B-meson Ioffe-time distribution amplitude at short distances

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We propose the approach for a lattice investigation of light cone distribution amplitudes (LCDA) of heavy-light mesons, such as the B meson, using the formalism of parton pseudodistributions. A basic ingredient of the approach is the study of short-distance behavior of the B-meson Ioffe-time distribution amplitude (ITDA), which is a generalization of the B-meson LCDA in coordinate space. We construct a reduced ITDA for the B meson, and derive the matching relation between the reduced ITDA and the LCDA. The reduced ITDA is ultraviolet finite, which guarantees that the continuum limit exists on the lattice.

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I. INTRODUCTION

The B-meson physics plays a remarkable role in particle physics, both in a detailed examination of the Standard Model and in the search of new physics beyond the Standard Model. One of the most important functions describing the structure of the B-meson is its light cone distribution amplitude (LCDA) [1]. It is an inherent part of hard-collinear factorization theorems for many exclusive B decay reactions [2–8], where the amplitude is factorized into a convolution of the hard scattering kernel and the B-meson LCDA. It is also an essential element in the light cone sum-rule studies [9–15] of the B-meson decays.

The perturbative structure of the B-meson LCDA may be studied in a model-independent way, e.g., using the renormalization group equation [16–19] and constraints on the perturbative tail of the leading-twist LCDA $\phi_B^+(\omega,\mu)$ [20,21]. On the other hand, the nonperturbative aspects of B-meson LCDA has been mainly explored within models based on QCD sum rules [22,23].

A first-principle approach to study the nonperturbative aspects of the B-meson LCDA may be provided by lattice gauge simulations. However, there was not much work in this direction. The difficulties arise from the fact that, in the heavy quark effective theory (HQET), the B-meson LCDA is defined through the matrix element of a nonlocal operator in which the heavy and light quarks are separated along the light cone.

Thus it cannot be calculated directly on the Euclidean lattice. Moreover, unlike in the case of the parton distribution functions of the nucleon, it is impossible to access B-meson LCDA by computing its moments, just because the operator product expansion does not exist in this case [22]. One might propose to calculate instead the inverse moments of LCDA, which are more relevant to phenomenology. However, they are not related to matrix elements of local operators.

The recent developments in the study of parton distribution functions (PDFs) on the lattice (e.g., quasi-PDFs [24–26], pseudo-PDFs [27–29], lattice cross sections [30,31]) provide the possibility of studying light cone parton distributions directly with lattice simulations. In particular, there were attempts of accessing the leading twist B-meson LCDA within the quasidistribution amplitude (quasi-DA) approach, either in coordinate space [32] or momentum space [33]. Although the matching relation that links quasi-DA and LCDA has been investigated, it is still not clear how one can approach the continuum limit because of the existence of ultraviolet (UV) singularities.

In this paper, we propose to deal with the UV singularities using the pseudo-PDF approach [27]. Its essential idea is that, if the operator is multiplicatively renormalizable, one can choose a proper ratio that defines an UV finite reduced Ioffe-time distribution. To this end, we will study the short-distance behavior of the B-meson Ioffe-time distribution amplitude (ITDA) and construct a reduced ITDA.

Using the results of the one-loop calculation, we will show that the reduced ITDA can be factorized into the position-space LCDA and a hard function. Furthermore, the UV finiteness allows the reduced ITDA calculated on the lattice to approach its continuum limit. This result is

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crucial for building a practical method of accessing B-meson LCDA on the lattice.

II. B-MESON IOFFE-TIME DISTRIBUTION AMPLITUDE

Our starting object is a nonlocal heavy-light operator $O^\mu(z,0;v) \equiv \bar{q}(z)S(z,0)\gamma^\mu\gamma_5 h_v(0)$ in HQET, where $S(z,0) \equiv P\exp[igz_\nu\int_0^1 dtA^\nu(tz)]$ is a Wilson line, and h_v is the heavy quark field in HQET, with v denoting its velocity, $v^2 = 1$ and h_v satisfying $\not{v}h_v = h_v$. The light quark is located at z, where z is a spacelike vector. By Lorentz covariance, the meson-to-vacuum matrix element can be parametrized as

