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# The SU(3) Vector Currents in BChPT $\times\, 1/\mathrm{N_c}$

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## Abstract

Baryon Chiral Perturbation Theory (BChPT) combined with the $1/N_c$ expansion is applied to the $SU(3)$ vector currents. In terms of the $\xi$ power counting linking the low energy and $1/N_c$ expansions according to $\mathcal{O}(\xi) = \mathcal{O}(p) = \mathcal{O}(1/N_c)$, the study is carried out to next-to-next-to-leading order, and it includes $SU(3)$ breaking corrections to the $|\,\Delta S\,| = 1$ vector charges, charge radii, and magnetic moments and radii. The results are obtained for generic $N_c$, allowing for investigating the various scalings in $N_c$.

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## I. INTRODUCTION

Vector currents, being intimately related to the flavor $SU(3)$ symmetry of QCD, represent a fundamental probe for hadron structure as well as for the breaking of $SU(3)$ by quark masses. This is particularly interesting for baryons, where the electromagnetic current for nucleons, known empirically to remarkable accuracy [1], along with the magnetic moments of hyperons allow for an almost complete description of all the $SU(3)$ vector currents to the order in the low energy expansion considered in the present work. The charged vector currents are relevant in $\beta$ decays, where both $SU(3)$ breaking in the $|\Delta S|= 1$ charges and weak magnetism are still open problems. To the present level of experimental accuracy in hyperon $\beta$ decays, there is not sufficient sensitivity to the $SU(3)$ breaking in the charges [2]. The reason is that $\beta$ decay has a branching fraction of about $10^{-3}$, being dominated by the non-leptonic component. Fortunately, lattice QCD is producing results [3–6] which can be compared with the predictions of the approach in the present work. The experimental information on charge form factors is limited to the electric form factors of nucleons and the charge radius of the $\Sigma^-$. This is however sufficient to predict the rest of the charge radii, whose $SU(3)$ breaking effects are, at the order of the present calculation, finite non-analytic in quark masses. The octet baryons' EM magnetic moments and nucleons' magnetic radii give an almost complete prediction for the rest of the currents but for one low energy constant (LEC) which requires knowledge of at least one weak magnetic moment of a $\Delta S = 1$ current. In the approach followed here, results automatically extend to the vector current observables of the decuplet baryons and to EM transitions, e.g. the $M_1$ transition $\Delta \to N\gamma$, most of which remain empirically unknown or poorly known. The study of electric currents in BChPT with inclusion of the spin 3/2 baryons dates back a quarter century [7, 8], and numerous works have since been produced in various versions of that framework, among those close in spirit to the present one are found in Refs. [9–15], and works with additional constraints imposed by consistency with the $1/N_c$ expansion are those of Refs. [16–21]. The present work formalizes the combination of BChPT and the $1/N_c$ expansion [22] for the vector currents following the rigorous power counting scheme of the $\xi$ expansion [23, 24] based on the linking $\mathcal{O}(p) = \mathcal{O}(1/N_c) = \mathcal{O}(\xi)$. The combined framework was first applied to the $SU(3)$ vector charges in Ref. [20], where the $\xi$ expansion was not strictly implemented, however for the purpose of calculating the corrections of $SU(3)$ breaking to the vector charges, restricted by the Ademollo-Gatto theorem (AGT), such omission has no very significant effect [1]. Here a complete study is presented to $\mathcal{O}(\xi^3)$ and $\mathcal{O}(\xi^4)$ (depending on the observable) of the $SU(3)$ vector currents. The present work provides results for generic $N_c$, permitting in this way to sort out in particular the large $N_c$ behavior of non-analytic terms in $\xi$ stemming from one loop corrections, which gives additional understanding, as it has been shown for instance in the case of the Gell-Mann-Okubo relation and the $\sigma$ terms discussed in Refs. [25, 26]. The subject of magnetic moments has been addressed in the context of the $1/N_c$ expansion in works limited to a tree level expansion in composite operators [27–30], and in works including one loop corrections in BChPT Refs. [17–19, 21]. In addition to the BChPT, dispersive approaches implementing constraints of chiral dynamics have been implemented [31–33] and where in addition consistency with the $1/N_c$ expansion has also been required [34–38]. Such works naturally give a range of applicability beyond the present one, which is limited up to the form factor radii.

This work is organized as follows: Section II presents the baryon chiral Lagrangians needed for the present work, Section III summarizes the one loop corrections to the vector currents, Section IV presents the analysis of the vector charges and radii, and Section V does the same for the magnetic moments and radii. A summary is presented in Section VI. Several appendices are included for the benefit of readers intending to implement similar calculations.

## II. BARYON CHIRAL LAGRANGIAN

This section summarizes the pieces of the Baryon Chiral Lagrangian up to $\mathcal{O}(\xi^4)$ relevant to the calculations in this work. The details on the construction of the Lagrangians and the notations are given in Ref. [24]. In order to ensure the validity of the OZI rule for the quark mass dependency of baryon masses, namely, that the non-strange baryon mass dependence on $m_s$ be $\mathcal{O}(N_c^0)$, the following combination of the source $\chi_+$ is defined by [24]:

$$\hat{\chi}_+ \equiv \tilde{\chi}_+ + N_c\, \chi_+^0, \tag{1}$$

which is $\mathcal{O}(N_c)$ but has dependence on $m_s$ which is $\mathcal{O}(N_c^0)$ for all states with strangeness $\mathcal{O}(N_c^0)$. For convenience a scale $\Lambda$ is introduced, which can be chosen to be a typical QCD

[1] In [20] the baryon-GB vertices included higher order terms in $1/N_c$.

scale, in order to render most of the LECs dimensionless. In the calculations $\Lambda = m_\rho$ will be chosen. The quark mass matrix is defined by $\mathcal{M}_q = m^0 + m^a \frac{\lambda^a}{2}$, where in the physical case $m^0 = \frac{1}{3}(m_u + m_d + m_s)$, $m^3 = m_u - m_d$ and $m^8 = \frac{1}{\sqrt{3}}(m_u + m_d - 2m_s)$ and the rest of the $M^a$ s vanish.

Collecting the baryons in a spin flavor multiplet denoted by $\mathbf{B}$, and using standard notation for the chiral building blocks (for details see [24]), the LO $\mathcal{O}(\xi)$ Lagrangian reads:

$$\mathcal{L}_B^{(1)} = \mathbf{B}^\dagger (iD_0 - \frac{C_{HF}}{N_c}\hat{S}^2 + \mathring{g}_A u^{ia} G^{ia} + \frac{c_1}{2\Lambda}\hat{\chi}_+)\mathbf{B}, \tag{2}$$

where the hyperfine mass shifts are given by the second term, $G^{ia}$ are the spin-flavor generators (see Appendix A), and the axial coupling is at LO $\mathring{g}_A = \frac{6}{5} g_A$, being $g_A = 1.2732(23)$ the nucleon's axial coupling. The relevant terms in the $\mathcal{O}(\xi^2)$ Lagrangian are:

$$\mathcal{L}_B^{(2)} \; = \; \mathbf{B}^\dagger \left( \frac{c_2}{\Lambda}\chi_+^0 + \frac{C_1^A}{N_c} u^{ia} S^i T^a + \frac{\kappa}{2\Lambda} B_+^{ia} G^{ia} + \cdots \right) \mathbf{B}, \tag{3}$$

where the flavor $SU(3)$ electric and magnetic fields are denoted by $E_+$ and $B_+$ and given by $E_+^i = F_+^{0i}$ and $B_+^i = \frac{1}{2}\epsilon^{ijk} F_+^{jk}$ [24]. The term proportional to $\kappa$ gives at LO the magnetic moments associated with all vector currents. The $\mathcal{O}(\xi^3)$ and $\mathcal{O}(\xi^4)$ Lagrangians needed for the one-loop renormalization of the vector currents are the following:

$$\begin{aligned}
\mathcal{L}_B^{(3)} \; &= \; \mathbf{B}^\dagger \Big( \frac{g_1}{\Lambda^2} D_i E_{+i}^a T^a + \frac{\kappa_1}{2\Lambda N_c} B_+^{ia} S^i T^a + \cdots \Big) \mathbf{B} \\
\mathcal{L}_B^{(4)} \; &= \; \mathbf{B}^\dagger \left( \frac{1}{N_c \Lambda^2} (g_2 D_i E_{+i}^a S^j G^{ja} + g_3 D_i E_{+j}^a \{S^i, G^{ja}\}^{\ell=2}) + \frac{\kappa_r}{\Lambda^3} D^2 B_+^{ia} G^{ia} \right. \\
&+ \; \frac{1}{2\Lambda^3} (\kappa_2 \chi_+^0 B_+^{ia} G^{ia} + i\kappa_F f^{abc} \chi_+^a B_+^{ib} G^{ic} + \kappa_D d^{abc} \chi_+^a B_+^{ib} G^{ic} + \kappa_3 \chi_+^a B_+^{ia} S^i) \\
&+ \; \left. \frac{1}{2\Lambda N_c^2} (\kappa_4 B_+^{ia} \{\hat{S}^2, G^{ia}\} + \kappa_5 B_+^{ia} S^i S^j G^{ja}) + \cdots \right) \mathbf{B}
\end{aligned} \tag{4}$$

The LECs $g_1$ and $g_2$ will be determined by charge radii, the term proportional to $g_3$ gives electric quadrupole moments for decuplet baryons and for transitions between decuplet to octet baryons, which will not be discussed here, and the term proportional to $\kappa_r$ gives a contribution to magnetic radii ($D^2 B_+ \equiv D_\mu D^\mu B_+$ being the covariant divergence of the magnetic field). The rest are quark mass and higher order in $1/N_c$ corrections to the magnetic moments.

