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The MARATHON (MeAsurement of the $F_{2}^{n} / F_{2}^{p}, d / u$ RAtios and A=3 EMC Effect in Deep Inelastic Scattering Off the Tritium and Helium MirrOr Nuclei) experiment ran in the Hall A Facility of the Thomas Jefferson National Accelerator Facility (JLab). The experiment used a 10.59 GeV electron beam from the Continuous Electron Beam Accelerator Facility (CEBAF) of JLab to measure the ratio of the $F_{2}$ structure functions of Helium-3 and Deuterium. Data were taken in the kinematic range of $0.195 \leq x \leq 0.825,2.5(\mathrm{GeV} / c)^{2} \leq Q^{2} \leq 13(\mathrm{GeV} / c)^{2}$, and $3.5\left(\mathrm{GeV} / c^{2}\right)^{2} \leq W^{2} \leq 13\left(\mathrm{GeV} / c^{2}\right)^{2}$. This is the first measurement of the ${ }^{3} \mathrm{He}$ EMC effect that is purely in the Deep Inelastic Scattering region. The results of this experiment constitute an important, complimentary addition, to the limited data presently available for the EMC effect of light nuclei. The MARATHON results are compared to the world data for light, medium, and heavy nuclei. This measurement is considered an essential component to uncovering the nature of the EMC effect.

# MEASUREMENT OF THE EMC EFFECT OF THE HELIUM-3 NUCLEUS AT JEFFERSON LAB 

A dissertation submitted<br>to Kent State University<br>in partial fulfillment of the requirements for the degree of Doctor of Philosophy

by Tyler Hague

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## CHAPTER 1

## Introduction

A pioneering experiment at the Stanford Linear Accelerator Center (SLAC) [1] showed that, at sufficiently high values of momentum transfer, electron-nucleon scattering is sensitive to the partonic structure of the nucleon. This led to a renaissance of lepton scattering experiments, all vying to better our understanding of nuclear structure and the constituent parts that make up the nucleus. High momentum transfer lepton-nucleon scattering, dubbed Deep Inelastic Scattering (DIS), has proven to be an invaluable tool for the study of nuclear physics.

One puzzle still unsolved, born out of the use of DIS to study the nuclear structure functions, is that of the EMC effect. The European Muon Collaboration, the namesake of the EMC effect, studied the ratio of the per nucleon cross-sections of Iron (corrected for neutron excess) to Deuterium in an effort to understand their experimental systematics [2]. What was found was clear evidence for nucleon modification within the nucleus. Where they expected a measurement of unity for the ratio, the ratio exhibited a downward slope in the region of the data. This is the EMC effect.

Since then, many experiments have contributed data sets to the study of the EMC effect. These data have shown correlations between the strength of the EMC effect and various nuclear quantities, such as mass number $A$, nuclear density, and short range correlations within the nucleus. The available measurements have primarily focused on heavy nuclei, with the exception of the JLab E03-103 experiment which focused on light nuclei.

The MARATHON (MeAsurement of the $F_{2}^{n} / F_{2}^{p}, d / u$ RAtios and $A=3$ EMC Effect in Deep Inelastic Scattering Off the Tritium and Helium MirrOr Nuclei) experiment ran in the winter and spring of 2018 in the Hall A facility of Thomas Jefferson National Accelerator Facility (JLab) using a 10.59 GeV electron beam from the CEBAF accelerator. One of the primary goals for MARATHON was to measure the EMC effect in the $A=3$ mirror nuclei, ${ }^{3} \mathrm{He}$ and ${ }^{3} \mathrm{H}[3]$. The measurements of the $A=3$ EMC effects are considered critical to a more complete understanding of the EMC effect.

This thesis will present the study of the ${ }^{3} \mathrm{He}$ EMC effect from the MARATHON data. Chapter 2 will give an overview of electron scattering with a focus on Deep Inelastic Scattering. Chapter 3 will discuss the history of the EMC effect, as well as a selection of models that aim to describe it. Chapter 4 describes the experimental setup of the MARATHON experiment at JLab. Chapter 5 shows the methods used to analyze the data acquired from the experiment. Finally, Chapter 6 presents the results of this study, as well as a look into how the measured strength of the ${ }^{3} \mathrm{He}$ EMC effect fits in with correlations that are useful for understanding the source of this effect.

## CHAPTER 2

## Electron Scattering and Nuclear Structure

### 2.1 History

Ernest Rutherford performed what is often considered the first scattering experiment. This experiment fired an $\alpha$-particle beam at a gold foil. The result of this experiment saw most particles pass through the foil completely undeflected. Those that did deflect were scattered at a large range of angles. This gave the world a new view of the atom: that of a largely empty space with hard scattering centers. We now know that this scattering center is the nucleus of the atom, formed by a dense combination of protons and neutrons [4].

Since that time, many experiments have been conducted that expanded our view of the nucleus. The evidence of quarks at the Stanford Linear Accelerator Center (SLAC) once again revolutionized our understanding of the nucleus. In this experiment, electrons were scattered off protons over a large momentum transfer, $Q^{2}$, and final hadronic invariant mass, $W$, range. This experiment noted a "surprisingly weak" $Q^{2}$ dependence once the kinematics reached the $W>2 \mathrm{GeV}$ range, a key feature of Deep Inelastic Scattering [1]. In this view, the nucleons, protons and neutrons, are comprised of quarks. The quarks are elementary particles that define the characteristics and structure of the nucleon [5].

SLAC paved the way for a wave of Deep Inelastic Scattering experiments that have refined our understanding of the nucleus and its constituent components. Deep Inelastic Scattering has proven to be one of the most powerful tools available when one seeks to study nuclear structure.

### 2.2 Electron Scattering

Electron scattering allows for a finely tuned analysis of the nucleus. By manipulating the energy of the incoming electron and the kinematic variables accepted by the detectors used, experimenters can choose what aspect of the nucleus will be probed. This technique has been used to study the structure of the nucleus all the way down to the properties of the constituent quarks.


Figure 1: SLAC cross section data plotted versus invariant mass of the final hadronic state, $W$, showing the regions of scattering [6].

There are four kinematic regimes of electron scattering that can be explored: elastic, quasielastic, resonance, and deep inelastic scattering. Each of these regimes are defined by the kinematics and the underlying physical structure that they are sensitive to. Figure 1 shows SLAC electron scattering data plotted against the invariant mass of the final hadronic state, $W$. Seen is the elastic peak at $W=M$, followed by resonance peaks, and finally the Deep Inelastic Scattering region when $W \geq 2 \mathrm{GeV} / c^{2}$.

Elastic scattering occurs when the electron scatters coherently off of the nucleus. This occurs at low momentum and energy transfer. At these kinematics, the electron is sensitive to the size of the nucleus. The size of the nucleus is accessed by extracting nuclear "form factors" from the measured cross sections. From this we can learn about the charge distributions, magnetic moments, and charge radius of the nucleus.

Quasi-elastic scattering occurs at larger energy transfers when the electron scatters elastically off of a nucleon inside a nucleus, rather than a nucleus itself. At these kinematics, the electron is sensitive to the form factors of the nucleon.

Resonance scattering occurs at even larger momentum and energy transfers. In this region, some of the energy is used to excite the nucleon into a higher energy state, called a resonance. A resonance is a short-lived particle. The resonance will quickly decay back into the nucleon and emit
the excess energy as an additional particle. For example, resonance scattering off of a proton can lead to the emission of a neutral pion via $e p \rightarrow e \Delta^{+} \rightarrow e p \pi^{0}$.

As we push the momentum and energy transfer further, we enter the Deep Inelastic Scattering (DIS) region. As $Q^{2}$ is increased into the transition region between resonance and DIS, the resonance peaks begin to smooth out. Here the electron becomes sensitive to the constituent parts of the nucleon. The wavelength of the exchanged photon is inversely proportional to $Q^{2}$. DIS scattering is of particular interest because the wavelength is sufficiently small enough to discern the parton structure. Through careful measurement, we can access the nuclear structure functions and the parton distributions in the nucleons $[6,7,8]$.

The MARATHON experiment seeks to study the nuclear and nucleon structure functions, as well as nucleon parton distributions. The kinematics used are in the DIS region in order to facilitate this study. The remainder of this chapter will be focused on the DIS cross section and what we can learn from it.

### 2.2.1 Deep Inelastic Scattering Cross Section

Deep Inelastic Scattering, shown in Figure 2, involves a high energy electron scattering off of a nucleon. In the lowest order perturbation theory, a virtual photon is exchanged between the electron and nucleon. This momentum transfer then excites the nucleon into a hadronic final state, $X$. Though the final hadronic state is undetected, the detection of the scattered electron can yield insight into the interaction.

The reaction for scattering off a proton, is written as:

$$
e^{-}+P \rightarrow e^{-}+X
$$

In this section, there will be many variables defined. For clarity, the meaning of these variables are defined in Table 1. Mathematical definitions will follow, as necessary.