$$\langle 0|\bar{q}(z)S(z,0)\gamma^\mu\gamma_5 h_v(0)|\bar{B}(v)\rangle = iF(\mu)[v^\mu M_{B,v}(\nu,-z^2,\mu) + z^\mu M_{B,z}(\nu,-z^2,\mu)], \quad (1)$$

where $M_{B,v}(\nu,\mu)$ and $M_{B,z}(\nu,\mu)$ are two scalar functions and $\nu \equiv v\cdot z$ will be referred to as the "Ioffe time" of the B meson (note that in the QCD case the Ioffe time is the inner product of momentum p and z [34,35]). $F(\mu)$ is the decay constant of the B meson defined by the matrix element of the local current

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 h_v(0)|\bar{B}(v)\rangle = iv^\mu F(\mu). \quad (2)$$

Unlike the QCD case, decay constant in HQET is scale dependent.

When $z^2 \to 0$, the $M_{B,v}$ term gives the twist-2 distribution while $M_{B,z}$ is a higher-twist contribution. Note that the local limit has been included in the decay constant, so $z^2 \to 0$ infers the light cone limit for the distributions $M_{B,v}$ and $M_{B,z}$. Because we are only interested in the leading-twist distribution at present, we rename $M_{B,v}$ as M_B for short, and call $M_B(\nu,-z^2,\mu)$ the ITDA of the B meson.

If z is a lightlike vector, e.g., only the minus component of z is nonzero, then ITDA will reduce to the light cone ITDA $\mathcal{I}_B^+(\nu,\mu)$, i.e., $M_B(\nu,0,\mu) = \mathcal{I}_B^+(\nu,\mu)$, which is actually the LCDA in position space. The B-meson LCDA that appears in the factorization theorems of B-meson exclusive decay is defined by the Fourier transform of $\mathcal{I}_B^+(\nu,\mu)$ [1]

$$\phi_B^+(\omega,\mu) = \frac{v^+}{2\pi}\int_{-\infty}^{\infty} dz^- e^{-i\omega v^+ z^-}\mathcal{I}_B^+(v^+z^-,\mu). \quad (3)$$

There are no light cone separations on the Euclidean lattice, but as proposed in Refs. [24,36], one can study equal-time separations $z = (0,0,0,z_3)$. The same idea may also be applied for the B-meson LCDA. In this case, $\nu = -v_3 z_3$ and $z^2 = -z_3^2$. One can choose the Lorentz index $\mu = 0$ in Eq. (1), so that the higher-twist part $z^\mu M_{B,z}$ disappears. In the quasi-PDF-based approaches [32,33] one deals with the B-meson quasi-DA $\tilde{\phi}_B^+(\omega,v_3,\mu)$ that can be expressed in terms of ITDA as

$$\tilde{\phi}_B^+(\omega,v_3,\mu) = \frac{|v_3|}{2\pi}\int_{-\infty}^{\infty} dz_3 e^{i\omega v_3 z_3} M_B(-v_3 z_3, z_3^2,\mu). \quad (4)$$

A matching relation linking the quasi-DA and LCDA was derived in Ref. [33].

However, integration over the parameter z_3 present in both arguments of the ITDA $M_B(-v_3z_3, z_3^2,\mu)$ mixes two distinct phenomena: the ν dependence that governs the ω shape of the LCDA, and the z_3^2 dependence that corresponds to the probing scale for the LCDA. For this reason, we propose to proceed along the lines of the pseudo-PDF approach [27,29] in which these phenomena are clearly separated.

III. HARD CORRECTION AT ONE LOOP

Formally, the LCDA in coordinate space $\mathcal{I}_B^+(\nu,\mu)$ can be approached by taking $z^2 \to 0$ limit of ITDA $M_B(\nu,-z^2,\mu)$. However, logarithmic dependence on z^2 will be generated when hard corrections of ITDA are included. As a result, the $z^2 \to 0$ limit cannot be approached directly, and a perturbative matching is needed.

