# Comparative analysis of $\omega p, \phi p$, and $J / \psi p$ scattering lengths from A2, CLAS, and GlueX threshold measurements 

Igor Strakovsky, ${ }^{1, *}$ Lubomir Pentchev, ${ }^{2}$ and Alexander Titov ${ }^{3}$<br>${ }^{1}$ Institute for Nuclear Studies, Department of Physics, The George Washington University, Washington, DC 20052, USA<br>${ }^{2}$ Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA<br>${ }^{3}$ Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna 141980, Russia<br>(Dated: January 23, 2020)


#### Abstract

The high accuracy $\phi$-meson photoproduction data from the CLAS experiment in Hall B of Jefferson Laboratory allow us to determine the near-threshold total cross section of the $\gamma p \rightarrow \phi p$ reaction and use it for evaluating the $\phi p$ scattering length $\alpha_{\phi p}$. These data result in an absolute value of $\left|\alpha_{\phi p}\right|=(0.063 \pm 0.010) \mathrm{fm}$, which is smaller than the typical hadron size. A comparative analysis of $\alpha_{\phi p}$ with the previously determined scattering lengths for $\omega p$ and $J / \psi p$ from the A2 and GlueX experiments is performed.


[^0]Since the discovery of the vector mesons $\left(\rho(770)\right.$ in 1961 [1], $\omega(782)$ in 1961 [2], $K^{\star}(892)$ in 1961 [3], $\phi(1020)$ in $1962[4,5], J / \psi(1 S)$ in $1974[6,7], D^{\star}$ in $1976[8,9]$, and $\Upsilon(1 S)$ in $\left.1977[10]\right)$, they became attractive probes for the investigation of different aspects of the properties of hadronic matter and hadronic interactions. In particular, exclusive vector-meson photoproduction allows for the study of vector meson - proton scattering and the evaluation of the corresponding scattering lengths $\alpha_{V p}$, which may serve as a unique input for QCD-motivated models of vector meson-nucleon interactions. The absolute value of the scattering length may be determined from the near threshold vector-meson photoproduction total cross section by making use of the vector-meson dominance (VMD) model [11]. VMD assumes that a real photon can fluctuate into a virtual vector meson, which subsequently scatters off the target proton [12]. This method was used for the determination of the $\omega p$ and $J / \psi p$ scattering lengths [13, 14].

Recently, the CLAS Collaboration reported the first differential cross section measurements for the exclusive reaction $\gamma p \rightarrow \phi p$ near threshold [15]. This is a unique experiment that measured the differential cross sections from threshold at a photon energy of $E_{\gamma}=1.63 \mathrm{GeV}$ up to 2.82 GeV . The CLAS experiment used tagged real photons produced from 4.023 GeV electrons by coherent Bremsstrahlung on a thin diamond radiator. The full acceptance of the detector in $\cos \theta$ span from -0.80 to 0.93 , where $\theta$ is the $\phi$ meson center-of-mass (c.m.) production angle and is achieved by means of the CEBAF Large Acceptance Spectrometer (CLAS) [16] and the Bremsstrahlung photon tagging facility ("photon tagger") [17] in Hall B of the Thomas Jefferson National Accelerator Facility (JLab). The $\phi$-meson was studied in both, the charged $\left(\phi \rightarrow K^{+} K^{-}\right)$and neutral $\left(\phi \rightarrow K_{S}^{0} K_{L}^{0}, K \bar{K}\right)$ decay modes.

In this work, we report our determination of the total cross section $\sigma_{t}$ for the reaction $\gamma p \rightarrow \phi p$ near threshold using the recent differential cross sections measured by the CLAS Collaboration [15], and the estimation of the $\phi p$ scattering length $\left|\alpha_{\phi p}\right|$, which is compared with the previously evaluated scattering lengths for the $\omega p$ and $J / \psi p$ reaction [13, 14].