Figure 2: Feynman Diagram of Deep Inelastic Scattering.

| $Q^{2}$ | Negative of the 4-momentum transfer |
| :--- | :--- |
| $W^{2}$ | Square of the invariant mass of the final hadronic state |
| $E$ | Beam energy |
| $E^{\prime}$ | Scattered electron energy |
| $L_{\mu \nu}$ | Electron tensor |
| $W^{\mu \nu}$ | Symmetric hadronic tensor |
| $k$ | 4-momentum of the incident electron |
| $k^{\prime}$ | 4-momentum of the scattered electron |
| $p$ | 4-momentum of the target nucleon |
| $\theta$ | Scattered electron angle |
| $\nu$ | Energy difference between incoming and scattered electron |
| $M$ | Proton mass |
| $m$ | Electron mass |

Table 1: Variable definitions introduced in this section.

The kinematics of scattering are typically defined by the Lorentz-invariant kinematic variables $Q^{2}$ and $W^{2}$, as well as the energy difference $\nu$. These variables are defined as:

$$
\begin{gather*}
q^{2} \equiv-Q^{2}=\left(k-k^{\prime}\right)^{2},  \tag{2.1}\\
W^{2}=(p+q)^{2}, \tag{2.2}
\end{gather*}
$$

and

$$
\begin{equation*}
\nu=E-E^{\prime} . \tag{2.3}
\end{equation*}
$$

Now it is useful to consider the scattering in the laboratory rest frame and to note that MARATHON is a fixed target experiment. In this frame, the target is "at rest" (i.e. having a four-momentum of $(M, 0))$. This leads to a simplification of $Q^{2}$ and $W^{2}$ with

$$
\begin{equation*}
Q^{2}=4 E E^{\prime} \sin ^{2} \frac{\theta}{2} \tag{2.4}
\end{equation*}
$$

and

$$
\begin{equation*}
W^{2}=M^{2}+2 M \nu-Q^{2} . \tag{2.5}
\end{equation*}
$$

The electron-nucleon scattering cross section can be generally expressed as [6]:

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\frac{\alpha^{2}}{Q^{4}} \frac{E^{\prime}}{E} L_{\mu \nu} W^{\mu \nu} \tag{2.6}
\end{equation*}
$$

The electron tensor is expressed as:

$$
\begin{equation*}
L_{\mu \nu}=2\left(k^{\prime \mu} k^{\nu}+k^{\prime \nu} k^{\mu}-\left(k^{\prime} \cdot k-m^{2}\right) g^{\mu \nu}\right) . \tag{2.7}
\end{equation*}
$$

The DIS hadronic tensor is expressed in terms of the structure functions $W_{1}$ and $W_{2}$ :

$$
\begin{equation*}
W^{\mu \nu}=W_{1}\left(-g^{\mu \nu}+\frac{q^{\mu} q^{\nu}}{q^{2}}\right)+\frac{W_{2}}{M^{2}}\left(p^{\mu}-\frac{p \cdot q}{q^{2}} q^{\mu}\right)\left(p^{\nu}-\frac{p \cdot q}{q^{2}} q^{\nu}\right) . \tag{2.8}
\end{equation*}
$$

Combining all of the above, we arrive at the $e p$ DIS cross section in the laboratory frame:

$$
\begin{equation*}
\left.\frac{d^{2} \sigma}{d \Omega d E^{\prime}}\right|_{\text {lab }}=\frac{\alpha^{2}}{4 E^{2} \sin ^{4} \frac{\theta}{2}}\left[W_{2} \cos ^{2} \frac{\theta}{2}+2 W_{1} \sin ^{2} \frac{\theta}{2}\right] . \tag{2.9}
\end{equation*}
$$

### 2.3 Bjorken Scaling

As $Q^{2}$ is pushed higher, inelastic scattering begins to give way to DIS. In this kinematic region, the wavelength of the virtual photon is sufficiently short enough to resolve the internal structure of the nucleon. This transition sees the system begin to behave like a free Dirac particle, the parton. As the Bjorken limit is approached, $Q^{2} \rightarrow \infty$ and $\nu \rightarrow \infty$, the scattering center approaches a structureless parton [9]. With this in mind, it is useful to look at the cross section for scattering off of a structureless target, which is given by:

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\frac{\alpha^{2}}{4 E^{2} \sin ^{4} \frac{\theta}{2}}\left[\cos ^{2} \frac{\theta}{2}+\frac{Q^{2}}{2 m^{2}} \sin ^{2} \frac{\theta}{2}\right] \delta\left(\nu-\frac{Q^{2}}{2 M}\right) . \tag{2.10}
\end{equation*}
$$

Noting that DIS is scattering off of a structureless parton, we can compare Equations 2.9 and 2.10. By equating these two cross sections, we can clearly extract two equations for the DIS structure functions:

$$
\begin{equation*}
2 M W_{1}\left(Q^{2}, \nu\right)=\frac{Q^{2}}{2 M \nu} \delta\left(1-\frac{Q^{2}}{2 M \nu}\right) \tag{2.11a}
\end{equation*}
$$

and

$$
\begin{equation*}
\nu W_{2}\left(Q^{2}, \nu\right)=\delta\left(1-\frac{Q^{2}}{2 M \nu}\right) . \tag{2.11b}
\end{equation*}
$$

In this kinematic region, we see that the structure functions are dependent on the ratio $Q^{2} / 2 M \nu$, rather than $Q^{2}$ and $\nu$ independently while the target mass sets the scale. Noting this dependency, the scaling variable Bjorken $x$ is defined as:

$$
\begin{equation*}
x=\frac{Q^{2}}{2 M \nu} . \tag{2.12}
\end{equation*}
$$

As the Bjorken limit is approached, DIS is only dependent on $x$, showing little to no scaling with $Q^{2}$ or $\nu$ [10]. Two new structure functions, $F_{1}$ and $F_{2}$, are also defined in terms of $x$ to clearly show the lack of scaling with $Q^{2}$ and $\nu$ independently:

$$
\begin{equation*}
2 M W_{1}\left(Q^{2}, \nu\right) \rightarrow F_{1}(x) \tag{2.13a}
\end{equation*}
$$

and

$$
\begin{equation*}
\nu W_{2}\left(Q^{2}, \nu\right) \rightarrow F_{2}(x) . \tag{2.13b}
\end{equation*}
$$

The independence of structure functions with respect to $Q^{2}$ has been experimentally tested [6]. These data were taken at fixed $x$ with varying $Q^{2}$. All measurements were consistent with no $Q^{2}$ dependence. Proton data showing this effect for the structure function $F_{2}$ can be seen in Figure 3. Substituting Equations 2.13a and 2.13b into Equation 2.9, we find the cross section in terms of the structure functions $F_{1}(x)$ and $F_{2}(x)$ :

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\frac{4 \alpha^{2}\left(E^{\prime}\right)^{2}}{Q^{4}} \cos ^{2}\left(\frac{\theta}{2}\right)\left[\frac{F_{2}(x)}{\nu}+\frac{2 F_{1}(x)}{M} \tan ^{2}\left(\frac{\theta}{2}\right)\right] . \tag{2.14}
\end{equation*}
$$



Figure 3: SLAC proton data showing no $Q^{2}$ dependence of $\nu W_{2}=F_{2}$ in the DIS region [11].

### 2.4 Nuclear Structure Functions and The Quark Parton Model

Having shown that the nucleons consist of structureless partons, we can define physics quantities in terms of the Quark-Parton Model (QPM) [6]. The QPM defines kinematic properties of the quarks and the nucleon structure functions in terms of constituent quark properties in the Bjorken limit. In the DIS regime, Bjorken $x$ is the fraction of the nucleon momentum and energy carried by the struck quark, when the scattering is viewed in a reference frame where the nucleon moves with infinite momentum.

To analyze the structure functions of the nucleon, we first look at elastic scattering off of a parton. In this setup, we imagine that we have the means to determine what parton type the electron was scattered from. This is the same equation as Equation 2.10, but with $\alpha$ multiplied by the charge of the parton being scattered from, $e_{i}$, and replacing $M$ with $x M$ for the mass of the parton. The resulting cross section is:

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}=\frac{\alpha^{2} e_{i}^{2}}{4 E^{2} \sin ^{4} \frac{\theta}{2}}\left[\cos ^{2} \frac{\theta}{2}+\frac{Q^{2}}{2 x^{2} M^{2}} \sin ^{2} \frac{\theta}{2}\right] \delta\left(\nu-\frac{Q^{2}}{2 x M}\right) . \tag{2.15}
\end{equation*}
$$

Comparing this with the nuclear inelastic cross section, it is clear that the nuclear structure functions can be written in terms of parton structure functions. The parton structure functions can be derived using the same method as the nuclear structure functions, resulting in

$$
\begin{equation*}
W_{1}^{i}=\frac{e_{i}^{2} Q^{2}}{4 M^{2} x^{2} \nu} \delta\left(1-\frac{Q^{2}}{2 M x \nu}\right) \tag{2.16a}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{2}^{i}=\frac{e_{i}^{2}}{\nu} \delta\left(1-\frac{Q^{2}}{2 M x \nu}\right) . \tag{2.16b}
\end{equation*}
$$

Defining $f_{i}(x)$ as the probability that a parton of type $i$ has the momentum fraction $x$, or parton distribution, we can then write the nucleon structure functions in terms of the parton distribution functions. The delta function makes the integrals trivial, yielding:

$$
\begin{equation*}
W_{1}\left(Q^{2}, \nu\right)=\sum_{i} \int_{0}^{1} \frac{e_{i}^{2} Q^{2}}{4 M^{2} x^{2} \nu} f_{i}(x) \delta\left(1-\frac{Q^{2}}{2 M x \nu}\right) d x=\sum_{i} \frac{e_{i}^{2}}{2 M} f_{i}(x) \tag{2.17a}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{2}\left(Q^{2}, \nu\right)=\sum_{i} \int_{0}^{1} \frac{e_{i}^{2}}{\nu} f_{i}(x) \delta\left(1-\frac{Q^{2}}{2 M x \nu}\right) d x=\sum_{i} \frac{e_{i}^{2}}{\nu} f_{i}(x) . \tag{2.17b}
\end{equation*}
$$