Under quantum correction, the hard part will be generated by gluon exchanges. As indicated in Refs. [27,37], the hard contribution can be determined at operator level with coordinate representation. The Feynman diagrams are presented by Fig. 1. A calculation has been performed in Ref. [32], where the UV and IR singularities are regularized by dimensional regularization (DR). To distinguish the UV and IR singularities for ITDA, we will adopt Polyakov regularization [38] for UV singularities, in which the gluon propagator in coordinate representation is replaced by $-g_{\mu\nu}/4\pi^2z^2 \to -g_{\mu\nu}/4\pi^2(z^2-a^2)$. Collinear singularities are regularized by the mass of light quark m; the soft singularity is regularized by DR. We will work in Feynman gauge but the results are gauge invariant.

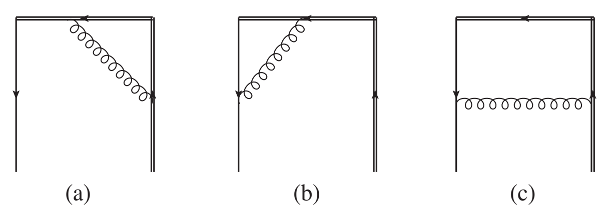

FIG. 1. One-loop hard contribution to nonlocal heavy-light operator in HQET. The horizontal double line represents the gauge link, while the vertical double line denotes the heavy quark in HQET.

According to Eq. (1), to study the hard contribution of $M_B(\nu, -z^2, \mu)$, one should consider the one-loop correction to both the nonlocal operator and decay constant. We consider the decay constant first. Note that the B-meson decay constant in HQET is UV divergent and scale dependent, which is different from the pion decay constant case. Under Polyakov regularization, the one-loop hard correction to decay constant is

$$F(a) = F(a)^{(0)}\left[1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{4}\ln\frac{a^2 m^2 e^{2\gamma_E}}{4} + \frac{21}{8}\right)\right], \qquad (5)$$

where γ_E is the Euler-Mascheroni constant and $F(a)^{(0)}$ denotes the decay constant without the hard correction, while $F(a)$ is the decay constant in which the hard correction is included.

Now we turn to the one-loop hard contribution of the nonlocal operator. To begin with, we consider the heavy quark and light quark self energies. Up to one loop, we have

$$\begin{aligned}\delta Z_h &= -\frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\rm IR}} + \ln\frac{a^2 e^{2\gamma_E}}{4} + \ln 4\pi\mu_{\rm IR}^2 e^{-\gamma_E}\right),\\ \delta Z_2 &= -\frac{\alpha_s C_F}{2\pi}\left(-\frac{1}{2}\ln\frac{a^2 m^2 e^{2\gamma_E}}{4} + \frac{1}{\epsilon_{\rm IR}}\right.\\ &\quad\left. + \ln\frac{4\pi\mu_{\rm IR}^2 e^{-\gamma_E}}{m^2} + \frac{9}{4}\right) \qquad (6)\end{aligned}$$

for heavy and light quarks, respectively. Here $d = 4 - 2\epsilon_{\rm IR}$ is the dimension of space-time in DR, and $\mu_{\rm IR}$ denotes the infrared (IR) scale associated with the soft singularity $1/\epsilon_{\rm IR}$. The self-energy of the gauge link has already been calculated in PDF case. The result reads [37,39]

$$\Gamma_\Sigma(z,a) = \frac{\alpha_s C_F}{2\pi}\left(-\frac{\pi}{a}\sqrt{-z^2} + \ln\frac{-z^2}{a^2} + 2\right) + \mathcal{O}(z^2). \qquad (7)$$

The heavy-quark Wilson line vertex is presented in Fig. 1 (a). In HQET, the heavy quark can be expressed as a Wilson line along the v direction. At one-loop level, the exchange of gluon between Wilson lines along v and n directions contributes

$$\begin{aligned}O^\mu(z,0;v) &= \frac{\alpha_s C_F}{2\pi}\bar{q}(z)\gamma^\mu\gamma_5 h_v(0)\\ &\quad\times\left[\ln a^2\left(\ln 2iv\cdot z - \frac{1}{2}\ln(-z^2)\right)\right.\\ &\quad\left. + \frac{1}{4}\ln^2(-z^2) - \ln^2 2iv\cdot z - \frac{\pi^2}{6}\right] + \mathcal{O}(z^2). \qquad (8)\end{aligned}$$

Note that the exchange of the gluon between the two Wilson lines generates a cusp singularity [38], which is represented by $\ln a^2$. Another interesting feature here is that, because of the existence of cusp singularity, there is a double logarithmic dependence on z^2, which is very different from the nucleon Ioffe-time distribution function case.