To determine the total cross sections from the CLAS differential cross-section $d \sigma / d \Omega\left(E_{\gamma}, \cos \theta\right)$, we use a series of Legendre polynomials $P_{j}(\cos \theta)$ (see, for instance, Ref. [18]):

$$
\begin{equation*}
d \sigma / d \Omega\left(E_{\gamma}, \cos \theta\right)=\sum_{j=0}^{6} A_{j}\left(E_{\gamma}\right) P_{j}(\cos \theta) \tag{1}
\end{equation*}
$$

with integer $j$. As expected for such a fit using orthogonal polynomials, the Legendre coefficients $A_{j}\left(E_{\gamma}\right)$ decrease markedly for large $j$. For the CLAS energies and precisions, a maximum value of $j=6$ was found to be sufficient to describe the threshold data for $\cos \theta$ between -0.80 and 0.93 . Recall that $\sigma_{t}=4 \pi A_{0}\left(E_{\gamma}\right)$. The best-fit results are summarized in Table I. Note that the large uncertainties at low energies result from the incomplete angular coverage in $\cos \theta$.

TABLE I. The total cross section $\sigma_{t}$ for the reaction $\gamma p \rightarrow \phi p$ near threshold as a function of the beam energy $E_{\gamma}$. The errors represent the total uncertainties resulting from the quadrature sum of the statistical and systematic uncertainties of the CLAS differential cross sections [15]. The uncertainty of the beam energy is 12 MeV .

| $E_{\gamma}$ <br> $(\mathrm{MeV})$ | $\sigma_{t}$ <br> $(\mu \mathrm{~b})$ | $E_{\gamma}$ <br> $(\mathrm{MeV})$ | $\sigma_{t}$ <br> $(\mu \mathrm{~b})$ |
| :---: | :---: | :---: | :---: |
| 1673.1 | $0.1563 \pm 0.0856$ | 1959.9 | $0.3042 \pm 0.0153$ |
| 1694.5 | $0.1460 \pm 0.0686$ | 1982.7 | $0.3467 \pm 0.0141$ |
| 1716.1 | $0.1973 \pm 0.0586$ | 2005.6 | $0.3759 \pm 0.0180$ |
| 1737.7 | $0.2126 \pm 0.0386$ | 2028.7 | $0.3434 \pm 0.0152$ |
| 1759.4 | $0.2125 \pm 0.0743$ | 2051.8 | $0.3670 \pm 0.0167$ |
| 1781.3 | $0.1862 \pm 0.0709$ | 2075.0 | $0.3485 \pm 0.0146$ |
| 1803.2 | $0.2468 \pm 0.0271$ | 2098.4 | $0.3453 \pm 0.0155$ |
| 1825.3 | $0.2237 \pm 0.0166$ | 2121.8 | $0.3578 \pm 0.0165$ |
| 1847.5 | $0.2485 \pm 0.0281$ | 2145.4 | $0.3436 \pm 0.0162$ |
| 1869.7 | $0.2856 \pm 0.0223$ | 2169.0 | $0.3367 \pm 0.0164$ |
| 1892.1 | $0.2727 \pm 0.0160$ | 2192.8 | $0.3306 \pm 0.0142$ |
| 1914.6 | $0.3140 \pm 0.0197$ | 2216.7 | $0.3266 \pm 0.0133$ |
| 1937.2 | $0.3052 \pm 0.0133$ |  |  |

As mentioned above, the near-threshold total cross sections of good accuracy allow for the extraction of the vector meson - proton scattering lengths as was done in Refs. [13, 14]. Below, we use the same approach for the extraction of the $\phi$-meson - proton scattering length.

In general, the total cross section of a binary reaction $a b \rightarrow c d$ with particle masses $m_{a}+M_{b}<m_{c}+M_{d}$ can be written as $\sigma_{t}=\frac{q}{k} \cdot F(q, k, s)$, where $s$ is the square of the total c.m. energy, and $q$ and $k$ are the c.m. momenta of the initial and final states, respectively. The factor $F(q, k, s)$ is proportional to the square of the invariant amplitude and does not vanish at threshold, where $E_{\gamma} \rightarrow E_{\mathrm{thr}}, q \rightarrow 0$, and $k$ is finite. Thus, near threshold, $\sigma_{t} \rightarrow 0$ and is at least proportional to $q$.