Using the definitions of the $F_{1}$ and $F_{2}$ structure functions in the previous sections, this formalism allows us to write them in terms of parton quantities:

$$
\begin{equation*}
M W_{1}\left(Q^{2}, \nu\right)=\sum_{i} \frac{e_{i}^{2}}{2} f_{i}(x) \equiv F_{1}(x) \tag{2.18a}
\end{equation*}
$$

and

$$
\begin{equation*}
\nu W_{2}\left(Q^{2}, \nu\right)=\sum_{i} e_{i}^{2} x f_{i}(x) \equiv F_{2}(x) . \tag{2.18b}
\end{equation*}
$$

These equations lead to a relation between the structure functions (in the Bjorken limit) called the Callan-Gross relation:

$$
\begin{equation*}
F_{2}(x)=2 x F_{1}(x) . \tag{2.19}
\end{equation*}
$$

Deriving the structure functions in terms of parton quantities also allows us to place constraints on the ratio of $F_{2}$ for the two nucleons. Due to mass constraints, we can restrict this analysis to up $(u)(q=2 / 3)$, down $(d)(q=-1 / 3)$, and strange $(s)(q=-1 / 3)$ quarks. Since the proton $(p)$ and neutron $(n)$, along with the up and down quarks, form an isospin doublet, we can relate their quark distributions and extend this to their antiquark distributions:

$$
\begin{array}{r}
u^{p}(x)=d^{n}(x) \equiv u \\
d^{p}(x)=u^{n}(x) \equiv d \\
s^{p}(x)=s^{n}(x) \equiv s \tag{2.20c}
\end{array}
$$

Using the above relations, we can write the nucleon structure functions and their ratio as:

$$
\begin{align*}
F_{2}^{p} & =x\left[\frac{4}{9}(u+\bar{u})+\frac{1}{9}(d+\bar{d})+\frac{1}{9}(s+\bar{s})\right],  \tag{2.21a}\\
F_{2}^{n} & =x\left[\frac{4}{9}(d+\bar{d})+\frac{1}{9}(u+\bar{u})+\frac{1}{9}(s+\bar{s})\right] \tag{2.21b}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{F_{2}^{n}}{F_{2}^{p}}=\frac{[(u+\bar{u})+(s+\bar{s})]+4(d+\bar{d})}{[(d+\bar{d})+(s+\bar{s})]+4(u+\bar{u})} \tag{2.22}
\end{equation*}
$$

where $\bar{u}, \bar{d}$, and $\bar{s}$ denote antiquark distributions.
The latter equation can be evaluated noting that by definition quark distributions must be positive. This naturally leads to a constraint on $F_{2}^{n} / F_{2}^{p}$ known as the Nachtmann inequality:

$$
\begin{equation*}
\frac{1}{4} \leq \frac{F_{2}^{n}}{F_{2}^{p}} \leq 4 \tag{2.23}
\end{equation*}
$$

## $2.5 R=\sigma_{L} / \sigma_{T}$

If we instead approach DIS as the production and absorption of a virtual photon by a parton, we can extract a different structure function $R=\sigma_{L} / \sigma_{T}$, referred to as photonuclear $R$. That is, the ratio of the cross sections for absorbing longitudinal photons, $\sigma_{L}$, to transverse photons, $\sigma_{T}$. In the Bjorken limit, as in the previous section, $R \rightarrow 0$. In practice, the Bjorken limit is an imperfect approximation and it is useful to consider the effects of large, but finite, $Q^{2}$ and $\nu$.

We can write the DIS cross section in terms of these cross sections as:

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}\left(E, E^{\prime}, \theta\right)=\Gamma\left[\sigma_{T}\left(x, Q^{2}\right)+\epsilon \sigma_{L}\left(x, Q^{2}\right)\right] \tag{2.24}
\end{equation*}
$$

In this equation, $\epsilon$ is the degree of the longitudinal polarization of the virtual photons and $\Gamma$ is the equivalent flux of virtual photons. These two quantities are defined by

$$
\begin{equation*}
\Gamma=\frac{\alpha K E^{\prime}}{2 \pi^{2} Q^{2} E_{0}(1-\epsilon)} \tag{2.25}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon=\frac{1}{1+2\left(1+\nu^{2} / Q^{2}\right) \tan ^{2}\left(\frac{\theta}{2}\right)} . \tag{2.26}
\end{equation*}
$$

Where, $K$ is the laboratory photon energy producing a final state of total mass $W$ upon absorption by a nucleus at rest:

$$
\begin{equation*}
K=\frac{W^{2}-M^{2}}{2 M} \tag{2.27}
\end{equation*}
$$

By comparing these equations to Equation 2.14, $F_{1}$ and $F_{2}$ can be related to $\sigma_{L}$, $\sigma_{T}$, and each other by:

$$
\begin{gather*}
\sigma_{T}=\frac{4 \pi \alpha^{2}}{K M} F_{1},  \tag{2.28}\\
\sigma_{L}=\frac{4 \pi \alpha^{2}}{K M} \frac{1}{2 x}\left[F_{2}-2 x F_{1}\right], \tag{2.29}
\end{gather*}
$$

and

$$
\begin{equation*}
F_{1}=\frac{F_{2}\left(1+Q^{2} / \nu^{2}\right)}{2 x(1+R)} \tag{2.30}
\end{equation*}
$$

Substituting $F_{1}$ from Equation 2.30 into our DIS cross section equation, Equation 2.14, we can eliminate $F_{1}$. This also makes it clear that we can easily access the $F_{2}$ structure functions by measuring cross section ratios because Equation 2.14 now becomes:

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega d E^{\prime}}\left(E, E^{\prime}, \theta\right)=\frac{4 \alpha^{2}\left(E^{\prime}\right)}{Q^{4}} \cos ^{2}\left(\frac{\theta}{2}\right) F_{2}\left[\frac{1}{\nu}+\frac{\left(1+Q^{2} / \nu^{2}\right)}{x M(1+R)} \tan ^{2}\left(\frac{\theta}{2}\right)\right] . \tag{2.31}
\end{equation*}
$$

If we measure the cross section ratios of two different nuclear targets, $A$ and $B$, at the same kinematics (that is at the same $E, E^{\prime}$, and $\theta$ ), we find:

$$
\begin{equation*}
\frac{\sigma_{A}}{\sigma_{B}}=\frac{F_{2}^{A}}{F_{2}^{B}} \frac{\left[\frac{1}{\nu}+\frac{\left(1+Q^{2} / \nu^{2}\right)}{x M\left(1+R^{A}\right)} \tan ^{2}\left(\frac{\theta}{2}\right)\right]}{\left[\frac{1}{\nu}+\frac{\left(1+Q^{2} / \nu^{2}\right)}{x M\left(1+R^{B}\right)} \tan ^{2}\left(\frac{\theta}{2}\right)\right]} \tag{2.32}
\end{equation*}
$$

As shown in Figure 4, historical data suggest that the photonuclear $R$ has no nuclear dependence to within $10 \%$. If we assume that that there is no nuclear dependence, Equation 2.32 simplifies to:

$$
\begin{equation*}
\frac{\sigma_{A}}{\sigma_{B}}=\frac{F_{2}^{A}}{F_{2}^{B}} . \tag{2.33}
\end{equation*}
$$

Therefore, measuring the cross section ratios of nuclear targets in the DIS region, we can easily access the $F_{2}$ nuclear structure functions of the targets.

## $2.6 F_{2}^{n} / F_{2}^{p}$

The structure functions of the nucleons are common inputs to models. Making use of a Hydrogen target, the $F_{2}^{p}$ structure function is easily experimentally accessible. Unfortunately, there is no free neutron target. This absence means that there is no direct way to measure $F_{2}^{n}$. However, with the proper input, we can extract the ratio $F_{2}^{n} / F_{2}^{p}$.


Figure 4: Historical data of $R=\sigma_{L} / \sigma_{T}$. This data shows measurements of the difference in $R$ between two nuclei. The data is consistent with no nuclear dependence [12].