The light-quark Wilson line vertex is presented in Fig. 1(b). This contribution is the same as the vertex contribution in the PDF operator (see, e.g., Ref. [37]). Direct calculation gives

$$\begin{aligned}O^\mu(z,0;v) &= \frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2}\left(\ln\frac{-z^2}{a^2} - 1\right)\bar{q}(z)\gamma^\mu\gamma_5 h_v(0)\right.\\ &\quad - \int_0^1 du\left[\ln\frac{-z^2 m^2 e^{2\gamma_E}}{4}\frac{\bar{u}}{u} + \frac{\bar{u} + (2-u)\ln u^2}{u}\right]_+\\ &\quad\left.\times\bar{q}(\bar{u}z)\gamma^\mu\gamma_5 h_v(0)\right\} + \mathcal{O}(z^2), \qquad (9)\end{aligned}$$

where $\bar{u} \equiv 1 - u$. The plus distribution $[f(u)]_+$ is defined by $\int_0^1 du[f(u)]_+ T(u) \equiv \int_0^1 du f(u)[T(u) - T(u_0)]$, where u_0 is the pole of $f(u)$, and $T(u)$ is a test function.

Figure 1(c) represents the contribution from interaction between light and heavy quarks. To calculate this contribution, we adopt the nonrelativistic approximation, where the momentum of light quark is given by $p = mv$. Under this approximation, we have

$$\begin{aligned}O^\mu(z,0;v) &= -\frac{\alpha_s C_F}{2\pi}\left\{\left[\frac{2}{u} + \ln(iumv\cdot z e^{\gamma_E})\right]_+\right.\\ &\quad\left. - \left[\frac{1}{\epsilon_{\rm IR}} - 1 + \ln\frac{4\pi\mu_{\rm IR}^2 e^{-\gamma_E}}{m^2} - \ln(imv\cdot z e^{\gamma_E})\right]\delta(u)\right\}\\ &\quad\times\bar{q}(\bar{u}z)\gamma^\mu\gamma_5 h_v(0) + \mathcal{O}(z^2). \qquad (10)\end{aligned}$$

The Lorentz structure of the type $\gamma^\mu \not{z}\gamma_5$ has been neglected because it yields a higher-twist contribution to the ITDA. One may notice that the box diagram has no $\ln z^2$ dependence, so that it gives the same contribution to the light cone ITDA. This means that the box diagram does not contribute to the matching relation. This has been confirmed by the calculation in momentum space [33].

Adding all contributions together, the soft IR singularities $1/\epsilon_{\rm IR}$, as well as the logarithmic dependence on the soft scale $\mu_{\rm IR}$, are canceled. According to Eqs. (1) and (5), one can derive the one-loop hard contribution of the ITDA $M_B(\nu, -z^2, \mu)$:

$$
\begin{aligned}
M_B(\nu,-z^2,a) = M_B(\nu)^{(0)} + \frac{\alpha_s C_F}{2\pi}\Bigg\{\Bigg[-\frac{\pi}{a}\sqrt{-z^2}+\frac{3}{2}\ln\frac{-z^2}{a^2}+2 \\
+\ln a^2\left(\ln 2i\nu-\frac{1}{2}\ln(-z^2)\right)-\frac{\pi^2}{6}-\ln^2 2i\nu+\frac{1}{4}\ln^2(-z^2)+\frac{1}{2}\ln\frac{a^2}{4}-\ln i\nu\Bigg]M_B(\nu)^{(0)} \\
-\int_0^1 dw\left[\frac{w}{\bar{w}}\ln\frac{-z^2m^2e^{2\gamma_E}}{4}+\ln(i\bar{w}m\nu e^{\gamma_E})+\frac{2}{\bar{w}}+\frac{w+(2-\bar{w})\ln\bar{w}^2}{\bar{w}}\right]_+ M_B(w\nu)^{(0)}\Bigg\}+\mathcal{O}(z^2). \qquad (11)
\end{aligned}
$$