Throughout, spin-flavor operators in the Lagrangians are scaled by appropriate powers of $1/N_c$ such that all LECs start at zeroth order in $N_c$. Of course, LECs have themselves an expansion in $1/N_c$, kept implicit, which requires information for $N_c > 3$ to be determined. In that sense each Lagrangian term has a leading power in $1/N_c$ which is used to assign its order in the $\xi$ power counting, followed by sub-leading terms in $1/N_c$ due to the expansion of the corresponding LEC. In addition, each term in the Lagrangian is explicitly chiral invariant and its expansion in powers of the Goldstone Boson fields yields factors $1/F_\pi = \mathcal{O}(1/\sqrt{N_c})$ for each additional factor of a GB field.

For convenience the following definition is used:

$$\delta\hat{m} \equiv \frac{C_{\rm HF}}{N_c}\hat{S}^2 - \frac{c_1}{2\Lambda}\,\hat{\chi}_+. \tag{5}$$

Note that $\delta\hat{m}$ gives rise to mass splittings between baryons which are the $\mathcal{O}(1/N_c)$ hyperfine term in Eqn.(2) and the $\mathcal{O}(p^2)$ quark mass term. The $\mathcal{O}(m_q N_c)$ term in $\hat{\chi}_+$ becomes immaterial in the loop calculations as only differences of baryon masses appear for which such terms exactly cancel.

## III. ONE LOOP CORRECTIONS TO CURRENTS

The one-loop corrections to the vector currents involve the two sets of gauge invariant diagrams $A$ and $B$ in Fig. 1, where the vertices are given in Appendix C. The explicit results are the following:

$$\begin{aligned}
V^{\mu a}(A_1) &= i\left(\frac{\mathring{g}_A}{F_\pi}\right)^2 \sum_{n_1,n_2} G^{ib}\mathcal{P}_{n_2}\Gamma^{\mu a}\mathcal{P}_{n_1}G^{jb}\,\frac{1}{q_0-\delta m_{n_2}+\delta m_{n_1}} \\
&\quad\times\left(H^{ij}(p_0-\delta m_{n_1},M_b)-H^{ij}(p_0+q_0-\delta m_{n_2},M_b)\right) \\
V^{\mu a}(A_2) &= \frac{1}{2}\{\Gamma^{\mu a},\delta\hat{Z}_{1-loop}\} \\
V^{\mu a}(A_3) &= \left(\frac{\mathring{g}_A}{F_\pi}\right)^2 f^{abc}\sum_n G^{ib}\mathcal{P}_n G^{jc}H^{ij\mu}(p_0-\delta m_n,q,M_b,M_c) \\
V^{\mu a}(B_1) &= -\frac{i}{2F_\pi^2}f^{abc}f^{bcd}\Gamma^{\mu d}I(0,1,M_b^2) \\
V^{\mu a}(B_2) &= g^{\mu 0}\frac{i}{4F_\pi^2}f^{abc}f^{bcd}T^d({q_0}^2K(q,M_b,M_c)+4q_0K^0(q,M_b,M_c)+4K^{00}(q,M_b,M_c)),
\end{aligned} \tag{6}$$

where $\mathcal{P}_n$ are projectors onto the corresponding baryon in the loop, $p_0$ is the residual energy of the initial baryon, $q_0$ is the incoming energy in the current, and $\Gamma^{\mu a} = g^{\mu 0}T^a + i\frac{\kappa}{\Lambda}\epsilon^{0\mu ij}f^{abc}f^{cbd}q^iG^{jd}$ contains both the electric charge and magnetic moment components. The one-loop wave function renormalization factor $\delta\hat{Z}_{1-loop}$ can be found in [24], and the loop integrals $I$, $K$, $K^\mu$, $K^{\mu\nu}$, $H^{ij}$ and $H^{ij\mu}$ are given in Appendix B. Since the temporal component of the current can only connect baryons with the same spin, $q_0$ is equal to the $SU(3)$ breaking mass difference between them plus the kinetic energy transferred by the current, which are all $\mathcal{O}(\xi^2)$ or higher and must therefore be neglected in this calculation. In the evaluations one sets $p_0 \to \delta m_{in}$ and $p_0 + q_0 \to \delta m_{out}$. In particular, for diagram $A_1$, if it requires evaluation at $q_0 = 0$ such a limit must be taken in the end of the evaluation. The $U(1)$ baryon number current can used to check the calculation: only diagrams $A_{1+2}$ contribute, and as required they cancel each other.

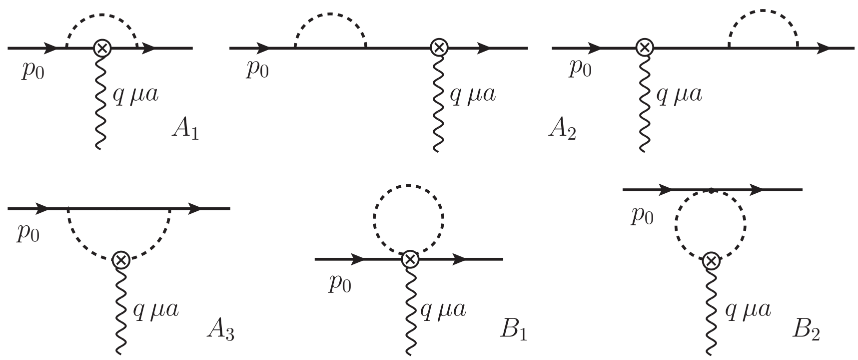


FIG. 1: Diagrams contributing to the 1-loop corrections to the vector currents.

For a generic current vertex $\Gamma$, the combined UV divergent and polynomial piece of diagrams $A_{1+2}$ can be written as:

$$\begin{aligned}\Gamma(A_{1+2})^{\rm poly} &= \frac{1}{(4\pi)^2}\left(\frac{\mathring{g}_A}{F_\pi}\right)^2\left(\frac{1}{2}(\lambda_\epsilon+1)M^2_{ab}[G^{ia},[G^{ib},\Gamma]]\right.\\ &+ \left.\frac{1}{3}(\lambda_\epsilon+2)\left(2[[G^{ia},\Gamma],[\delta\hat{m},[\delta\hat{m},G^{ia}]]]+[[\Gamma,[\delta\hat{m},G^{ia}]],[\delta\hat{m},G^{ia}]]\right)\right), \quad (7)\end{aligned}$$

where $\lambda_\epsilon = \frac{1}{\epsilon} - \gamma + \log 4\pi$. The first term is proportional to quark masses through the GB mass-square matrix $M^2_{ab} = m^0\delta^{ab} + \frac{1}{2}d^{abc}m^c$, and the second involves the baryon hyperfine mass splittings $\delta\hat{m}$ which are $\mathcal{O}(1/N_c)$ and, following the strict $\xi$ power counting, the $\mathcal{O}(p^2)$ terms due to $SU(3)$ breaking in $\delta\hat{m}$ are disregarded. The consistency with the $1/N_c$ power counting can be readily checked. Diagrams $A_3$ and $B_{1,2}$ are separately consistent with the