To extract the latter ratio, we first define "EMC-type" ratios. These are simply the ratio of the nuclear structure function to the sum of the structure functions of its constituent nucleons. The "EMC-type" ratios for ${ }^{3} \mathrm{He}$ and ${ }^{2} \mathrm{H}$ are:

$$
\begin{align*}
R_{\text {EMC }}\left({ }^{3} \mathrm{He}\right) & =\frac{F_{2}^{3} \mathrm{He}}{2 F_{2}^{p}+F_{2}^{n}}  \tag{2.3}\\
R_{\mathrm{EMC}}\left({ }^{2} \mathrm{H}\right) & =\frac{F_{2}^{2} \mathrm{H}}{F_{2}^{p}+F_{2}^{n}} \tag{2.35}
\end{align*}
$$

The two ratios above can be used to create a "Super-Ratio", $\mathcal{R}$, as the ratio of "EMC-type" ratios:

$$
\begin{equation*}
\mathcal{R}=\frac{R_{\text {EMC }}\left({ }^{3} \mathrm{He}\right)}{R_{\mathrm{EMC}}\left({ }^{2} \mathrm{H}\right)}=\frac{F_{2}^{3} \mathrm{He}}{2 F_{2}^{p}+F_{2}^{n}} \cdot \frac{F_{2}^{p}+F_{2}^{n}}{F_{2}^{2} \mathrm{H}} . \tag{2.36}
\end{equation*}
$$

Solving Equation 2.36 for $F_{2}^{n} / F_{2}^{p}$ makes it clear that this quantity can be easily extracted with a cross section ratio measurement and a model input for $\mathcal{R}$, as follows:

$$
\begin{equation*}
\frac{F_{2}^{n}}{F_{2}^{p}}=\frac{F_{2}^{3} \mathrm{He} / F_{2}^{2} \mathrm{H}-2 \mathcal{R}}{\mathcal{R}-F_{2}^{{ }^{3} \mathrm{He}} / F_{2}^{2} \mathrm{H}} . \tag{2.37}
\end{equation*}
$$

## CHAPTER 3

## The EMC Effect

### 3.1 History

The EMC effect was first discovered by its namesake, the European Muon Collaboration (EMC group), in 1983 [2]. The EMC group measured the structure functions of hydrogen, deuterium, and iron. After correcting for the neutron excess in iron, the per nucleon $F_{2}$ structure function ratio of iron to deuterium was calculated. As seen in Figure 5, the data showed a clear $x$ dependence, contrary to expectations.

Prior to this original measurement, nucleons were assumed to be quasi-free within the nucleus. In this understanding, the nuclear $F_{2}$ structure function would be described as

$$
\begin{equation*}
F_{2}^{A}=Z F_{2}^{p}+(A-Z) F_{2}^{n}, \tag{3.1}
\end{equation*}
$$

where $A$ is the nuclear mass number and $Z$ is the nuclear charge number. This description leads to the prediction that the per nucleon structure function ratio of any two isoscalar targets (nuclear targets with the same number of protons and neutrons) will be unity. At that time, the only other expected nuclear effect was Fermi motion, which would cause a sharp rise in the per nucleon structure function ratio at high $x$, but would leave the ratio largely unchanged at low $x$.

The original EMC experiment did not originally set out to measure the EMC effect, rather the data were a byproduct of efforts to achieve higher luminosity. Because of this, the data had very large uncertainties. However, the uncertainties were small enough that the anomaly could be exhibited.

Shortly after the original EMC measurement, a Rochester-SLAC-MIT group analyzed previous SLAC data to confirm the phenomenon. The data analyzed not only confirmed the effect in iron, but also in aluminum. The iron data showed the EMC effect, as well as the expected rise from Fermi motion at high $x$. The aluminum data showed these phenomena, as well as low $x$ shadowing and anti-shadowing (see Sections 3.3 .1 and 3.3.2) [13, 14, 15].


Figure 5: Results from the EMC collaboration showing a clear $x$ dependence of the iron per-nucleon $F_{2}$ structure function ratio [2].

### 3.2 Further Results

Since that time, numerous experiments have taken additional nuclear $F_{2}$ structure function data in order to better understand the nature of this anomaly. These searches have primarily focused on heavy isoscalar nuclei. The following represents a non-exhaustive presentation of these experiments.

### 3.2.1 SLAC

At SLAC, a new experiment, E139, was set up with the explicit goal of measuring the EMC effect in a wide range of nuclei. The data cover a large kinematic range, $0.089 \leq x \leq 0.8$ and $2(\mathrm{GeV} / c)^{2} \leq Q^{2} \leq 15(\mathrm{GeV} / c)^{2}$, and confirm the phenomenon seen by the EMC group in the $x>0.3$ region, but also exhibit a downturn in the ratios at low $x$.

The targets studied were ${ }^{4} \mathrm{He},{ }^{9} \mathrm{Be},{ }^{12} \mathrm{C},{ }^{27} \mathrm{Al},{ }^{40} \mathrm{Ca},{ }^{56} \mathrm{Fe},{ }^{108} \mathrm{Ag}$, and ${ }^{197} \mathrm{Au}$. The SLAC E139 results are shown in Figure 6. This large target ensemble allowed them to study the mass number $A$-dependence of the EMC effect. These data suggest an approximately $\ln (A)$ dependence on the strength of the EMC effect, with notable outliers of ${ }^{4} \mathrm{He}$ and ${ }^{9} \mathrm{Be}[16,17]$.


Figure 6: Results from SLAC experiment E139 showing an $A$-dependent EMC effect [17].

### 3.2.2 BCDMS

The BCDMS experiment at CERN measured the ${ }^{14} \mathrm{~N}$ and ${ }^{56} \mathrm{Fe}$ EMC ratios. The iron data are consistent with the original EMC measurement within a normalization discrepancy. The nitrogen data are consistent with the SLAC carbon data. The BCDMS results show no $Q^{2}$ dependence in the EMC effect. However, BCDMS does not demonstrate an $A$-dependence of the EMC effect, as SLAC did [15].

### 3.2.3 EMC

The EMC group performed three more experiments to study the EMC effect.
The first of these experiments set out to improve the systematics of the original experiment. This followup measured the EMC ratio of ${ }^{12} \mathrm{C},{ }^{63} \mathrm{Cu}$, and Sn . These data agree with the original data for $x \geq 0.08$. However, below this threshold the data indicate a downturn, the shadowing region, that the original experiment did not reveal [18].

The second experiment (NA28) focused on studying the EMC ratio of ${ }^{12} \mathrm{C}$ and ${ }^{40} \mathrm{Ca}$ at low $x$. The data confirm the shadowing effect seen in the previous EMC data, where the ratio drops
below unity in the region of $x<0.1$. These results also show that the shadowing region has no $Q^{2}$ dependence. The data overlap well with previous measurements.

The last EMC group experiment to study the EMC effect remeasured the copper EMC ratio. To minimize systematics, two ${ }^{2} \mathrm{H}$ targets and three ${ }^{63} \mathrm{Cu}$ targets were used. These results agree with the results from the first followup and are of greater precision.

### 3.2.4 NMC

The New Muon Collaboration (NMC) continued the study of the EMC effect at CERN. Initially, NMC measured the EMC ratio of ${ }^{6} \mathrm{Li},{ }^{12} \mathrm{C}$, and ${ }^{40} \mathrm{Ca}$ to high precision at low $x[19]$. These data were taken with two goals: to confirm the EMC data in the shadowing region and to study the effect of nuclear size and density on the EMC ratio. The data confirm the previous EMC measurement and found a very weak $Q^{2}$ dependence. Lithium and carbon have approximately the same nuclear size, but different nuclear densities. Calcium and carbon have approximately the same nuclear density, but calcium is larger. It was found that both of these factors play a part in the suppression of the EMC ratio in the shadowing region. Increases in nuclear size or density show an increase in the suppression of the EMC ratio due to nuclear shadowing.

NMC then set out to study the difference between the photonuclear $R$ of different targets in the region of $0.01 \leq x \leq 0.3$. The results were found, within uncertainties, to be compatible with zero. This result confirms that $\sigma_{A} / \sigma_{D}=F_{2}^{A} / F_{2}^{D}$.

Finally, the NMC group studied the EMC effect on ${ }^{9} \mathrm{Be},{ }^{12} \mathrm{C},{ }^{27} \mathrm{Al},{ }^{40} \mathrm{Ca},{ }^{56} \mathrm{Fe}, \mathrm{Sn}$, and ${ }^{208} \mathrm{~Pb}$. These results again confirm that there is no $Q^{2}$ dependence of the EMC ratio above $x=0.06$. These data agree with the SLAC E139 finding that the EMC effect is, to a good approximation, logarithmic with $A$ [20].

### 3.2.5 HERMES

The HERMES experiment ran at the HERA collider in Hamburg, Germany. This experiment collided positrons with nuclei to study the EMC effect on ${ }^{3} \mathrm{He}$ and ${ }^{14} \mathrm{~N}$. Data were taken for $0.013 \leq x \leq 0.65$ and $0.5(\mathrm{GeV} / c)^{2} \leq Q^{2} \leq 15(\mathrm{GeV} / c)^{2}$. In the $x<0.06$ region, the HERMES results differed drastically from the NMC results. Initially, this was misreported as an $A$ dependence of photonuclear $R$. This reporting was later amended when it was found that the difference could be attributed to a previously unaccounted for systematic effect [21, 22].

### 3.2.6 JLab

E03-103
The E03-103 experiment ran in Hall C at Jefferson Lab. This experiment studied the EMC effect in ${ }^{3} \mathrm{He},{ }^{4} \mathrm{He},{ }^{9} \mathrm{Be}$, and ${ }^{12} \mathrm{C}$. The kinematics covered the range of $0.3<x<0.9$ and $3(\mathrm{GeV} / \mathrm{c})^{2}<$ $Q^{2}<6(\mathrm{GeV} / c)^{2}$. The data measured was not purely DIS, but also included data in the resonance region. This led the experiment to extensively verify that their data was indeed independent of $Q^{2}$. The data for Beryllium are noted not to match the previous SLAC E139 data. This is caused by the use of a different isoscalar correction and is further rectified by noting normalization uncertainties. These data show a significantly larger EMC effect in Beryllium than expected by the $\ln (A)$ prediction, which is consistent with SLAC noting that Beryllium is an outlier. A suggested explanation is that the EMC effect may be dependent on local nuclear density rather than mean nuclear density. The ${ }^{3} \mathrm{He}$ results are displayed in Figure 7 [23].