IV. REDUCED IOFFE-TIME DISTRIBUTION AMPLITUDE

The hard contributions above involve UV singularities that are regularized by a. In lattice computations, the matrix elements are calculated on discrete space-time. The UV singularities correspond to the singularities in the continuum limit (i.e., the lattice spacing $a \to 0$). Although the ITDA can be computed on the lattice, however, the UV divergences obstruct to approach the result in continuum space-time from lattice data. Thus one should renormalize the UV singularities for a practical lattice evaluation.

Based on the auxiliary field formalism [40], it has been shown that the off light cone operator defining the B-meson quasi-DA is multiplicatively renormalizable [33]. So, the bare and renormalized operators are related by

$$[\bar{q}(z)\gamma^\mu\gamma_5 h_v(0)]^R = Z(z\cdot v, z^2;\Lambda)\bar{q}(z)\gamma^\mu\gamma_5 h_v(0), \qquad (12)$$

where Z is a renormalization factor and Λ denotes a cutoff. A similar equation can be written down for the decay constant. The operator with a superscript "R" denotes the renormalized operator while the operator without it denotes a bare one. The multiplicative renormalizability verified in Ref. [33] will be the foundation of establishing a practically calculable quantity on the lattice.

Since the renormalization relation holds at operator level, it is valid for any matrix element of the operator. For example, one can replace B-meson state with the leading Fock state of B meson. Similar to the definition of B-meson ITDA, such matrix element can be parametrized as

$$\langle 0|\bar{q}(z)\gamma^\mu\gamma_5 h_v(0)|b(v)\bar{q}(\omega v)\rangle = iv^\mu f(\mu)m_B(\omega\nu,-z^2), \qquad (13)$$

where $f(\mu)$ and $m_B(\omega\nu, z^2)$ are the "decay constant" and ITDA of the Fock state $|b(v)\bar{q}(\omega v)\rangle$, respectively; ωv is the momentum carried by the light quark. Note that the higher-twist contribution that is proportional to z^α has been neglected. $f(\mu)$ is defined through matrix element of local operator

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 h_v(0)|b(v)\bar{q}(\omega v)\rangle = iv^\mu f(\mu). \qquad (14)$$

As discussed above, the UV divergence only depends on the operator, i.e., the matrix elements of hadron state and its Fock state should involve the same UV structure. Furthermore, the HQET operator is multiplicatively renormalizable, so the ratios of hadron and Fock-state matrix elements should be UV finite:

$$\frac{F^R(\mu)}{F(\mu;a)} = \frac{f^R(\mu)}{f(\mu;a)}, \qquad (15)$$

$$\frac{F^R(\mu)M_B^R(\nu,-z^2)}{F(\mu;a)M_B(\nu,-z^2;a)} = \frac{f^R(\mu)m_B^R(\omega\nu,-z^2)}{f(\mu;a)m_B(\omega\nu,-z^2;a)}. \qquad (16)$$

These relations indicate that for the ratio of meson and Fock-state ITDAs, the continuum limit exists on the lattice, therefore the ratio can be evaluated with lattice simulations. For the sake of simplicity, we define a reduced ITDA $\bar{M}(\nu,-z^2)$ by dividing Fock-state ITDA at $\omega = 0$:

$$\bar{M}_B(\nu,-z^2) = \frac{M_B(\nu,-z^2;a)}{m_B(\omega\nu,-z^2;a)}\bigg|_{\omega=0}. \qquad (17)$$