$1/N_c$ power counting. Their polynomial contributions are the following:

$$
\begin{aligned}
V^{\mu a}(A_3)^{Poly} &= -\frac{1}{(4\pi)^2}\left(\frac{\mathring{g}_A}{F_\pi}\right)^2 \frac{1}{6}\, i f^{abc} \\
&\times \left(g^{\mu 0}\Big((\lambda_\epsilon q^i q^j + \frac{1}{2}(\lambda_\epsilon+1)q^2 g^{ij})\delta^{bd} - 3g^{ij}(\lambda_\epsilon+1)M^2_{bd}\Big)[G^{id}, G^{jc}]\right. \\
&- g^{\mu 0}\,(\lambda_\epsilon+2)\Big(\frac{1}{2}[G^{ib},[[G^{ic},\delta\hat{m}],\delta\hat{m}]] - [[G^{ib},\delta\hat{m}],[G^{ic},\delta\hat{m}]]\Big) \\
&+ \left. g^\mu_i(\lambda_\epsilon+2)\Big(\frac{1}{2}g^{jk}q^i[[G^{kb},G^{jc}],\delta\hat{m}] + 2g^{ij}q^k(3[G^{kb},[G^{jc},\delta\hat{m}]] + [[G^{jc},G^{kb}],\delta\hat{m}])\Big)\right) \\
V^{\mu a}(B_1)^{Poly} &= \frac{1}{(4\pi)^2}(\lambda_\epsilon+1)\frac{1}{2F_\pi^2} f^{abd} f^{cde}\,\Gamma^{\mu e} M^2_{bc} \\
&= -\frac{1}{(4\pi)^2}\frac{3}{F_\pi^2}(\lambda_\epsilon+1)g^{\mu 0}B_0(m^0\,\Gamma^{\mu a} + \frac{1}{4}d^{abc}m^b\,\Gamma^{\mu c}) \\
V^{\mu a}(B_2)^{Poly} &= -\frac{1}{(4\pi)^2}\lambda_\epsilon\frac{1}{4F_\pi^2}(g^{\mu 0}\,\vec{q}^{\,2} + g^\mu_i q^i q_0)\,T^a - g^{\mu 0}V^{0a}(B_1)^{Poly}
\end{aligned}
\tag{8}
$$

Reduction formulas that can be found in [25] are used to express the above in a base of irreducible operators, Eqns.(9) and (12) below.

## IV. VECTOR CHARGES

In this section the SU(3) vector current charges and corresponding radii are analyzed. The SU(3) breaking corrections to the charges already presented in [20] and [24] are discussed for completeness. At lowest order the charges are represented by the flavor generators $T^a$. The one-loop corrections are UV finite at $Q^2 \equiv -q^2 = 0$, and since up to $\mathcal{O}(\xi^3)$ the AGT is satisfied, the corrections to the charges are unambiguously given by UV finite one-loop contributions. Note that the AGT applies to the whole baryon spin-flavor multiplet. On the other hand, at finite $Q^2$ the one-loop correction has an UV divergent piece which is independent of quark masses and is renormalized via the terms $g_1$ and $g_2$ in $\mathcal{L}_B$, one of them removes the UV divergence ($g_1$) and the other one is a finite counterterm ($g_2$).

Combining the polynomial pieces in Eqns.(7) and (8) and using that, $[\delta\hat{m}, T^a] = [\delta\hat{m}, \hat{G}^2] = [\delta\hat{m}, G^{ib}T^aG^{ib}] = 0$ one obtains the polynomial loop contributions to vector charges, which are proportional to $Q^2 = \vec{q}^{\,2}$:

$$
\begin{aligned}
f_1^a(A_{1+2+3})^{\rm poly} &= \frac{\lambda_\epsilon - 3}{(4\pi)^2}\left(\frac{\mathring{g}_A}{4F_\pi}\right)^2 Q^2\,T^a \\
f_1^a(B_{1+2})^{\rm poly} &= -\frac{\lambda_\epsilon+1}{(4\pi)^2}\frac{Q^2}{4F_\pi^2}\,T^a,
\end{aligned}
\tag{9}
$$

where $f_1^a \equiv V^{0a}$.

The corrections to the $\mid \Delta S \mid= 1$ charges, already discussed in [20], are evaluated using the physical values $\mathring{g}_A = \frac{6}{5} \times 1.27$ and $F_\pi = 92$ MeV, however one needs to be aware that their values are effected by the NLO corrections, leading to a theoretical uncertainty. With the usual notation for those charges [20], evaluating the ratios $\delta f_1/f_1$ in the large $N_c$ limit one finds that $\delta f_1/f_1 = \mathcal{O}(1/N_c)$. However, this behavior sets in rather slowly at $N_c \sim 20$, emphasizing the fact that the non commutativity of the low energy and $1/N_c$ expansions is very important at the physical $N_c = 3$. The results are shown in Table I, where the errors are estimated from the above mentioned theoretical uncertainty. The agreement with recent LQCD calculations [4] is encouraging, and further improvement in the precision of those calculations would be very useful.

| | $\frac{\delta f_1}{f_1}$ | |
|---|---|---|
| | One-loop | LQCD |
| $\Lambda p$ | $-0.067(15)$ | $-0.05(2)$ |
| $\Sigma^- n$ | $-0.025(10)$ | $-0.02(3)$ |
| $\Xi^- \Lambda$ | $-0.053(10)$ | $-0.06(4)$ |
| $\Xi^- \Sigma^0$ | $-0.068(17)$ | $-0.05(2)$ |

TABLE I: SU(3) breaking corrections to the $\Delta S = 1$ vector charges. The LQCD results are from Ref. [4].

For the charge radii the loop contributions are from diagrams $A_3$ and $B_2$ and the renormalization is provided by the LECs $g_1$ and $g_2$ in $\mathcal{L}_B^{(3)}$ and $\mathcal{L}_B^{(4)}$ respectively, of which only $g_1$ is required for canceling the loop UV divergence according to Eqn. (9) [2]. As is the case with form factors in ChPT, the charge radii depend logarithmically in the GB masses. They can be determined by fitting to the known electric charge radii of proton, neutron and $\Sigma^-$, or simply fixed using the first two. If one wishes to study also the large $N_c$ limit, an assignment at generic $N_c$ of the quark electric charges has to be done. One such an assignment that respects all gauge and gauge-gravitational anomaly cancellations in the Standard Model is is given by [39] $\hat{Q} = T^3 + \frac{1}{\sqrt{3}} T^8 + \frac{3-N_c}{6N_c} B$. The last term comes from the baryon number

[2] In Ref. [24] the finite term proportional to $g_2$ was overlooked.

charge $B$, and can be implemented by simply adding to the Lagrangians the corresponding terms with an $SU(3)$ singlet vector source field. This charge operator gives for the states identified with the physical octet and decuplet the same electric charges as the physical ones for any $N_c$. The analysis of the charge radii in the present framework is revealing: in the strict large $N_c$ limit one finds that the non-analytic loop contributions to the $T^3$ charge radius of nucleons by Diagram $A_3$ is $\mathcal{O}(N_c^0)$, where the contribution is driven by the hyperfine mass splitting term, i.e, for $C_{HF} \to 0$ the contribution becomes $\mathcal{O}(1/N_c)$, and Diagram $B_2$ gives only contributions $\mathcal{O}(1/N_c)$. For the charge $T^8$ the loop contributions are $\mathcal{O}(N_c^0)$. One however notes that for the physical $\pi$ and $K$ meson masses the non-analytic terms join the large $N_c$ scaling at rather large $N_c$. The charge radii of the neutral baryons receive only UV finite loop contributions and are renormalized only by the finite $g_2$ term.

Using the three known charge radii, $g_{1,2}$ are determined modulo the main uncertainty stemming from the value used for $\mathring{g}_A$. At the renormalization scale $\mu = m_\rho$, using the value of $\mathring{g}_A \sim 1$ obtained by the analysis of the axial couplings [24], $C_{HF} \sim 200$ MeV, and with $\Lambda = m_\rho$ one finds $g_1 \simeq 1.33$ and $g_2 \simeq 0.74$. $g_2$ is sensitive to $C_{HF}$, which is understood as a result that the non-analytic contributions to the neutron radius is very important, and thus sensitive to that parameter, while $g_1$ is not. One also observes that both LECs are crucial for obtaining a good description of the radii. For the used value of $\mu$, the fraction of the loop contribution to $\langle r^2 \rangle$ of the proton is 15%, and for the neutron it is about 60%. The short distance contributions are thus very important in both cases. The dominant non-analytic contributions to the radii are proportional to $\log m_q$, with other non-analytic terms involving the LEC $C_{HF}$ giving almost negligible contributions, making the results insensitive to it. Table II shows the results for the charge radii of the baryon octet along with the contributions by the CTs. The latter contributions to $\langle r^2 \rangle$ satisfy the exact linear relation, in obvious notation: $a\Lambda + p + \Sigma^+ + \frac{1}{3}(a-4)(n + \Sigma^0 + \Xi^0) + \Sigma^- + \Xi^- = 0$ valid for any $a$ and resulting from the electric charge being a U-spin singlet; it is violated only by finite $SU(3)$ breaking loop contributions. The isotriplet nucleon charge radius is $\mathcal{O}(N_c^0)$, while the isosinglet one receives loop and $g_2$ contributions $\mathcal{O}(N_c^0)$ and a $g_1$ contribution $\mathcal{O}(N_c)$, where the $\mathcal{O}(N_c)$ term contribution to the EM charge radius must be cancelled by adding to the Lagrangian a finite charge-radius CT proportional to baryon number and weighted according to the electric charge assignment at arbitrary $N_c$ mentioned above.