## CLAS

A recent study of electron scattering data in Hall B at Jefferson Lab using CLAS examined the relationship between the EMC effect and Short-Range Correlations [24]. This data measured the strength of the EMC effect and the Short-Range Correlations scaling coefficient $a_{2}$ in ${ }^{12} \mathrm{C},{ }^{27} \mathrm{Al},{ }^{56} \mathrm{Fe}$, and ${ }^{208} \mathrm{~Pb}$. This study, which also analyzed SLAC and E03-103 data, found very good agreement between data and the Short-Range Correlations model [24].

### 3.3 Structure Function Ratio Regions

In DIS $F_{2}$ structure function data, there are four phenomenological regions. In each region, different physics dominates the shape of the structure function ratio. Each kinematic region provides a test bed for our understanding of nuclear physics. Studying these nuclear effects is the driving force behind many experiments [25, 26]. Figure 8 shows a combination of iron or copper to deuterium structure function ratio data from EMC, SLAC, and BCDMS where the four phenomenological regions are clearly visible.

### 3.3.1 Shadowing

Nuclear shadowing is a phenomenon that occurs in the region of $x<0.1$. Here, there is a depletion of the structure function when compared to deuterium. This depletion increases with mass number

E03-103 Helium-3 EMC Ratio


Figure 7: Helium-3 results from JLab experiment E03-103. The upper red squares are the raw ratio and the lower blue circles have an isoscalar correction applied. The error bars include both statistical and systematic uncertainties, though they do not include an overall normalization uncertainty of 1.84\% [23].
$A$ and is weakly dependent on $Q^{2}$. This effect is typically explained by hadronic behavior of the virtual photon, which can be further investigated in reference [28].

### 3.3.2 Anti-shadowing

The Anti-shadowing, or enhancement, region is from $0.1 \leq x \leq 0.3$. In this region, the EMC structure function ratios are enhanced to greater than 1. Within experimental uncertainties, there is no $Q^{2}$ dependence in the anti-shadowing region. This effect is typically explained through overlapping partons in the nucleus [12].

### 3.3.3 EMC Effect

The EMC effect region spans from $0.3 \leq x \leq 0.8$. In this kinematic area, the EMC structure function ratio falls off and reaches a minimum around $x=0.65$. Since the discovery of the effect by the European Muon Collaboration, extensive EMC effect region data have been taken over a large


Figure 8: Iron or copper to deuterium structure function ratio data from EMC, SLAC, and BCDMS. The four phenomenological structure regions (shadowing, anti-shadowing, EMC effect, and Fermi motion) are clearly visible [27].
$Q^{2}$ range. The data suggest that the EMC effect is largely independent of $Q^{2}$. The EMC effect does appear to be logarithmically dependent on mass number $A$.

### 3.3.4 Fermi Motion

As $x$ is pushed past 0.8 , the EMC structure function ratio sharply increases far beyond unity. In this region, it is known that $F_{2}^{n}$, the free nucleon structure function, drops as $(1-x)^{3}$. Fermi motion increases the structure function of the bound nucleon, causing the ratio to show this sharp increase.

### 3.4 Theories

There are many theories as to the origin of the EMC effect. To cover them all is beyond the scope of this thesis; however, this section will discuss the broad classes of models which can be investigated further in $[15,12,29]$.

There are two primary groups of theories: i) nuclear structure, which focuses on the physics of scattering from a nucleus and ii) nucleon modification, which focuses on changes to quark momenta due to confinement effects.

### 3.4.1 Nuclear Structure

## Nucleon Models

Traditional scattering calculations assume that the scattered nucleon was on-shell. This class of models gives the struck nucleon an average separation energy $\langle\epsilon\rangle$, resulting in a shift of its average energy. The inclusion of this term moves the nucleon off-shell. This energy shift causes a rescaling of $x$, which can explain the EMC effect region and the Fermi motion region. However, it is not capable of reproducing the anti-shadowing region.

## Pion Enhancement

Pion Enhancement models describe an enhancement of the nuclear pion field due to nucleon-nucleon interactions. In these models, the pion contribution is concentrated to low $x$. The creation of pions also requires the creation of $\Delta$ resonances in the nucleus.

Alone, this class of models has several problems. To reproduce high- $x$ data requires the presence of significantly more $\Delta \mathrm{s}$ than calculations suggest are plausible. In addition to this, matching antishadowing data causes a mismatch in high- $x$ data.

### 3.4.2 Nucleon Modification

## Quark Bags

In quark cluster models, quarks are confined to "bags" as defined by the MIT bag model [15]. Each bag must create a color-singlet state with multiples of 3 quarks. The most common quark bag models rely on 6 -quark bags. 6 -quark bags are larger than a nucleon, which consists of 3 quarks, and thus lead to partial deconfinement of the quarks. This increase in size of the quark confinement leads to a decrease in quark momenta due to the Heisenberg uncertainty principle. A decrease in
quark momenta in this way will suppress the structure functions in the EMC region, giving rise to the EMC effect [15, 29].

This quark bag model alone can explain many nuclear effects. It is hampered by the need for an additional free parameter to compute each new observable. This model has fallen out of favor due to failed predictions in the nuclear Drell-Yan process [29].

## Mean Field Enhancement

Mean Field Enhancement suggests that the structure functions of the nucleons are modified by the rest of the nucleus surrounding them. Nucleons confined within the nucleus exchange mesons between the quarks of other confined nucleons. This modifies the structure of the nucleon as it changes the size of the quark confinement. The predicted increase in confinement size yields a smaller quark momentum [29, 30].

## Short Range Correlations

Short-Range Correlations (SRCs) greatly modify a few nucleons, rather than the small modification to all nucleons in mean-field enhancement. SRCs are inferred from the notion that there is a probability that two nucleon wavefunctions will overlap. In this scenario, the overlapping wavefunctions will cause the size of quark confinement within the correlated nucleons to greatly increase, drastically decreasing the quark momenta.

SRCs also predict an observed high momentum cross section tail at $x>1$. Studies of this effect have noted a correlation between the SRC "scale factor" and the strength of the EMC effect, i.e., the slope of the EMC ratio in the EMC region [29, 31, 32].

## Discerning between Mean Field Enhancement and SRCs

Both Mean Field Enhancement and SRCs have been shown to have very good predictive power within the datasets available. This leads to the conundrum of finding an unmeasured quantity for which the two models make different predictions. Hen [29] and Thomas [33] argue that mean field theory and SRCs make seemingly contradictory predictions for the polarized EMC effect. Mean field enhancement predicts that polarization will enhance the effect; SRCs predict the polarization to minimize the effect. This will be tested in the future at Jefferson Lab in Hall B with the CLAS spectrometer by measuring the spin structure functions of ${ }^{7} \mathrm{Li}[34]$.

## CHAPTER 4

## The Experimental Setup

### 4.1 CEBAF Accelerator

The Continuous Electron Beam Accelerator Facility (CEBAF) accelerator is a recirculating accelerator at the Thomas Jefferson National Accelerator Facility (JLab). That is, there are two linear accelerators (linacs) connected by recirculation arcs. CEBAF has recently undergone an upgrade which increased the maximum possible energy to 12 GeV (to Hall D). A schematic of the 12-GeV configuration of CEBAF can be seen in Figure 9. The process of electrons traveling through both linacs a single time is called a "pass". Halls A, B, and C are capable of receiving up to 5-pass beams; Hall D can receive up to 5.5-pass beam. The beam provided is of a Continuous Wave (CW) form, comprised of a steady stream of electrons, rather than many electrons in short pulses.

### 4.2 Beamline Components

When the electrons from the CEBAF Accelerator have circulated the desired number of passes, they then enter the Hall A beamline. The Hall A Beamline has several measurement devices that allow the experimenter to fully control the properties of the beam that is being delivered to the hall. A schematic of Hall A with the beamline components that are present in the hall can be seen in Figure 10. High beam quality and understanding the beam characteristics are critical for proper execution and accurate analysis of an experiment. In the MARATHON experiment, the critical beamline components (which will be described in this section) are:

- Beam Arc Energy Measurement
- Beam Current Monitor
- Raster
- Beam Position Monitor


Figure 9: The current $12-\mathrm{GeV}$ configuration of the CEBAF accelerator with the upgrades that were made to the $6-\mathrm{GeV}$ configuration. Shown also are the four experimental Halls: A, B, C, and D [35].

### 4.2.1 Arc Energy Measurement

Knowing the energy of the beam into the hall is critical for controlling the rest of the kinematics of the scattered electrons that are measured. This is done by measuring the deflection of the beam when passing through a series of eight dipoles in the beam arc leading to the hall. This measurement requires wire scanners to measure the bend angle of the beam through the arc and a suitable probe to measure the magnetic field integral of the dipole magnets.

The wire scanners are "harp" forks in the beamline, two before and two after the arc. A harp consists of three tungsten wires that are introduced sequentially into the path of the beam using a stepper motor. When the beam is incident on a wire, an electromagnetic shower is induced on the wire which is read by a photomultiplier tube (PMT). By determining when each wire is struck by the beam, the position of the beam can be determined very accurately. Using two harps in each position also allows for beam direction measurement. Using a harp is a destructive measurement; it cannot be done simultaneously to beam delivery into the hall for data taking.