Because Eq. (11) is a general relation which is valid for ITDAs of both meson state and its leading Fock state ITDAs, one can immediately get the one-loop correction to the denominator of Eq. (17). The result reads

$$
\begin{aligned}
& m_B(\omega\nu,-z^2,a)|_{\omega=0} \\
& = m_B(\omega\nu)^{(0)}|_{\omega=0} \\
& \quad + \frac{\alpha_s C_F}{2\pi}\left[-\frac{\pi}{a}\sqrt{-z^2}+\frac{3}{2}\ln\frac{-z^2}{a^2}+2\right. \\
& \quad +\ln a^2\left(\ln 2i\nu-\frac{1}{2}\ln(-z^2)\right)-\frac{\pi^2}{6}-\ln^2 2i\nu \\
& \quad \left.+\frac{1}{4}\ln^2(-z^2)+\frac{1}{2}\ln\frac{a^2}{4}-\ln i\nu\right]m_B(\omega\nu)^{(0)}\bigg|_{\omega=0} \\
& \quad +\mathcal{O}(z^2). \qquad (18)
\end{aligned}
$$

Then, the one-loop correction of the reduced ITDA is

$$\bar{M}_B(\nu,-z^2) = \bar{M}_B(\nu)^{(0)} - \frac{\alpha_s C_F}{2\pi}\int_0^1 dw\left[\frac{w}{\bar{w}}\ln\frac{-z^2m^2e^{2\gamma_E}}{4} + \ln(i\bar{w}m\nu e^{\gamma_E}) + \frac{2+w+(2-\bar{w})\ln\bar{w}^2}{\bar{w}}\right]_+ \bar{M}_B(w\nu)^{(0)} + \mathcal{O}(z^2). \tag{19}$$

It is easy to see that the linear and logarithm divergences that are related to the link are canceled; the cusp singularity that arises from the gluon exchange between heavy quark and link is canceled as well. This indicates the UV finiteness of the reduced ITDA. Thus it can be evaluated on the lattice, and the continuum limit can be approached. Furthermore, it was pointed out that the denominator of the reduced-ITDA is not IR sensitive [27], thus the IR structure is not modified in the ratio. This is also verified by the one-loop result (18).

In addition, the one-loop correction of the reduced ITDA is a plus distribution. If we define a quasi-DA from the reduced ITDA by taking Fourier transform with z_3, this will lead to a rapidly decreasing behavior of the corresponding quasi-DA $\tilde{\phi}(\omega, v_3, \mu)$ at large ω.

On the lattice, the denominator in the reduced ITDA will be evaluated nonperturbatively. However, at short distances, it can be calculated in perturbation theory. Thus, this ratio defines a nonperturbative renormalization scheme for the *B*-meson ITDA.

V. MATCHING RELATION

The reduced ITDA and the $\overline{\text{MS}}$ LCDA can be linked by a matching relation. Similar to the PDF case, one can use the nonlocal light cone operator product expansion. To determine the hard function, we also need the one-loop correction to the light cone ITDA, which can be extracted from the one-loop correction to the light cone operator [41]. The result reads

$$\mathcal{I}_B^+(\nu,\mu) = \mathcal{I}_B^+(\nu,\mu)^{(0)}\left\{1 - \frac{\alpha_s C_F}{2\pi}\left[\ln^2(i\mu\nu e^{\gamma_E}) + \ln(i\mu\nu e^{\gamma_E}) + \frac{5\pi^2}{24}\right]\right\} + \frac{\alpha_s C_F}{2\pi}\int_0^1 dw\left[\frac{w}{\bar{w}}\ln\frac{\mu^2}{\bar{w}^2m^2} - \frac{2}{\bar{w}} - \ln(i\bar{w}e^{\gamma_E}m\nu)\right]_+ \mathcal{I}_B^+(w\nu,\mu)^{(0)} + \mathcal{O}(\alpha_s^2). \tag{20}$$

One can find that the singularities regularized by ln m^2 are the same for the reduced-ITDA and light cone ITDA. Then, a matching formula for reduced ITDA and light cone ITDA can be written down:

$$\bar{M}_B(\nu,z_3^2) = \mathcal{I}_B^+(\nu,\mu) + \frac{\alpha_s C_F}{2\pi}\left\{\left[\ln^2(i\tilde{\mu}\nu) + \ln(i\tilde{\mu}\nu) + \frac{5\pi^2}{24}\right]\mathcal{I}_B^+(\nu,\mu) - \int_0^1 du\left[\frac{u}{\bar{u}}\left(\ln\frac{z_3^2\tilde{\mu}^2}{4}+1\right) + 2\frac{\ln\bar{u}}{\bar{u}}\right]_+\right\}\mathcal{I}_B^+(u\nu,\mu) + \mathcal{O}(\alpha_s^2), \tag{21}$$

where $\tilde{\mu} \equiv \mu e^{\gamma_E}$. We have chosen $z = (0,0,0,z_3)$ so that the reduced ITDA can be computed on the lattice. This relation allows one to convert the reduced ITDA calculated on the lattice to the LCDA in coordinate representation. The Fourier transformation of the latter enters the factorization theorems of *B*-meson exclusive decay.