At the present order in the $\xi$ expansion, the curvature of the form factors, proportional

| | $\langle r^2\rangle[\mathrm{fm}^2]$ | | |
|---|---|---|---|
| | Full | CT | Exp |
| p | 0.707 | 0.596 | 0.7071(7) |
| n | −0.116 | −0.049 | −0.116(2) |
| $\Lambda$ | −0.029 | −0.024 | $\cdots$ |
| $\Sigma^+$ | 0.742 | 0.596 | $\cdots$ |
| $\Sigma^0$ | 0.029 | 0.024 | $\cdots$ |
| $\Sigma^-$ | 0.683 | 0.548 | 0.608(156) |
| $\Xi^0$ | −0.016 | −0.049 | $\cdots$ |
| $\Xi^-$ | 0.633 | 0.548 | $\cdots$ |

TABLE II: Electric charge radii of octet baryons. The proton and neutron radii are inputs. The proton radius used is the one resulting from the muonic Hydrogen Lamb shift [40]. The second column shows the contribution by contact terms $g_{1,2}$ for $\mu = m_\rho$.

to $\langle r^4\rangle = 60\frac{d^2 f_1}{d(Q^2)^2}$, is given by the one-loop non-analytic terms with contributions that are inversely proportional to quark masses. The curvature is nominally an effect $\mathcal{O}(\xi^4)$ in the form factor, which therefore receives contributions from terms $\mathcal{O}(\xi^6)$ in the Lagrangian, and only in the limit of sufficiently small quark masses will the non-analytic contributions obtained here be dominant. In the recent work of Ref. [38] the electric charge higher moments have been studied, where t-channel elastic unitarity has been implemented in the EFT along with the constraints of the $1/N_c$ expansion [34–38]. In particular, for the curvature they find $\langle r^4\rangle^p = 0.735(35)\,\mathrm{fm}^4$ and $\langle r^4\rangle^n = -0.540(35)\,\mathrm{fm}^4$, to be compared with the one-loop contributions found here, 0.032 and $-0.021\,\mathrm{fm}^4$ respectively, roughly a factor 25 smaller in magnitude in each case. Clearly the description of the curvature must be primarily given by higher order contact terms, and to the order of the expansion followed here, the failure to account for the curvature limits the present description of charge form factors to the expected range given by the radii, $Q^2 \lesssim 0.05$ GeV$^2$.

## V. MAGNETIC MOMENTS

As mentioned earlier, at lowest order the magnetic moments of all vector currents are given in terms of the single LEC $\kappa$. In particular, using the EM current the LO value of $\frac{\kappa}{\Lambda}$ can be fixed from the proton's magnetic moment $\mu_p$ in units of the nuclear magneton $\mu_N$, namely $e\,\frac{\kappa}{2\Lambda} = \mu_p = 2.7928\,\mu_N$. Also, the $M_1$ radiative decay width of the $\Delta$ at LO is given by:

$$\Gamma_{\Delta\to N\gamma} = \frac{e^2}{9\pi}\left(\frac{\kappa}{\Lambda}\right)^2 \frac{m_N}{m_\Delta}\omega^3, \tag{10}$$

where $\omega$ is the photon energy. Using the above result for $\frac{\kappa}{\Lambda}$ gives $\Gamma^{\rm LO}_{\Delta\to N\gamma} = 0.38$ MeV, to be compared with the experimental value $0.70 \pm 0.06$ MeV. In terms of the transition magnetic moment, the LO result is $\mu_{\Delta^+ p} = \frac{2\sqrt{2}}{3}\,\mu_p$ while the experimental one from Eqn.(10) and from the helicity $N-\Delta$ photo-couplings [41] are $3.58(10)\mu_N$ and $3.46(3)\mu_N$ respectively. This shows the need for a significant spin-symmetry breaking effect of 30% to be accounted for by the higher order corrections.

The LO magnetic moment operator $G^{ia}$ is proportional to the LO axial currents, and the NLO effects stem from quark masses and spin symmetry breaking. In the strict large $N_c$ limit those corrections scale as follows: $SU(3)$ breaking corrections $\mathcal{O}((m_s-\hat{m})N_c)$, i.e. the same scaling in $N_c$ as the LO term, and spin symmetry breaking corrections $\mathcal{O}(1/N_c)$, i.e. $\mathcal{O}(1/N_c^2)$ with respect to the LO term, well known from tree level analyses in Refs. [42, 43].

The experimentally available magnetic moment ratios and the corresponding LO results are represented in Table III. It is evident that there are significant $SU(3)$ breaking effects, which together with the important spin-symmetry breaking observed. in particular in the $\Delta N$ $\mathrm{M}_1$ amplitude indicate the relevance of the NNLO calculation. Note that all weak magnetic moments, i.e., magnetic moments associated with the $\Delta S = 1$ currents are also fixed at LO, as they are empirically unknown. In the case of the neutron $\beta$ decay the weak magnetic term is obtained from the isovector part of the EM magnetic moments of proton and neutron, which in this case, due to isospin symmetry, is quite accurate. On the other hand, in hyperon beta decay the effect of weak magnetism is too small to be at present experimentally accessible. Fortunately the advent of LQCD calculations of magnetic moments with increasing accuracy will allow the study of weak magnetism.

The one loop corrections to the magnetic moments are obtained from the spatial components of the vector currents depicted in Fig. 1, where the contributions stem from diagrams

| | Exp | LO |
|---|---|---|
| $p/n$ | $-1.46$ | $-1.5$ |
| $\Sigma^+/\Sigma^-$ | $-2.12$ | $-3$ |
| $\Lambda/\Sigma^+$ | $-0.25$ | $-\frac{1}{3}$ |
| $p/\Sigma^+$ | 1.14 | 1 |
| $\Xi^0/\Xi^-$ | 1.92 | 2 |
| $p/\Xi^0$ | $-2.23$ | $-1.5$ |
| $\Delta^{++}/\Delta^+$ | 1.4(2.8) | 2 |
| $\Omega^-/\Delta^+$ | $-0.75$ | $-1$ |
| $p/\Delta^+$ | 1.03 | 1 |
| $p/(\Delta^+ p)$ | 0.78 | $\frac{3}{2\sqrt{2}}$ |
| $p/(\Sigma^{*0}\Lambda)$ | 1.02 | $\sqrt{\frac{3}{2}}$ |
| $p/(\Sigma^{*+}\Sigma^+)$ | $-0.88$ | $-\frac{3}{2\sqrt{2}}$ |

TABLE III: LO ratios of magnetic moments.

$A$ and $B_1$. Diagrams $A_{1,2}$ involve $\Gamma \propto G^{ia}$, which is similar to the axial currents already analyzed in Ref. [24]. The loop contributions to the $Q^2$ dependence of the magnetic form factors stem from diagram $A_3$.

The UV divergencies of the one loop diagrams contributing to the magnetic moments after reduction of the corresponding expressions Eqns.(7) and (8) using a basis of spin-flavor operators read as follows:

$$\begin{aligned}
V^{\mu a}_{Mag}(A_{1+2})^{UV} &= i\frac{\lambda_\epsilon}{(4\pi)^2}\frac{\kappa}{2\Lambda}\left(\frac{\mathring{g}_A}{F_\pi}\right)^2\epsilon^{ijk}q^j\left(-B_0\Big(\frac{23}{6}m^0G^{ka}+\frac{11}{24}d^{abc}m^bG^{kc}+\frac{5}{18}m^aS^k\Big)\right.\\
&\quad\left.+\ \frac{2}{3}\left(\frac{C_{HF}}{N_c}\right)^2\Big((N_c(N_c+6)-3)G^{ka}+8\{\hat{S}^2,G^{ka}\}+8S^kS^mG^{ma}-\frac{11}{2}(N_c+3)S^kT^a\Big)\right)\\
V^{\mu a}_{Mag}(A_3)^{UV} &= i\frac{\lambda_\epsilon}{(4\pi)^2}\left(\frac{\mathring{g}_A}{F_\pi}\right)^2\frac{C_{HF}}{N_c}\epsilon^{ijk}q^j\left(\frac{N_c+3}{2}G^{ka}-2S^kT^a\right)\\
V^{\mu a}_{Mag}(B_1)^{UV} &= -i\frac{\lambda_\epsilon}{(4\pi)^2}\frac{\kappa}{2\Lambda}\frac{1}{F_\pi^2}\epsilon^{ijk}q^jB_0\left(6\,m^0G^{ka}+\frac{3}{2}d^{abc}m^bG^{kc}\right),
\end{aligned}\tag{11}$$