Figure 10: An overhead schematic of the Hall A Facility of JLab [36].


Figure 11: A schematic drawing of a harp wire scanner. The harp is introduced into the beamline with a stepper motor. The three wires create a signal when they interact with the beam, allowing for highly accurate beam position determination [37].

The magnetic field integral of the eight magnets is measured in a ninth reference dipole that is not in the beamline. This ninth dipole is identical to the eight dipoles in the arc and is powered in series with the other dipoles. Measuring the field integral of the dipole requires a probe to be within the magnet, necessitating the use of this reference magnet [38].

After measuring the field integral $\int \vec{B} \cdot \overrightarrow{d l}$ (in Tm) and angle $\theta$ (in radians), the beam momentum (in $\mathrm{GeV} / c$ ) can be calculated using the well-known formula

$$
\begin{equation*}
p=k \frac{\int \vec{B} \cdot \overrightarrow{d l}}{\theta} \tag{4.1}
\end{equation*}
$$

where $k=0.299792 \mathrm{GeV} \cdot \mathrm{rad} /(\mathrm{Tm} c)$.

### 4.2.2 Beam Current Monitor

The Hall A Beam Current Monitor (BCM) is comprised of an Unser monitor and two radio frequency (RF) cavities. The Unser, a Parametric Current Transformer, provides an absolute reference for the RF cavities. Each RF cavity is tuned to the frequency of the beam ( 1.497 GHz ). The resonance then produces a voltage proportional to the beam current. The signals are then split to be either sampled or integrated. The sampled signal outputs the RMS of the voltage over a 1 second period. This is equivalent to the average beam current for that second. The signal that is integrated is first sent to an RMS-to-DC converter which is then fed to a Voltage-to-Frequency converter. This signal is then sent to a scaler that accumulates over a run. The final scaler value is proportional to the total accumulated charge in the run.

## Beam Current Monitor Calibration

The Unser is calibrated by putting a current on a wire that is inside of the Unser cavity and measuring the signal that is output. The calibration of the Unser drifts quite quickly, so it is used to calibrate the RF cavities but cannot be used for long-term monitoring. Once the Unser is properly calibrated, the reading can be used to determine the calibration for the RF cavities. The RF cavity calibration is a linear relationship between the RF cavity reading and the beam current.


Figure 12: The Hall A raster consists of four dipole magnets on the beamline.

When the beam enters Hall A, it has very little spread, meaning that all of the electrons will strike the target in one small location (typically within $80-200 \mu \mathrm{~m}$ ). This poses an issue for the targets in use. Depending on the beam current in use, a localized beam spot can significantly heat up the target. In the case of solid targets, a focused beam spot risks melting the target. For gas targets, there is a potential for cell rupture. The raster, shown in Figure 12, exists in the beamline
to mitigate this risk by spreading the beam over a larger area on the target. The larger beam spread helps to reduce localized heating of the target due to the incident beam. The raster is a set of four dipole magnets: two for steering horizontally (X) and two for steering vertically (Y) [39].


Figure 13: The X and Y raster pairs are each synced to produce the maximum kick. The X and Y directions are uncorrelated so that the beam travels uniformly over the target. These plots are the from the "online analysis script" that allows shift takers to ensure that all systems are working correctly. The output shown here is a properly working raster spectrum.

The magnet pairs that work in the same direction ( X or Y ) are synced, which ensures that they maximize the beam spread and do not work against each other. This characteristic can be seen in Figure 13. Each raster magnet is powered by a triangle wave of different frequencies to minimize harmonics. The horizontal rasters are set to 24.5 kHz and the vertical rasters are set to 25 kHz [40]. The triangle wave ensures that equal time is spent at all points in the rastered area. Figure 14 shows a typical raster spectrum as recorded by the High Resolution Spectrometer (HRS).


Figure 14: An example of a raster current spectrum. The range and size will change with ADCs used, beam energy, and raster size. The shape should always stay the same. The "bedposts" on the edges are due to rounding of the triangular waveform by a low-pass filter.

## Raster Calibration

The raster is calibrated by defining a line that maps the raster current to positions at each BPM and the target. To do this, the slope and intercept of this line had to be determined. The slope corresponds to the conversion of raster current to position displacement. The intercept is then determined from the central position that the beam is displaced from. This section will be a general presentation of the techniques used to calibrate the raster. For a more in-depth discussion of how the raster was calibrated, see Appendix A.

For the horizontal raster, this was done by optimizing the reconstructed $z$-vertex on the target. When properly calibrated, there should be no correlation between the horizontal raster and the $z$-vertex. Linear interpolation between two "bad" calibrations is a simple way to determine the correct calibration slope.

The vertical raster could be calibrated in a similar way by minimizing the correlation between the vertical raster and a known momentum phenomenon (e.g. a $W^{2}$ peak). Unfortunately, such a feature does not exist within the kinematics of MARATHON data. The vertical calibration was


Figure 15: A schematic of the target coordinate system. Note that X and Y in the raster coordinate system are reversed from x and y in the target coordinate system. $z$ is along the axis of the target, $y$ is horizontal, and $x$ is vertical.
determined using the available carbon hole target. The hole is known to have a 2 mm diameter. By using the raster data, the hole can be fit in order to determine the vertical calibration slope.

The intercepts are determined by looking at the mean BPM position readings and projecting these to the target. This position will correspond to the mean value of the rasters as well. Using the beam position, raster current, and calibration slope the calibration intercept can easily be determined.

The raster and the target each have a different coordinate system. Raster X and Y are reversed with the target y and x. The Hall A Analyzer software transforms the coordinate system of the raster to the coordinate system of the target, so no special steps need to be taken in order to facilitate this. A diagram of the target coordinate system can be seen in Figure 15.

### 4.2.4 Beam Position Monitors

The Beam Position Monitors (BPMs) are a pair of measurement devices that consist of four sensing wires, as diagrammed in Figure 16. These four sensing wires are tuned to the fundamental frequency of the beam. Using the signal received from each wire, the experimenter can reconstruct the position of the beam as it passed the BPM. Using both BPMs in conjunction allows the experimenter to determine the beam trajectory and where the electrons are incident on the target.


Figure 16: The Beam Position Monitor (BPM) uses four sensing wires to determine the beam position. Since the wires do not actually touch the beam, this measurement can be done during data taking [37].

The BPM electronics have a phase lag between the BPM measurement and the actual beam position. This means that the BPMs cannot provide position information on an event-by-event basis. However, they do provide a measure of the average position of the beam with a record of beam spread. This information is a critical component to calibrating the raster to provide accurate event-by-event position information.

## Beam Position Monitor Calibration

Using the BPMs alone provides only a reference position. The BPMs must be calibrated using a harp in order to measure the absolute beam position. This is done by a "bullseye scan", shown in Figure 17. This is accomplished by moving the beam to five positions corresponding to the corners


Figure 17: A bullseye scan maps five positions of the beam with the harp, shown as runs 805-809. The BPM calibration is then adjusted to make the reconstructed beam position, shown as gray blocks, match the harp data.
of a square and the center of the square. The harp will give an absolute position measurement of the beam at these positions. The BPMs are also used to measure the beam position as well. The calibration is done by determining the transformation coefficients that will convert the BPM signals to the positions returned by the harps.

### 4.3 Tritium Target System

When the beam reaches the center of Hall A, it will meet the target. Here the beam will either interact with the target and scatter, allowing for detection of events that are within the acceptance of the spectrometer, or pass through the target and be deposited in the beam dump. Figure 18 shows a side view of the hall, with the beam going from left to right. This view of the hall clearly shows these two possible paths for the beam (along the dashed line). For clarity, the spectrometer is drawn at a $0^{\circ}$ scattering angle, which is not a position the spectrometer can physically occupy.

### 4.3.1 Gas Cell Design

The gas targets used in this experiment were housed in a specially designed cell. This cell deviates from typical cells used in Hall A in that the gas is not circulated. The need for such a design was to meet safety protocols when using a tritium target, specifically to minimize tritium material and


Figure 18: A side-view schematic of Hall A showing the beamline, target, a High Resolution Spectrometer, and beam dump. The beam will either interact with the target or pass through to the beam dump [36].
to mitigate the risk of tritium leakage.
The target cells were 25 centimeters long and made of Aluminum 7075. The cells were sealed and utilized conductive cooling. The beam heating of the aluminum was approximately 11 W . This heat was recovered by a copper heat sink that was actively cooled by 15 K helium gas [41]. Figure 19 shows a 3 -dimensional rendering of the target cell. In the figure, the beam comes in from the bottom-left and passes through the center of the target along the axis of the cell.

The lack of gas circulation allowed for localized heating of the gas in the cell. This was caused by the energy being deposited into the gas by the incident beam. The heating caused a density change in the gas, changing the effective target thickness. This characteristic had to be addressed in the analysis of gas target data and is discussed in further detail in Section 5.5.3.


Figure 19: 3D rendering of the target cell. The black circular plate in the front is the upstream window of the cell [41].