Traditionally, the $\sigma_{t}$ behavior of a near-threshold binary inelastic reaction is described as a series of odd powers in $q$ (for details see Ref. [14]). In the energy range under our study, we use:

$$
\begin{equation*}
\sigma_{t}(q)=b_{1} q+b_{3} q^{3}+b_{5} q^{5} \tag{2}
\end{equation*}
$$

which assumes contributions from only the lowest $S-, P-$, and $D$-waves. Very close to threshold, the higher-order terms can be neglected and the linear term is determined by the $S$-wave only with a total spin of $1 / 2$ and/or $3 / 2$. The fit of the total cross section using Eq. (2) is shown in Fig. 1 by the solid magenta curve. The best-fit results are summarized in Table II.

TABLE II. The fit of the total cross section data using Eq. (2). The errors represent the total uncertainties (summing statistical and systematic uncertainties in quadrature).

| Parameter $b_{i}$ | Value |
| :---: | :---: |
| $b_{1}[\mu \mathrm{~b} /(\mathrm{MeV} / \mathrm{c})]$ | $(3.40 \pm 1.15) \times 10^{-4}$ |
| $b_{3}\left[\mu \mathrm{~b} /(\mathrm{MeV} / \mathrm{c})^{3}\right]$ | $(4.58 \pm 1.10) \times 10^{-9}$ |
| $b_{5}\left[\mu \mathrm{~b} /(\mathrm{MeV} / \mathrm{c})^{5}\right]$ | $(-12.48 \pm 2.53) \times 10^{-15}$ |
| $\chi^{2} /$ d.o.f. | 0.88 |



FIG. 1. The total $\gamma p \rightarrow \phi p$ cross section $\sigma_{t}$ (blue filled circles) derived from the CLAS data using Eq. (1) (numerical results are available in Table I) is shown as a function of the c.m. momentum $q$ of the final-state particles. The vertical error bars represent the total uncertainties of the data summing statistical and systematic uncertainties in quadrature, while the horizontal error bars reflect the energy binning. The magenta solid curve shows the fit of the CLAS data with Eq. (2). Note that the first data bin has a weighted average of $q=216 \mathrm{MeV} / \mathrm{c}$.

For the evaluation of the absolute value of the vector meson - proton scattering length, we apply the commonly used and effective VMD approach, which links the near-threshold cross sections of the vector-meson photoproduction $(\gamma p \rightarrow V p)$ and the elastic scattering $(V p \rightarrow V p)$ processes via:

$$
\begin{align*}
\left.\frac{d \sigma^{\gamma p \rightarrow V p}}{d \Omega}\right|_{\mathrm{thr}} & =\frac{q}{k} \cdot \frac{1}{64 \pi}\left|T^{\gamma p \rightarrow V p}\right|^{2} \\
& =\left.\frac{q}{k} \cdot \frac{\pi \alpha}{g_{V}^{2}} \frac{d \sigma^{V p \rightarrow V p}}{d \Omega}\right|_{\mathrm{thr}}=\frac{q}{k} \cdot \frac{\pi \alpha}{g_{V}^{2}}\left|\alpha_{V p}\right|^{2}, \tag{3}
\end{align*}
$$

where $k$ is the photon c.m. momentum $k=\left(s-M_{p}^{2}\right) / 2 \sqrt{s}, T^{\gamma p \rightarrow V p}$ is the invariant amplitude of the vector-meson photoproduction, $\alpha$ is the fine-structure constant, and $g_{V}$ is the VMD coupling constant, related to the vector-meson
electromagnetic (EM) decay width $\Gamma_{V \rightarrow e^{+} e^{-}}$