adding up to:

$$
\begin{aligned}
V_{Mag}^{UV\mu a} &= \frac{i\lambda_\epsilon q^j \epsilon^{ijk}}{16\pi^2 {F_\pi}^2 \Lambda} \Big( -\frac{1}{12}\kappa B_0 \Big( (\frac{11}{4}\mathring{g}_A^2 + 9) m^b G^{kc} d^{abc} + \big(23\mathring{g}_A^2 + 36\big) m^0 G^{ka} + \frac{5}{3}\mathring{g}_A^2 m^a S^k \Big) \\
&+ \frac{C_{HF}\mathring{g}_A^2}{6{N_c}^2} \Big( 2\,\kappa\, C_{HF}((N_c(N_c+6)-3)G^{ka} + 8\{\hat{S}^2, G^{ka}\}) + 3\Lambda N_c(N_c+3)G^{ka} \\
&+ 16\,\kappa\, S^m G^{ma} S^k - S^k T^a (11\,\kappa\, C_{HF}(N_c+3) + 12\Lambda N_c) \Big) \Big) \qquad (12)
\end{aligned}
$$

The renormalization of the magnetic moments is provided by the Lagrangians with the LECs $\kappa_{D,F,1,\cdots,5}$, and the magnetic radii receive only finite one-loop contributions and a finite renormalization by the term $\kappa_r$. The $\beta$ functions of the magnetic LECs resulting from Eqn.(12) are shown in Table IV.

| *LEC* | $\beta \times F_\pi^2$ |
|---|---|
| $\kappa$ | $\Lambda\,\mathring{g}_A^2\,\frac{C_{HF}}{N_c}\left(\frac{1}{2}(N_c+3) + \frac{1}{3}(N_c(N_c+6)-3)\frac{\kappa}{\Lambda}\frac{C_{HF}}{N_c}\right)$ |
| $\kappa_1$ | $-\Lambda\,\mathring{g}_A^2\,C_{HF}\left(2 + \frac{11}{6}(N_c+3)\frac{\kappa}{\Lambda}\frac{C_{HF}}{N_c}\right)$ |
| $\kappa_2$ | $-\Lambda^2\,\kappa\,\left(3 + \frac{23}{12}\,\mathring{g}_A^2\right)$ |
| $\kappa_D$ | $-\Lambda^2\,\kappa\,\left(\frac{3}{4} + \frac{11}{48}\,\mathring{g}_A^2\right)$ |
| $\kappa_F$ | $0$ |
| $\kappa_3$ | $-\Lambda^2\,\kappa\,\frac{5}{36}\,\mathring{g}_A^2$ |
| $\kappa_4$ | $\frac{8}{3}\mathring{g}_A^2\,\kappa\,C_{HF}^2$ |
| $\kappa_5$ | $\frac{8}{3}\mathring{g}_A^2\,\kappa\,C_{HF}^2$ |
| $\kappa_r$ | $0$ |

TABLE IV: $\beta$ functions of LECs associated with magnetic moments and radii. The renormalized LECs are defined according to $X = X(\mu) + \frac{\beta_X}{(4\pi)^2}\lambda_\epsilon$.

For $N_c = 3$ the set of local terms that contribute to the magnetic moments remains linearly independent. If one only considers the EM current, the term proportional to $\kappa_F$ does not contribute, and for the known magnetic moments together with the information on the $M_1$ transition $\Delta \to N\gamma$ one can fit the rest of the LECs. Note that in the absence of information on the $SU(3)$ singlet quark mass $m^0$ dependence, the LEC $\kappa_2$ is subsumed into $\kappa$, and the lack of knowledge on the $\Delta S = 1$ weak magnetic moments does prevents at present a determination of $\kappa_F$.

The results of the fits are shown in Table V. Since the input magnetic moments have errors (much) smaller than the theoretical error of the present calculation estimated to be of the order of NNNLO corrections or about 5%, the $\chi^2$ has been normalized for estimating the LECs' errors. Important correlation is found between the following pairs of LECs: $\kappa_4 - \kappa_5$, $\kappa_4 - \kappa_6$ and $\kappa_5 - \kappa_6$.

| LEC$\times\frac{m_N}{\Lambda}$ | LO | NNLO |
|---|---|---|
| $\kappa$ | 2.80 | 2.87(2) |
| $\kappa_1$ | 0 | 3.18 (10) |
| $\kappa_2$ | 0 | 0. |
| $\kappa_D$ | 0 | 0.46 (5) |
| $\kappa_F$ | 0 | $\cdots$ |
| $\kappa_3$ | 0 | 0.51(6) |
| $\kappa_4$ | 0 | −2.84(40) |
| $\kappa_5$ | 0 | 1.19(20) |

| | $\mu_{LO}$ | $\mu_{NNLO}$ | $\mu_{Exp}$ |
|---|---|---|---|
| p | 2.691 | 2.797 | 2.7928(23) |
| n | −1.794 | −1.929 | −1.9130(45) |
| $\Sigma^+$ | 2.691 | 2.359 | 2.46(1) |
| $\Sigma^0$ | 0.897 | 0.834 | $\cdots$ |
| $\Sigma^-$ | −0.897 | −0.691 | −1.16(3) |
| $\Lambda$ | −0.897 | −0.595 | −0.613(4) |
| $\Xi^0$ | −1.794 | −1.245 | −1.250(14) |
| $\Xi^-$ | −0.897 | −0.657 | −0.6507(25) |
| $\Delta^+ p$ | 2.537 | 3.580 | 3.58(10) |
| $\Sigma^0\Lambda$ | 1.553 | 1.562 | 1.61(8) |
| $\Sigma^{*0}\Lambda$ | 2.197 | 2.685 | 2.73(25)[a] |
| $\Sigma^{*+}\Sigma^+$ | −2.537 | −2.326 | −3.17(36)[b] |

| | $\mu_{LO}$ | $\mu_{NNLO}$ | $\mu_{Exp}$ |
|---|---|---|---|
| $\Delta^{++}$ | 5.381 | 5.979 | 3.7 − 7.5 |
| $\Delta^+$ | 2.691 | 3.027 | 2.7(1.2) |
| $\Delta^0$ | 0 | 0.074 | $\cdots$ |
| $\Delta^-$ | −2.691 | −2.879 | $\cdots$ |
| $\Sigma^{*+}$ | 2.691 | 3.163 | $\cdots$ |
| $\Sigma^{*0}$ | 0 | 0.315 | $\cdots$ |
| $\Sigma^{*-}$ | −2.691 | −2.534 | $\cdots$ |
| $\Xi^{*0}$ | 0 | 0.496 | $\cdots$ |
| $\Xi^{*-}$ | −2.691 | −2.242 | $\cdots$ |
| $\Omega$ | −2.691 | −2.005 | −2.02(5) |

TABLE V: Results from fits to the electric current magnetic moments, in units of the nuclear magneton $\mu_N$. The renormalization scale was set to $\mu = \Lambda = m_\rho$. $\kappa_F$ requires $\Delta S = 1$ weak magnetic moments to be determined. Empirical results from PDG and references [a][44], [b][45].

As mentioned earlier, the $\Delta N\gamma$ amplitude at LO is too small by roughly 30%, a manifestation of an important spin-symmetry breaking effect. The effect receives a small non-analytic contribution (at $\mu = m_\rho$), and the contributions from the contact terms are as follows: $\kappa_D : \mathcal{O}((m_s - \hat{m})N_c)$, and $\kappa_4 : \mathcal{O}(1/N_c)$. From the fit one finds a modest contribution from $\kappa_D$ and a dominant contribution from $\kappa_4$. Since the latter is a $1/N_c^2$ correction with respect to the LO magnetic moment, it seems to be unnaturally large. This is a bit surprising as a similar kind of effect in the $\Delta N$ axial vector coupling is actually unnaturally small. This contrast remains to be understood. Finally, a fit where the $\Delta N$ transition is not an input shows an enhancement but only by about half of what is needed.