### 4.3.2 Target Ladder

The MARATHON target ladder can be seen in Figure 20. This target ladder contained five targets that utilized the gas cell design described in the previous subsection. These are:

- Tritium
- Deuterium
- Hydrogen
- Helium-3
- Empty Cell

The cells that contain gas (i.e. all of the above except the empty target) were used for studying the physics goals of the MARATHON experiment. In particular, Helium-3 and Deuterium are the focus of this thesis. Table 2 lists the gas thicknesses and endcap thicknesses of the above gas targets.

| Target | Gas Thickness $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ | Entrance Window (mm) | Exit Window (mm) |
| :--- | :---: | :---: | :---: |
| Tritium | $77.00 \pm 0.01$ | $0.253 \pm 0.004$ | $0.343 \pm 0.047$ |
| Deuterium | $142.2 \pm 0.8$ | $0.215 \pm 0.004$ | $0.294 \pm 0.056$ |
| Hydrogen | $70.8 \pm 0.4$ | $0.311 \pm 0.001$ | $0.330 \pm 0.063$ |
| Helium-3 | $53.4 \pm 0.6$ | $0.203 \pm 0.007$ | $0.328 \pm 0.041$ |
| Empty cell | $N / A$ | $0.254 \pm 0.005$ | $0.279 \pm 0.005$ |

Table 2: Gas target thicknesses and cell wall thicknesses [42].

In addition to the gas cells, the target ladder also contained several solid targets that were used for other studies. Those relevant to this thesis are:

- 25 cm Dummy
- Carbon Hole - A carbon foil with a 2 mm diameter hole in the center
- Raster Target - An aluminum "straw" for ensuring the beam is not coming in at an angle

The Empty target is a gas cell that has a vacuum inside. The 25 cm Dummy is comprised of two Aluminum 7075 foils, the same material as the gas cells. The foils are spaced 25 cm apart, the same length as the gas cells. Each foil is $0.3495 \pm 0.0006 \mathrm{~g} / \mathrm{cm}^{2}$ thick, significantly thicker than the cell walls. These two targets are used to better understand the contribution of the gas cells to the electrons counted by the experiment. This study is discussed further in Section 5.5.4.

The Carbon Hole target is a foil made of carbon that is $0.8830 \pm 0.0002 \mathrm{~g} / \mathrm{cm}^{2}$ thick with a 2 mm diameter hole in the center. This target is used for centering the beam, as well as for determining the settings needed for a 2 mm by 2 mm raster setting. This is also used to assist calibrating the raster as documented in Appendix A.

After the beam is centered and the raster settings are determined, the Raster Target is used. This target is an aluminum "straw" that the beam should pass straight through. The goal of using this is to ensure that the beam is not approaching the target at an angle. If any counts are seen above background, then the beam is hitting the straw and can be assumed to be coming in at an angle that needs to be rectified. If the beam is angled, electrons would leave the physics targets


Figure 20: The target ladder for the MARATHON experiment. Shown are the 25 cm gas cells and several solid targets.
before passing through all of the target material. This would significantly reduce counting rates and make it very difficult to determine the effective target thickness seen by the beam.

### 4.4 The Hall A High Resolution Spectrometers

Hall A has two $4 \mathrm{GeV} / c$ High Resolution Spectrometers (HRSs), designated Left (LHRS) and Right (RHRS) corresponding to their orientation when looking downstream along the beam. In order to achieve Hall A's stated goal of $1 \%$ absolute cross section accuracy, the HRSs were designed to have $10^{-4}$ particle momentum resolution and 0.1 mrad scattering angle resolution.

Each HRS has four superconducting magnets: three $\cos (2 \theta)$ quadrupoles and a racetrack coil dipole. Utilizing a QQDQ magnet setup (named Q1, Q2, D, and Q3), each HRS has a $45^{\circ}$ bending angle in a vertical bending plane. Each HRS has a similar, but unique, detector package that accommodates precise tracking and particle identification (PID). Figure 21 shows the magnet layout of the HRSs. The setup is identical for both the Left and Right HRS.


Figure 21: A schematic view of the HRS magnet setup. Scattered particles enter from the left in Q1 and then pass through the magnets, exiting Q3 into the first VDC plane [36].

The two HRSs can be operated together for exclusive measurements or operated separately for inclusive measurements. MARATHON used each HRS separately in order to maximize counting rate. In particular, the RHRS was parked at a large angle for the highest- $x$ measurement, where the counting rate was very slow.

Each arm contains a pair of Vertical Drift Chambers (VDCs), two scintillator planes, a gas Cherenkov detector, and two segmented leaded glass calorimeters. This combination of detectors allows for fine tracking and powerful electron identification.

### 4.4.1 Vertical Drift Chambers

Each arm has two VDCs at the entrance of the detector stack. These detectors are used for fine tracking of particles. Drift chambers are comprised of high voltage planes and sense wires in a chamber with a gas mixture. When a particle passes through a drift chamber, it ionizes the gas. The high voltage planes keep a constant electric field within the drift chamber. The sense wires are held at ground potential. The ions produced by the incident particles then drift toward the sense wire, creating a build up of charge that can be measured. Using the drift speed of ions in the field


Figure 22: A schematic view of the VDCs. These are the first detectors in each HRS [36].
and the time that it takes for the ion to reach the sense wire, the position of the track through the VDC can be accurately determined.

The chambers used in the HRSs, as seen in Figure 22, are oriented parallel to the horizontal plane of the hall and $45^{\circ}$ to the detector stack. The active area of each VDC is 2118 mm by 288 mm . The gas used is an argon ( $62 \%$ ) and ethane ( $38 \%$ ) mixture. The electric field is created by gold-plated Mylar planes spaced 13 mm apart. These planes are held at a -4 kV voltage. Each chamber has two wire planes in a UV formation, $90^{\circ}$ to each other, that are separated by 335 mm . In each plane there are 368 wires with a wire spacing of 4.24 mm . This setup gives a position resolution of $100 \mu \mathrm{~m}$ and an angular resolution of 0.5 mrad .

### 4.4.2 Scintillator Planes

Each HRS has two scintillator planes that were used for MARATHON: S0 and S2. These two planes sandwich the gas Cherenkov detector. The scintillators are plastic (polystyrene) paddles with a Photomultiplier Tube (PMT) on each end. When scintillating material is struck by a particle, it is excited by absorbing a small amount of energy and emits it as light. The light then travels through


Figure 23: A schematic drawing of the S 2 scintillator plane [38].
the material to the PMTs on each end. When the light reaches the PMT, it knocks electrons out of the photocathode. These electrons are accelerated through a series of exceedingly higher voltage dynodes, where more electrons are released. Finally, this cascade reaches the anode with enough electrons to create a signal that can be read. The entire process is very quick, allowing the scintillators to be used for setting the timing of events. The time difference between the signal in the PMTs on each end allows for rough tracking.

S0 consists of a single paddle with the PMTs located on the top and bottom. The S0 paddle is made from BICRON 408 scintillating plastic which is 10 mm thick, 170 cm long, and 25 cm wide. The PMTs used are 3-inch XP2312B. There is a trade-off in timing resolution when using a large paddle. The timing resolution of S 0 is approximately 0.2 ns .

S2 consists of 16 paddles with the PMTs on the left and right. Each paddle is made from fast plastic scintillator EJ-230 and is 2 inches thick, 17 inches long, and 5.5 inches wide. The paddles are pressed together with a 60 pound force in order to minimize any space between them. The PMTs used are 2-inch Photonis 2282B. Figure 23 shows the layout of the S 2 scintillator plane. In this drawing, particles would pass through the plane of the page. The timing resolution of S2 has been measured to be smaller than 150 ps .

The signals measured in the scintillators form the basis for the HRS electronic trigger as described in Section 4.4.5. Since S2 has very good time resolution, it serves to set the timing of the event. Proper event timing is critical for the VDCs to accurately track a particle.

### 4.4.3 Gas Cherenkov Detectors

The gas Cherenkov is the first PID detector in the HRS. A gas Cherenkov detector functions by observing Cherenkov radiation from incident particles. Cherenkov radiation is light emitted by a particle that is traveling faster than the phase velocity of light in a medium. The Cherenkov radiation is emitted as an "electromagnetic shock wave" in the wake of the particle that is then guided to PMTs by mirrors. This property allows a gas Cherenkov detector to exclude low momentum particles [43].

The Cherenkov chamber is filled with $\mathrm{CO}_{2}$ at atmospheric pressure. This gas gives a $4.8 \mathrm{GeV} / c$ momentum threshold for pion detection, providing a very efficient rejection of pions, as the HRSs have a momentum acceptance set much lower than that. Each Cherenkov chamber has ten spherical mirrors with focal length 80 cm each aimed at a PMT (Burle 8854). The radiator length is 80 cm for the LHRS and 130 cm for the RHRS.

All of these characteristics ensure that analyzing the sum of all PMT signals allows for very efficient discernment between electrons and pions.

## Gas Cherenkov Calibration

The Gas Cherenkov is calibrated on a PMT-by-PMT basis. The ADC spectrum for each PMT has a Gaussian peak at low ADC values. This corresponds to a single photon knocking out an electron from the photocathode in the PMT, also known as the "single photo-electron peak". To determine this calibration, the peak is first fit with a Gaussian. Calibrating the PMT is done by determining a factor that will align these peaks at the same ADC value for all PMTs. MARATHON aligned the single photo-electron peak at ADC bin 300, as seen in Figure 24.