$$
\begin{equation*}
g_{V}=\sqrt{\frac{\pi \alpha^{2} m_{V}}{3 \Gamma_{V \rightarrow e^{+} e^{-}}}} \tag{4}
\end{equation*}
$$

where $m_{V}$ is the vector-meson mass.
Combining Eq. (2) (which is also valid for $\omega$ - and $J / \psi$-photoproduction $[13,14]$ ) and Eqs. $(3,4)$, one can express the absolute value of the scattering length as a product of the pure EM, VMD-motivated kinematic factor $R_{V}^{2}=$ $\alpha m_{V} k / 12 \pi \Gamma_{V \rightarrow e^{+} e^{-}}$and the factor $h_{V p}=\sqrt{b_{1}}$ that is determined by an interplay of strong (hadronic) and EM dynamics as

$$
\begin{equation*}
\left|\alpha_{V p}\right|=R_{V} h_{V p} \tag{5}
\end{equation*}
$$

In case of $\phi$-meson photoproduction, taking $\Gamma_{\phi \rightarrow e^{+} e^{-}}=(1.27 \pm 0.04) \mathrm{keV}$ [19] and $b_{1}$ from Table II, one gets $R_{\phi}=(343.0 \pm 5.4) \mathrm{MeV}^{1 / 2}$ and $h_{\phi p}=(0.000184 \pm 0.000032) \mathrm{fm} / \mathrm{MeV}^{1 / 2}$, which gives $\left|\alpha_{\phi p}\right|=(0.063 \pm 0.010) \mathrm{fm}$.

For the $\omega$ and $J / \psi$ mesons, Eq. (5) results in $\left|\alpha_{\omega p}\right|=(0.820 \pm 0.030) \mathrm{fm}$, and $\left|\alpha_{J / \psi p}\right|=(0.00308 \pm 0.00055) \mathrm{fm}$, respectively [13, 14]. The EM factors for the $\omega$ and $J / \psi$ mesons are close to each other, being $391 \mathrm{MeV}^{1 / 2}$ and $455 \mathrm{MeV}^{1 / 2}$, respectively. Therefore, such a big difference in scattering lengths is determined mainly by the hadronic factor $h_{V p}$, and reflects a strong weakening of the interaction in the $\bar{c} c-p$ system compared to that of the $\bar{q} q-p$ ( $q=u, d$ ) configurations. The interaction in the $\bar{s} s-p$ configuration has an intermediate strength that is manifested in an intermediate value of the $\phi p$ scattering length.

The value for $\left|\alpha_{\phi p}\right|$ as determined in this paper from the CLAS data, is smaller than the results given in the literature: 0.15 fm from forward coherent $\phi$-meson photoproduction from deuterons near threshold by the LEPS Collaboration $[20],(-0.15 \pm 0.02) \mathrm{fm}$ using a QCD sum rule analysis on the spin-isospin averaged $\rho, \omega$, and $\phi$ mesonnucleon scattering [21], and $\simeq 2.37 \mathrm{fm}$ using the QCD van der Waals attractive $\phi N$ potential for the analysis of the $\phi$-nucleus bound states [22]. The latter value is more than an order of magnitude greater than the results using experimental data and provides a problem for this particular potential model.


FIG. 2. Comparison of the $\left|\alpha_{V p}\right|$ scattering lengths estimated from vector-meson photoproduction at threshold vs. the inverse mass of the vector meson. The blue open triangle shows the analysis of the A2 $\omega$-meson data [14], the red filled circle shows the current analysis of the CLAS $\phi$-meson data [15], and the blue open circle shows the analysis of the GlueX $J / \psi$-meson data [13, 23].