An interesting case is the magnetic moment of $\Sigma^{*0}$: all LO and NLO tree level and quark mass independent contributions vanish, receiving only NNLO tree and loop contributions which vanish in the SU(3) symmetry limit. On the other hand, the experimental value of the magnetic moment of $\Sigma^-$ quoted as average by the PDG [40] cannot be described: U-spin symmetry implies that it must be equal to the magnetic moment of the $\Xi^-$ up to NNLO $SU(3)$ breaking by quark masses. The experimental results imply a very large effect which is very difficult to reconcile with the other U-spin multiplets, where the effect is between 12% and 25% per unit of strangeness, while for the pair $\Sigma^-$ $\Xi^-$ case it is 44%!.

One of the early tests of the magnetic moments in $SU(3)$ was provided by the Coleman-Glashow (CG) relation, namely $\mu_p - \mu_n - \mu_{\Sigma^+} + \mu_{\Sigma^-} + \mu_{\Xi^0} - \mu_{\Xi^-} = 0$. This relation remains valid at tree level NNLO and receives only a finite correction from the one loop contributions. Explicit calculation gives the deviation with estimated theoretical error $\Delta_{CG} = 1.09 \pm 0.25\,\mu_N$ to be compared with the experimental deviation $0.49 \pm 0.03\,\mu_N$, affected however by the $\Sigma^-$ issue.

Finally, the weak interaction magnetic moments for hyperon decays turn out to depend on the LEC $\kappa_F$ which does not appear in the EM case. The result for the LECs from the EM case gives the predictions: $\mu_{\Sigma^- n} = (0.516 - 0.180\,\kappa_F)\frac{g}{2m_N}$ and $\mu_{\Lambda p} = (-1.41 + 0.66\,\kappa_F)\frac{g}{2m_N}$, where $g = e/\sin\theta_W$. At LO one has the large hierarchy $\mu_{\Lambda p}/\mu_{\Sigma^- n} = -\sqrt{27/2}$. A determination of $\kappa_F$ will require a LQCD calculation.

### A. Magnetic radii

The magnetic radii are theoretically very constrained at the order of the present calculation. For all the vector currents and baryons they are determined only by UV finite loop contributions and the single available finite counterterm fixed by the LEC $\kappa_r$. Since only the magnetic radii of proton and neutron are experimentally known, one can use these to fit that LEC leading to the results shown in Table VI. The rest of the radii are then predictions which can hopefully be tested in the future with LQCD calculations. Note that the lion share of the magnetic radii is from the short distance terms proportional to $\kappa_r$ with the loop contribution from diagram $A_3$ in Fig. 1 giving up to 20% for proton, neutron and $\Sigma^-$ and less than 10% for the rest.

| $\kappa_r = -2.63$ | | $\langle r^2 \rangle$[fm$^2$] | |
|---|---|---|---|
| | Exp | Th | Loop |
| p | 0.724 | 0.718 | 0.134 |
| n | 0.746 | 0.747 | 0.179 |
| $\Sigma^+$ | $\cdots$ | 0.766 | 0.100 |
| $\Sigma^0$ | $\cdots$ | 0.698 | 0.061 |
| $\Sigma^-$ | $\cdots$ | 0.922 | 0.189 |
| $\Lambda$ | $\cdots$ | 0.895 | 0.079 |
| $\Xi^0$ | $\cdots$ | 0.872 | 0.081 |
| $\Xi^-$ | $\cdots$ | 0.796 | 0.035 |
| $\Delta^+ p$ | $\cdots$ | 0.875 | 0.226 |

TABLE VI: Magnetic radii from a fit to nucleons.

Finally, a calculation of the curvature of the EM magnetic moments yields: $\langle r^4 \rangle^p = 0.38\,\text{fm}^4$ and $\langle r^4 \rangle^n = 0.54\,\text{fm}^4$ to be compared with those obtained in Ref. [38], which are respectively 1.72(6) and 2.04(1) fm$^4$, leading to a similar assessment as in the case of the electric charge already discussed, although less dramatic.

## VI. SUMMARY

This work presented the study of the $SU(3)$ vector currents in baryons based on the combined chiral and $1/N_c$ expansion. It was carried out in the context of the $\xi$ power counting to one-loop. This corresponds to a calculation of the charges, magnetic moments and their radii for both octet and decuplet baryons. The calculations have been provided for generic $N_c$, which permits an exploration of the behavior of those observables with respect to the number of colors. Only two LECs are needed to determine all $SU(3)$ charge radii, while the magnetic moments need to be renormalized involving eight LECs, of which all but two can be fixed solely in terms of the known EM magnetic moments. Of the two remaining LECs, one needs information about $\Delta S = 1$ weak magnetic moments and the second requires knowledge of magnetic moments at different values of quark masses, which can be obtained from LQCD calculations. Finally the magnetic radii are all determined in terms of a single LEC. The fits indicate that the values LECs are within the range of natural magnitude, although there is a puzzling issue, namely the unnaturally large spin-symmetry breaking required for the description of the $\Delta N$ transition magnetic moment. Finally, the curvature of form factors is given at the order of the calculation by non-analytic terms in $m_q$, which turn out to be very small, and therefore requiring for their description an extension of the present work to higher order.

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### Appendix A: Spin-flavor algebra

The $4N_f^2-1$ generators of the spin-flavor group $SU(2N_f)$ consist of the three spin generators $S^i$, the $N_f^2-1$ flavor $SU(N_f)$ generators $T^a$, and the remaining $3(N_f^2-1)$ spin-flavor generators $G^{ia}$. The commutation relations are:

$$\begin{aligned}
[S^i,S^j]=i\epsilon^{ijk}S^k,\quad [T^a,T^b]=if^{abc}T^c,\quad [T^a,S^i]=0 \quad &,\\
[S^i,G^{ja}]=i\epsilon^{ijk}G^{ka},\quad [T^a,G^{ib}]=if^{abc}G^{ic} \quad &,\\
[G^{ia},G^{jb}]=\tfrac{i}{4}\delta^{ij}f^{abc}T^c+\tfrac{i}{2N_f}\delta^{ab}\epsilon^{ijk}S^k+\tfrac{i}{2}\epsilon^{ijk}d^{abc}G^{kc} \quad &.
\end{aligned} \tag{A1}$$

In spin-flavor representations with $N_c$ indices corresponding to baryons, the generators $G^{ia}$ have matrix elements $\mathcal{O}(N_c)$ on states with $S=\mathcal{O}(N_c^0)$. The ground state baryons furnish the totally symmetric irreducible representation of $SU(6)$ with $N_c$ Young boxes, which decomposes into the following $SU(2)_{\rm spin}\times SU(3)$ irreducible representations: $[S,(p,q)]=[S,(2S,\frac{1}{2}(N_c-2S))]$, $S=1/2,\cdots,N_c/2$ (assumed $N_c$ is odd). The baryon states can then be denoted by: $|SS_3,YII_3\rangle$, where the spin $S$ of the baryon determines its $SU(3)$ multiplet.

#### 1. Matrix elements of the SU(6) generators

In general the matrix elements of a $SU(2)_{\rm spin}\times SU(3)\subset SU(6)$ tensor operator between baryons ground state baryons are given by the Wigner-Eckart theorem, with obvious notation:

$$\begin{aligned}
\langle S'S_3',R'\,Y'I'I_3'\mid O^{\ell\ell_3}_{\tilde R\tilde Y\tilde I\tilde I_3}\mid SS_3,R\,YII_3\rangle \;=\; \frac{1}{\sqrt{2S'+1}\sqrt{{\rm dim}R'}}\,\langle SS_3,\ell\ell_3\mid S'S_3'\rangle \\
\times\sum_\gamma \langle S',R'\,||\,O^{\ell}_{\tilde R}\,||\,S,R\rangle_\gamma \quad \left\langle \begin{matrix} R & \tilde R \\ Y\;I\;I_3 & \tilde Y\;\tilde I\;\tilde I_3 \end{matrix}\right| \left.\begin{matrix} R' \\ Y'\;I'\;I_3'\end{matrix}\right\rangle_\gamma \quad ,
\end{aligned} \tag{A2}$$

where $R$ represents the $SU(3)$ multiplet of the baryon, and $\gamma$ indicates the possible recouplings in $SU(3)$. The matrix elements of interest are then given by:

$$
\begin{aligned}
\langle S'S'_3, Y'I'I'_3 \mid S^m \mid SS_3, YII_3\rangle &= \delta_{SS'}\delta_{YY'}\delta_{II'}\delta_{I_3I'_3}\sqrt{S(S+1)}\langle SS_3, 1m \mid S'S'_3\rangle \\
\langle S'S'_3, Y'I'I'_3 \mid T^{yii_3} \mid SS_3, YII_3\rangle &= \delta_{SS'}\delta_{S_3S'_3}\frac{1}{\sqrt{\dim(2S,\frac{1}{2}(N_c-2S))}}\langle S \mid\mid T \mid\mid S\rangle \\
&\times \left\langle \begin{matrix} (2S,\frac{1}{2}(N_c-2S)) & (1,1) \\ Y\ I\ I_3 & yii_3 \end{matrix} \right| \left. \begin{matrix} (2S,\frac{1}{2}(N_c-2S)) \\ Y'\ I'\ I'_3 \end{matrix} \right\rangle_{\gamma=1} \\
\langle S'S'_3, Y'I'I'_3 \mid G^{m,yii_3} \mid SS_3, YII_3\rangle &= \frac{\langle SS_3, 1m \mid S'S'_3\rangle}{\sqrt{2S'+1}\sqrt{\dim(2S,\frac{1}{2}(N_c-2S))}} \\
&\times \sum_{\gamma=1,2}\langle S' \mid\mid G \mid\mid S\rangle_\gamma \left\langle \begin{matrix} (2S,\frac{1}{2}(N_c-2S)) & (1,1) \\ Y\ I\ I_3 & yii_3 \end{matrix} \right| \left. \begin{matrix} (2S,\frac{1}{2}(N_c-2S)) \\ Y'\ I'\ I'_3 \end{matrix} \right\rangle_{\gamma}
\end{aligned} \tag{A3}
$$

where the reduced matrix elements are (here $p = 2S$, $q = \frac{1}{2}(N_c - 2S)$):

$$
\begin{aligned}
\langle S \mid\mid T \mid\mid S\rangle &= \sqrt{\dim(p,q)C_2(p,q)} \\
&= \frac{\sqrt{(2S+1)(N_c-2S+2)(N_c+2S+4)(N_c(N_c+6)+12S(S+1))}}{4\sqrt{6}} \\
\langle S' \mid\mid G \mid\mid S\rangle_{\gamma=1} &= \begin{cases} \text{if} \quad S = S'+1: & -\frac{\sqrt{(4S^2-1)((N_c+2)^2-4S^2)((N_c+4)^2-4S^2)}}{8\sqrt{2}} \\ \text{if} \quad S = S'-1: & -\frac{\sqrt{(4S(S+2)+3)(N_c-2S)(N_c-2S+2)(N_c+2S+4)(N_c+2S+6)}}{8\sqrt{2}} \\ \text{if} \quad S = S': & \mathrm{sign}(N_c-2S-0^+)\frac{(N_c+3)(2S+1)\sqrt{S(S+1)(N_c-2S+2)(N_c+2S+4)}}{\sqrt{6N_c(N_c+6)+12S(S+1)}} \end{cases} \\
\langle S' \mid\mid G \mid\mid S\rangle_{\gamma=2} &= -\delta_{SS'}\frac{(2S+1)\sqrt{(N_c-2S)(N_c+2S+6)\left((N_c+2)^2-4S^2\right)\left((N_c+4)^2-4S^2\right)}}{8\sqrt{2}\sqrt{N_c(N_c+6)+12S(S+1)}}
\end{aligned} \tag{A4}
$$

**Appendix B: Loop integrals**

The one-loop integrals needed in this work are provided here. The definition $\widetilde{d^dk} \equiv d^dk/(2\pi)^d$ is used.

The scalar and tensor one-loop integrals are:

$$
\begin{aligned}
I(n,\alpha,\Lambda) \equiv \int \widetilde{d^dk}\, \frac{k^{2n}}{(k^2-\Lambda^2)^\alpha} &= i(-1)^{n-\alpha}\frac{1}{(4\pi)^{\frac{d}{2}}}\frac{\Gamma(n+\frac{d}{2})\Gamma(\alpha-n-\frac{d}{2})}{\Gamma(\frac{d}{2})\Gamma(\alpha)}\left(\Lambda^2\right)^{n-\alpha+\frac{d}{2}} \\
I^{\mu_1,\cdots,\mu_{2n}}(\alpha,\Lambda) \equiv \int \widetilde{d^dk}\, \frac{k_{\mu_1}\cdots k_{\mu_{2n}}}{(k^2-\Lambda^2)^\alpha} &= i(-1)^{n-\alpha}\frac{1}{(4\pi)^{\frac{d}{2}}}\frac{1}{4^n n!}\frac{\Gamma(\alpha-n-\frac{d}{2})}{\Gamma(\alpha)}\left(\Lambda^2\right)^{n-\alpha+\frac{d}{2}} \\
&\times \sum_\sigma g_{\mu_{\sigma_1}\mu_{\sigma_2}}\cdots g_{\mu_{\sigma_{2n-1}}\mu_{\sigma_{2n}}} \qquad \text{(B1)} \\
&= \frac{1}{4^n n!}\frac{\Gamma(\frac{d}{2})}{\Gamma(n+\frac{d}{2})} I(n,\alpha,\Lambda)\sum_\sigma g_{\mu_{\sigma_1}\mu_{\sigma_2}}\cdots g_{\mu_{\sigma_{2n-1}}\mu_{\sigma_{2n}}} \quad ,
\end{aligned}
$$

where $\sigma$ are the permutations of $\{1,\cdots,2n\}$.

The Feynman parametrizations needed when heavy propagators are in the loop are as follows:

$$
\begin{aligned}
\frac{1}{A_1\cdots A_m B_1\cdots B_n} &= 2^m\Gamma(m+n)\int_0^\infty d\lambda_1\cdots d\lambda_m\int_0^1 d\alpha_1\cdots d\alpha_n\delta(1-\alpha_1-\cdots-\alpha_n) \\
&\times \frac{1}{(2\lambda_1 A_1+\cdots+2\lambda_m A_m+\alpha_1 B_1+\cdots+\alpha_n B_n)^{m+n}}, \qquad \text{(B2)}
\end{aligned}
$$

where the $A_i$ are heavy particle static propagators denominators, and the $B_i$ are relativistic ones.

The integration over a Feynman parameter $\lambda$ is of the general form:

$$
J(C_0,C_1,\lambda_0,d,\nu) \equiv \int_0^\infty (C_0+C_1(\lambda-\lambda_0)^2)^{-\nu+\frac{d}{2}}d\lambda, \qquad \text{(B3)}
$$

which satisfies the recurrence relation:

$$
\begin{aligned}
J(C_0,C_1,\lambda_0,d,\nu) &= \frac{-\lambda_0(C_0+C_1\lambda_0^2)^{1-\nu+\frac{d}{2}}+(3+d-2\nu)J(C_0,C_1,\lambda_0,d,\nu-1)}{(d-2\nu+2)C_0} \\
J(C_0,C_1,\lambda_0,d,\nu) &= C_0\frac{d-\nu}{d-2\nu+1}J(C_0,C_1,\lambda_0,d,\nu+1)+\frac{\lambda_0}{d-2\nu+1}(C_0+C_1\lambda_0^2)^{\frac{d}{2}-\nu}. \qquad \text{(B4)}
\end{aligned}
$$

Integrals with factors of $\lambda$ in the numerator are obtained by using

$$
\begin{aligned}
J(C_0,C_1,\lambda_0,d,\nu,n=1) &\equiv \int_0^\infty (\lambda-\lambda_0)^{n=1}(C_0+C_1(\lambda-\lambda_0)^2)^{-\nu+\frac{d}{2}}d\lambda \\
&= -\frac{1}{2\,C_1\,(\frac{d}{2}+1-\nu)}(C_0+C_1\lambda_0^2)^{\frac{d}{2}+1-\nu}, \qquad \text{(B5)}
\end{aligned}
$$

and the recurrence relations

$$
J(C_0,C_1,\lambda_0,d,\nu,n) = \frac{1}{C_1}(J(C_0,C_1,\lambda_0,d,\nu-1,n-1)-C_0J(C_0,C_1,\lambda_0,d,\nu,n-2)). \qquad \text{(B6)}
$$

For convenience in some of the calculations for the currents, the following integral is defined:

$$
\tilde{J}(C_0,C_1,\lambda_0,d,\nu,1) \equiv J(C_0,C_1,\lambda_0,d,\nu,1)+\lambda_0 J(C_0,C_1,\lambda_0,d,\nu) \qquad \text{(B7)}
$$

For the calculations in this work the following integrals are needed at $d=4-2\epsilon$:

$$
\begin{aligned}
J(C_0,C_1,\lambda_0,d,3) &= \frac{1}{\sqrt{C_0C_1}}\left(\frac{\pi}{2}+\arctan(\lambda_0\sqrt{\frac{C_1}{C_0}})\right) \\
J(C_0,C_1,\lambda_0,d,2) &= \frac{1}{d-3}(\lambda_0(C_0+C_1\lambda_0^2)^{\frac{d}{2}-2}+(d-4)C_0J(C_0,C_1,\lambda_0,d,3)) \\
J(C_0,C_1,\lambda_0,d,1) &= \frac{1}{d-1}(\lambda_0(C_0+C_1\lambda_0^2)^{\frac{d}{2}-1}+(d-2)J(C_0,C_1,\lambda_0,d,2)) \qquad \text{(B8)}
\end{aligned}
$$

**Specific integrals**

Here a summary of relevant one-loop integrals for the calculations in this work is provided for the convenience of the reader.

