits_{\bar{k}_0}^{k_0^{max}}\frac{dk_0}{k_0}(Q_m^2L_m-1). \tag{C17}$$

Since $k_0^{max\prime}=(p_x^2-(M+m_\pi)^2)/2\sqrt{p_x^2}$ the integration for δ_H is finally presented in the form of (38).

Appendix D: Calculation of Λ_μ and $\Pi^l_{\alpha\mu}$

The γ-matrix recombination, convolution over α-indexes in Eq. (47) for Λ_μ and calculation of the traces for $\Pi^l_{\alpha\mu}$ in n-dimension space result in:

$$\begin{aligned}\Lambda_\mu&=\frac{\alpha}{4\pi}\Big\{\gamma_\mu[(n-2)J_\delta^\delta-4J^\delta(k_{1\delta}+k_{2\delta})\\&+2Q_m^2J]+2\gamma_\delta[2J^\delta(k_{1\mu}+k_{2\mu})\\&-(n-2)J_\mu^\delta]-4mJ_\mu\Big\},\\ \Pi^l_{\alpha\mu}&=\frac{\alpha}{\pi}\frac{1}{Q^2}\Big(\sum_{i=e,\mu,\tau}\Big\{g_{\alpha\mu}(q_\delta J_i^\delta+m_i^2J_i-J_{i\delta}^\delta)\\&+2J_{i\alpha\mu}-q_\alpha J_{i\mu}-q_\mu J_{i\alpha}\Big\}\Big).\end{aligned} \tag{D1}$$

where:

$$\begin{aligned}J&=\frac{1}{i\pi^2}\lim_{n\to4}\int\frac{(2\pi\nu)^{4-n}d^nl}{l^2(l^2-2lk_2)(l^2-2lk_1)}=-2L_m\left(P_{IR}+\log\frac{m}{\nu}\right)-\frac{1}{2}\sqrt{\lambda_m}L_m^2+\frac{1}{2\sqrt{\lambda_m}}\left(\pi^2-4\mathrm{Li}_2\frac{2\sqrt{\lambda_m}}{\sqrt{\lambda_m}+Q^2}\right),\\ J_\delta&=\frac{1}{i\pi^2}\lim_{n\to4}\int\frac{l_\delta(2\pi\nu)^{4-n}d^nl}{l^2(l^2-2lk_2)(l^2-2lk_1)}=-L_m(k_{1\delta}+k_{2\delta}),\\ J_{\delta\rho}&=\frac{1}{i\pi^2}\lim_{n\to4}\int\frac{l_\delta l_\rho(2\pi\nu)^{4-n}d^nl}{l^2(l^2-2lk_2)(l^2-2lk_1)}=\frac{1}{4}\Big\{g_{\delta\rho}\left(3-2P_{UV}-2\log\frac{m}{\nu}-\frac{\lambda_m}{Q^2}L_m\right)+q_\delta q_\rho\frac{2Q^2-\lambda_mL_m}{Q^4}\\&\qquad-L_m(k_{1\delta}+k_{2\delta})(k_{1\rho}+k_{2\rho})\Big\},\\ J_i&=\frac{1}{i\pi^2}\lim_{n\to4}\int\frac{(2\pi\nu)^{4-n}d^nl}{(l^2-m_i^2)((l-q)^2-m_i^2)}=2-2P_{UV}-2\log\frac{m_i}{\nu}-\frac{\lambda_m^i}{Q^2}L_m^i,\\ J_{i\delta}&=\frac{1}{i\pi^2}\lim_{n\to4}\int\frac{l_\delta(2\pi\nu)^{4-n}d^nl}{(l^2-m_i^2)((l-q)^2-m_i^2)}=\frac{1}{2}q_\delta J_i,\\ J_{i\delta\rho}&=\frac{1}{i\pi^2}\lim_{n\to4}\int\frac{l_\delta l_\rho(2\pi\nu)^{4-n}d^nl}{(l^2-m_i^2)((l-q)^2-m_i^2)}=\frac{1}{72}\Big\{g_{\delta\rho}\left(6\left[Q^2-\frac{3\lambda_m^i}{Q^2}\right](P_{UV}+\log\frac{m_i}{\nu})+\left[21-\frac{6\lambda_m^i}{Q^2}L_m^i\right]\frac{\lambda_m^i}{Q^2}-5Q^2\right)\end{aligned}$$

$$+q_\delta q_\rho\biggl(40-48P_{UV}-48\log\frac{m_i}{\nu}+12\frac{\lambda^i_m}{Q^4}-6\frac{\lambda^i_m}{Q^2}\biggl[3+\frac{\lambda^i_m}{Q^4}\biggr]L_m\biggr)\biggr\}. \quad \text{(D2)}$$

The infrared divergent P_{IR} term is defined by Eq. (39) while the ultraviolet divergent term has the same structure $P_{UV}=P_{IR}$ and

$$L^i_m=\frac{1}{\sqrt{\lambda^i_m}}\log\frac{\sqrt{\lambda^i_m}+Q^2}{\sqrt{\lambda^i_m}-Q^2},\ \lambda^i_m=Q^2(Q^2+4m_i^2). \quad \text{(D3)}$$

After substituting (D2) into (D1) and using $nP_{UV}=4P_{UV}+1$ we find the final expressions for Λ_μ and $\Pi^l_{\alpha\mu}$ (48).

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