Note that our value of $\left|\alpha_{\phi p}\right|$ is much smaller than the result from the A2 Collaboration at MAMI for the $\omega p$ scattering length $\left|\alpha_{\omega p}\right|=(0.82 \pm 0.03) \mathrm{fm}[14]$ and much larger than the recent result from the GlueX data [23] for the $J / \psi p$ scattering length $\left|\alpha_{J / \psi p}\right|=(0.00308 \pm 0.00055$ (stat.) $\pm 0.00042$ (syst.)) fm [13]. All results are shown in Fig. 2 as a function of the inverse vector-meson mass. Such a small value of the $\left|\alpha_{\phi p}\right|$ scattering length compared to the typical hadron size of 1 fm , indicates that the proton is more transparent for $\phi$-mesons compared to $\omega$-mesons, and is much less transparent than for $J / \psi$ mesons. Moreover, our analysis shows a non-linear exponential increase $\alpha_{V p} \propto \exp \left(1 / m_{V}\right)$ with increasing $1 / m_{V}$.

In summary, an experimental study of $\phi$-meson photoproduction off the proton was performed by the CLAS Collaboration at JLab [15]. The quality of the CLAS data near-threshold allows for the determination of the total cross sections of the reaction $\gamma p \rightarrow \phi p$ and for an estimation of the $\phi p$ scattering length within the VMD model. This
results in an absolute value of the $\phi p$ scattering length that is smaller compared to the known theoretical prediction. We found $\left|\alpha_{J / \psi p}\right| \ll\left|\alpha_{\phi p}\right| \ll\left|\alpha_{\omega p}\right|$ and a strong exponential increase of $\alpha_{V p}$ with the inverse mass of the vector meson.

We thank Daniel Carman for valuable comments. This work was supported in part by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Award No. DE-SC0016583 and Contract No. DE-AC0506OR23177.
[1] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Lett. 6, 628 (1961).
[2] B. C. Maglic et al., Phys. Rev. Lett. 7, 178 (1961).
[3] M. Alston et al., Phys. Rev. Lett. 6, 300 (1961).
[4] P. L. Connolly et al., Phys. Rev. Lett. 10, 371 (1963).
[5] P. E. Schlein et al., Phys. Rev. Lett. 10, 368 (1963).
[6] J. J. Aubert et al. [E598 Collaboration], Phys. Rev. Lett. 33, 1404 (1974).
[7] J. E. Augustin et al. [SLAC-SP-017 Collaboration], Phys. Rev. Lett. 33, 1406 (1974) [Adv. Exp. Phys. 5, 141 (1976)].
[8] G. Goldhaber et al., Phys. Lett. 69B, 503 (1977).
[9] H. K. Nguyen et al., Phys. Rev. Lett. 39, 262 (1977).
[10] S. W. Herb et al., Phys. Rev. Lett. 39, 252 (1977).
[11] M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).
[12] A. I. Titov, T. Nakano, S. Date, and Y. Ohashi, Phys. Rev. C 76, 048202 (2007).
[13] I. Strakovsky, D. Epifanov, and L. Pentchev, arXiv:1911.12686 [hep-ph].
[14] I. I. Strakovsky et al. [A2 Collaboration at MAMI], Phys. Rev. C 91, no. 4, 045207 (2015).
[15] B. Dey et al. [CLAS Collaboration], Phys. Rev. C 89, no. 5, 055208 (2014).
[16] B. A. Mecking et al., Nucl. Inst. Meth. A 503, 513 (2003).
[17] D. I. Sober et al., Nucl. Inst. Meth. A 440, 263 (2000).
[18] Y. I. Azimov, I. I. Strakovsky, W. J. Briscoe, and R. L. Workman, Phys. Rev. C 95, no. 2, 025205 (2017).
[19] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no. 3, 030001 (2018).
[20] W. C. Chang et al. [LEPS Collaboration], Phys. Lett. B 658, 209 (2008).
[21] Y. Koike and A. Hayashigaki, Prog. Theor. Phys. 98, 631 (1997).
[22] H. Gao, T. S. H. Lee, and V. Marinov, Phys. Rev. C 63, 022201 (2001).
[23] A. Ali et al. [GlueX Collaboration], Phys. Rev. Lett. 123, no. 7, 072001 (2019).


[^0]:    * Corresponding author: igor@gwu.edu