1) Loop integrals involving only relativistic propagators

$$
\begin{aligned}
I(0,1,M) &= -\frac{i}{(4\pi)^{\frac{d}{2}}}\Gamma(1-\frac{d}{2})M^{d-2} \\
I(0,2,M) &= \frac{i}{(4\pi)^{\frac{d}{2}}}\Gamma(2-\frac{d}{2})M^{d-4} \\
K(q,M_a,M_b) &\equiv \int \widetilde{d^dk}\frac{1}{(k^2-M_a^2+i\epsilon)((k+q)^2-M_b^2+i\epsilon)} = \int_0^1 d\alpha\ I(0,2,\Lambda(\alpha)) \\
K^\mu(q,M_a,M_b) &\equiv \int \widetilde{d^dk}\frac{k^\mu}{(k^2-M_a^2+i\epsilon)((k+q)^2-M_b^2+i\epsilon)} = \int_0^1 d\alpha\ (\alpha-1)\, q^\mu\, I(0,2,\Lambda(\alpha)) \\
K^{\mu\nu}(q,M_a,M_b) &\equiv \int \widetilde{d^dk}\frac{k^\mu k^\nu}{(k^2-M_a^2+i\epsilon)((k+q)^2-M_b^2+i\epsilon)} \\
&= \int_0^1 d\alpha\ ((1-\alpha)^2\, q^\mu q^\nu\, I(0,2,\Lambda(\alpha)) + \frac{g^{\mu\nu}}{d}I(1,2,\Lambda(\alpha))), \qquad \text{(B9)}
\end{aligned}
$$

where:

$$
\Lambda(\alpha) = \sqrt{\alpha M_a^2 + (1-\alpha)M_b^2 - \alpha(1-\alpha)q^2}
$$

2) Loop integrals involving one heavy propagator

$$
\begin{aligned}
H(p_0,M) &\equiv \int \widetilde{d^dk}\frac{1}{(p_0-k_0+i\epsilon)(k^2-M^2+i\epsilon)} \\
&= \frac{2i}{(4\pi)^{\frac{d}{2}}}\Gamma(2-\frac{d}{2})J(M^2-p_0^2,1,p_0,d,2) \\
H^{ij}(p_0,M) &\equiv \int \widetilde{d^dk}\frac{k^ik^j}{(p_0-k_0+i\epsilon)(k^2-M^2+i\epsilon)} \\
&= -\frac{i}{(4\pi)^{\frac{d}{2}}}g^{ij}\Gamma(1-\frac{d}{2})J(M^2-p_0^2,1,p_0,d,1) \qquad \text{(B10)} \\
H^{ij\mu}(p_0,q,M_a,M_b) &\equiv \int \widetilde{d^dk}\frac{k^i(k+q)^j(2k+q)^\mu}{(p_0-k_0+i\epsilon)(k^2-M_a^2+i\epsilon)((k+q)^2-M_b^2+i\epsilon)} \\
&= i\frac{4}{(4\pi)^{\frac{d}{2}}}\int_0^1 d\alpha \left\{ -\frac{1}{2}\Gamma(3-\frac{d}{2})q^iq^j\alpha(1-\alpha)\right. \\
&\times \Big((1-2\alpha)q^\mu J(C_0,C_1,\lambda_0,d,3) - 2\,g^{\mu 0}\tilde{J}(C_0,C_1,\lambda_0,d,3,1)\Big) \\
&+ \Gamma(2-\frac{d}{2})\,\Big((-(1-2\alpha)g^{ij}q^\mu + 2(\alpha g^{\mu i}q^j - (1-\alpha)g^{\mu j}q^i))J(C_0,C_1,\lambda_0,d,2) \\
&+ \left. 2g^{ij}g^{\mu 0}\tilde{J}(C_0,C_1,\lambda_0,d,2,1)\Big)\right\},
\end{aligned}
$$

where:

$$
\begin{aligned}
C_0 &= \alpha M_a^2 + (1-\alpha)M_b^2 - p_0^2 - 2(1-\alpha)p_0 q_0 - (1-\alpha)(\alpha\, q^2 + (1-\alpha)q_0^2) \\
C_1 &= 1 \\
\lambda_0 &= p_0 + (1-\alpha)q_0.
\end{aligned}
\tag{B11}
$$

The polynomial pieces of the integrals are as follows:

$$
\begin{aligned}
H(p_0, M)^{\text{poly}} &= \frac{i}{(4\pi)^2} 2p_0(\lambda_\epsilon + 2) \\
H^{ij}(p_0, M)^{\text{poly}} &= \frac{i}{(4\pi)^2} \frac{p_0}{3}((3M^2 - 2p_0^2)\lambda_\epsilon + 7M^2 - \frac{16}{3}p_0^2) \\
H^{ij\mu}(p_0, q, M_a, M_b)^{\text{poly}} &= \frac{i}{96\pi^2}\left(\lambda_\epsilon \left(g^{ij}\left(g^{\mu 0}\left(-3(M_a^2 + M_b^2) + 12\, p_0(p_0+q_0) + q^2 + 4q_0^2\right) - q_0 q^\mu\right)\right.\right. \\
&- 2q^i(3p_0 + 2q_0)g^{\mu j} + 2q^j((3p_0+q_0)g^{\mu i} + q^i g^{\mu 0})) \\
&+ g^{ij}\left(g^{\mu 0}\left(-3(M_a^2 + M_b^2) + 24\, p_0(p_0+q_0) + q^2 + 8q_0^2\right) - 2q_0 q^\mu\right) \\
&- \left. 4q^i(3p_0 + 2q_0)g^{\mu j} + 4q^j(3p_0 + q_0)g^{\mu i}\right),
\end{aligned}
\tag{B12}
$$

where the UV divergence is given by the terms proportional to $\lambda_\epsilon \equiv 1/\epsilon - \gamma + \log 4\pi$, where $d = 4 - 2\epsilon$.

## Appendix C: Interaction and vector current vertices needed in loop calculations

The interaction and currents vertices needed in the one-loop calculations are given for completeness.

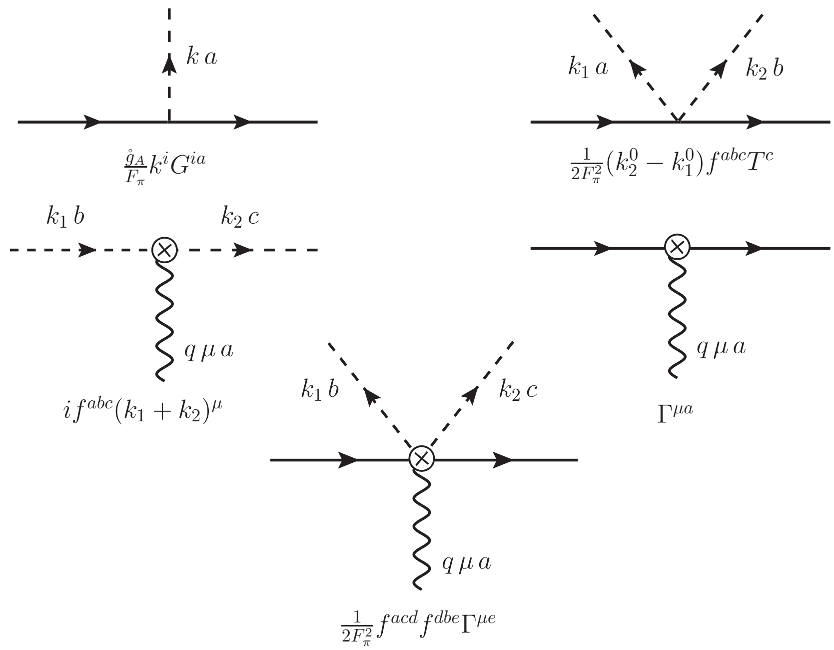


FIG. 2: The vector current vertices indicated with a square are the magnetic ones. The momentum $q$ is incoming, and $\Gamma^{\mu a} = g^{\mu 0} T^a + i\frac{\kappa}{\Lambda}\,\epsilon^{0\mu ij}\, f^{abc} f^{cbd}\, q^i\, G^{jd}$.


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