Finally, let us take a look at the evolution equations for the ITDAs. Since the light cone ITDA does not depend on z^2, one can write down the z^2-evolution equation for the reduced ITDA:

$$\frac{d}{d\ln z_3^2}\bar{M}_B(\nu,z_3^2) = -\frac{\alpha_s C_F}{2\pi}\int_0^1 du\left[\frac{u}{\bar{u}}\right]_+\bar{M}_B(u\nu,z_3^2). \tag{22}$$

On the other hand, the reduced ITDA does not depend on μ. Thus, by taking the derivative with respect to ln μ on both sides, one can get the renormalization group equation for light cone ITDA:

$$\mu\frac{d}{d\mu}\mathcal{I}_B^+(\nu,\mu) = -\frac{\alpha_s C_F}{\pi}\left\{\left[\ln(i\tilde{\mu}\nu)+\frac{1}{2}\right]\mathcal{I}_B^+(\nu,\mu) - \int_0^1 du\left[\frac{u}{\bar{u}}\right]_+\mathcal{I}_B^+(u\nu,\mu)\right\}. \tag{23}$$

By including the anomalous dimension of the decay constant, one will find that the above equation reproduces the RGE for heavy-light light cone operator (see, e.g., Refs. [41,42]).

VI. IMPLEMENTATIONS ON THE LATTICE

In recent lattice calculations, the typical lattice spacing a is around 0.1 fm. The Compton wavelength of the b quark is much smaller than the lattice spacing, $m_b \gg 1/a$. Hence HQET is a natural framework to study the B meson on the lattice.

The renormalization of full HQET is complicated, even in continuum theory. In fact, the full HQET Lagrangian is not renormalizable. However, since the operator we are going to measure is taken in the $m_b \to \infty$ limit, we can restrict ourselves to the static approximation of HQET. At the lowest order of $1/m_b$ expansion, there will be no higher dimensional operators getting mixed in, and the renormalization property is simple. The higher dimensional operators in lattice theory can also been excluded under static approximation, hence the continuum limit of the reduced ITDA can be approached, without considering the operator mixing.

Similar to the lattice calculation of hadron Ioffe-time distribution functions [28], a possible way to get the ITDA is to calculate $M_B(-v_3 z_3, z_3^2)$ for several values of v_3, and then to fit the data by a function of ν and z_3^2. A proper framework might be the leading-order moving HQET [43]. The meson and Fock state decay constants should also be calculated on the lattice, or using the phenomenological result. The Fock state ITDA at $\omega = 0$, i.e., $m_B(0 \cdot \nu, z_3^2)$ is necessary for the construction of reduced ITDA and should be calculated on the lattice as well.

We note that, in practical lattice HQET, a result with large noise-to-signal ratio might be expected. Still, a rough evaluation of the B-meson LCDA with lattice methods is of great value because there is very little knowledge on the B-meson LCDA, even from first principle calculations.

VII. SUMMARY

To access B-meson leading-twist light cone distribution amplitude from lattice QCD computations, we have proposed the approach based on the strategy of reduced Ioffe-time distributions. The reduced Ioffe-time distribution amplitude of a B meson is constructed by the ratio of meson ITDA and the ITDA of the meson's leading Fock state, in which the light-quark momentum is zero. According to the multiplicative renormalizability of the off light cone operator, the ratio is UV finite; hence, one can approach its continuum limit from the lattice data. A matching relation that maps LCDA to the reduced ITDA is also derived. These results provide a basis for a practical computation of B-meson LCDA with lattice methods.

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