### 4.4.4 Leaded Glass Calorimeters

Both arms have two leaded glass calorimeters, denoted as the preshower and shower detectors. When a particle enters a calorimeter, it interacts with the material by depositing energy. This energy is converted into an electromagnetic shower of photons which are detected by PMTs attached to the glass blocks. How much energy is deposited is dependent on the particle and its radiation


Figure 24: The Cherenkov PMTs are calibrated to have the single photo-electron peak centered at ADC bin 300. The peak at 0 is the ADC pedestal.
length within the material. In the case of the HRSs, the calorimeters are thick enough that electrons will completely deposit all of their energy into the preshower and shower detectors, primarily through the process of bremsstrahlung radiation. Each HRS has a slightly different configuration for the calorimeters.

In the LHRS, the preshower and shower blocks are all perpendicular to the path of the particle. Both layers are comprised of 34 blocks that alternate in size between $15 \mathrm{~cm} \times 15 \mathrm{~cm} \times 30 \mathrm{~cm}$ and $15 \mathrm{~cm} \times 15 \mathrm{~cm} \times 35 \mathrm{~cm}$, as shown in Figure 25 (top).

In the RHRS, the preshower blocks are perpendicular to the path of the particle while the shower blocks are parallel to the path of the particle. The preshower layer is composed of 48 blocks that measure $10 \mathrm{~cm} \times 10 \mathrm{~cm} \times 35 \mathrm{~cm}$. The shower layer is composed of 80 blocks that measure 15 cm x $15 \mathrm{~cm} \times 35 \mathrm{~cm}$, as shown in Figure 25 (bottom).

The signal from the calorimeters is directly correlated to the energy of the particle that was detected. Typically, for PID, a measure of $E / p$ is used. $E / p$ is the ratio of energy (from the calorimeter) to momentum (from tracking) of the detected particle. This allows for very efficient identification of particles because electrons will peak around 1 and higher-mass particles will have a much lower ratio.


Figure 25: Layout of the shower blocks. Particles enter from the bottom of the page. Left is top-view. Right is side-view [36].

### 4.4.5 Trigger

The HRSs read in data from the described detectors through a combination of Fastbus ADCs and TDCs and VME Flash ADCs. A Trigger Supervisor (TS) unit is used to distribute a trigger signal to this hardware. This trigger signals the ADCs and TDCs to record this signal and send it to be recorded. This process is overseen by CEBAF Online Data Acquisition (CODA) software written at JLab. The CODA software communicates with the TS crate to signal when it is ready to receive data and that triggers should begin being processed.

In order to distribute a signal to the ADCs and TDCs, the TS unit must receive an outside trigger. A trigger is a logic signal that is generated to indicate that the data to be recorded potentially come from a "good event". This trigger is generated from the signals that are produced by the detectors in the HRSs.

To produce this trigger, signals received from the scintillators and the Cherenkov detector are processed by NIM electronics. First, the scintillator PMT signals are "discriminated": each of these signals are passed through a discriminator which, provided the signals are large enough to exceed a set threshold signifying a real signal, converts the signal into a logic pulse. The length of these
logic pulses is set to allow for timing the coincidence of these three detectors. The PMTs in each scintillator plane are then checked for coincidence, that is any signal received within a designated window of time is defined as being from the same event. In the setup for S0, both PMTs are required to have a signal for the processing of an event. The S 2 setup requires that any single paddle must have a signal from both PMTs in order to trigger. For the Cherenkov, the PMT signals are summed and then discriminated. The logic pulses for each detector are then delayed to allow for timing of coincidence between detectors. The need for delaying the signals is due to the varying length of cables from the detectors to the processing hardware and differences in the internal response time of different detectors and electronic units.

These logic signals are finally combined into four different triggers:

- S0 || S2
- S0 \& \& S2
- (S0 || S2) \&\& Cherenkov
- (S0 \&\& S2) \&\& Cherenkov

Ultimately, the final three of these triggers were used in the experiment. These are referred to (sequentially) as Triggers T1-T3. A schematic for the formation of these signals can be seen in Figure 26. In the formation of these triggers, the scintillators are used to set the timing for the TDCs. Particularly, S2 always sets the timing of the trigger since it has the highest timing resolution. In the case of the triggers where S 0 and S 2 are "OR'd", S 0 will set the timing in the absence of an S 2 signal.


Figure 26: A schematic diagram of the trigger setup for MARATHON. In this diagram "disc." stands for discriminator and "FIFO" stands for "Fan in fan out", which is a unit that takes a signal and then outputs it to multiple channels. "NIM $=>$ ECL" denotes the conversion from NIM to ECL logic standards which is necessary to interface with the Trigger Supervisor. The drawings on the right show the relative width of each signal to facilitate event timing [44].

## CHAPTER 5

## Analysis

### 5.1 Data Collected

The MARATHON experiment ran from January through April of 2018 in Hall A at JLab. During the experiment, the HRSs were used in "inclusive" mode, that is each spectrometer was making an independent measurement. Throughout the discussion of the analysis, each measurement by an HRS is referred to as a "kinematic" point. To switch between kinematics, the LHRS was rotated around the target to different angles. The RHRS was parked at the highest angle kinematic for the entirety of the experiment since that kinematic had a very slow event rate. The beam energy, $E$, was fixed at 10.59 GeV .

For all kinematics, the LHRS momentum was set to $3.1 \mathrm{GeV} / c$. The RHRS momentum was set to $2.9 \mathrm{GeV} / c$ due to limitations in the operation of the dipole magnet. The kinematics are numbered based on the originally proposed kinematic settings. Due to time limitations, not every kinematic was measured. During the data taking, the $x$ and $Q^{2}$ values of the data were studied and it was found that there was sufficient overlap between kinematics without the skipped kinematics. Plots showing this overlap can be seen in Figure 27. A few kinematics were measured more than once when the run period was extended. Individual iterations of a kinematic are treated as independent measurements for the sake of analysis. Table 3 shows the variables that define each kinematic.

### 5.2 Analysis Outline

This chapter will go over the analysis of the data collected in the MARATHON experiment. Specifically examined are the cuts and corrections that go into the calculation of the results as well as the calibrations performed to make the measurements accurate. An outline of the analysis process is as follows:

| Kinematic | $E^{\prime}(\mathrm{GeV} / c)$ | $\theta\left({ }^{\circ}\right)$ | Central $x$ | No. of Iterations |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 3.1 | $16.811^{\circ}$ | 0.199 | 1 |
| 1 | 3.1 | $17.577^{\circ}$ | 0.218 | 1 |
| 2 | 3.1 | $19.115^{\circ}$ | 0.257 | 1 |
| 3 | 3.1 | $20.578^{\circ}$ | 0.298 | 1 |
| 4 | 3.1 | $21.940^{\circ}$ | 0.338 | 1 |
| 5 | 3.1 | $23.213^{\circ}$ | 0.378 | 1 |
| 7 | 3.1 | $25.594^{\circ}$ | 0.46 | 2 |
| 9 | 3.1 | $27.778^{\circ}$ | 0.538 | 2 |
| 11 | 3.1 | $29.917^{\circ}$ | 0.62 | 2 |
| 13 | 3.1 | $31.732^{\circ}$ | 0.70 | 2 |
| 15 | 3.1 | $33.562^{\circ}$ | 0.78 | 3 |
| 16 | 2.9 | $36.121^{\circ}$ | 0.82 | 2 |

Table 3: The key quantities that define each kinematic. The beam energy was fixed at 10.59 GeV .

1. For each run:
i. Apply cuts to data
ii. Calculate the electron yield and bin data in $x$
iii. Apply livetime correction
iv. Apply target boiling correction
v. Calculate target boiling uncertainty
vi. Calculate beam charge and add it to the kinematic total charge
2. For each kinematic:
i. Sum the yields from all runs in the kinematic
ii. Apply charge normalization
iii. Sum the target boiling uncertainties and apply them to the data


Figure 27: The $x, Q^{2}$, and $W^{2}$ coverage of all of the MARATHON kinematics. Each kinematic overlaps with the previous and successive kinematic, giving complete coverage of the proposed range of the experiment [45].
iv. Drop the first and last bins as they are near the edge of the acceptance
3. For production of a ratio, the kinematic yields for two targets are divided $\left({ }^{3} \mathrm{He}\right.$ and ${ }^{2} \mathrm{H}$ in the case of this thesis)
4. Combine all kinematics by weighted average, weighted by uncertainty
5. Apply remaining corrections:
i. Endcap contamination subtraction
ii. Charge symmetric background subtraction
iii. Target energy loss correction
iv. Coulomb correction
v. Radiative corrections
vi. Bin centering correction

This analysis procedure will yield the $F_{2}^{3} \mathrm{He} / F_{2}^{2} \mathrm{H}$ structure function ratio. From there, the $\mathrm{A}=3$ isoscalar EMC ratio can be extracted by applying an isoscalar correction. Also, the $F_{2}^{n} / F_{2}^{p}$ nucleon structure function ratio can be extracted using the methodology described in Section 2.6.

### 5.3 Yield and Yield Ratio Calculation

The MARATHON experiment measured yield ratios. This method allows us to cancel many systematics that would otherwise plague a full cross section analysis. By taking data for each target at the same kinematics, acceptance effects are identical. This data taking technique means that with reasonable acceptance cuts, a ratio of target yields is wholly equivalent to the ratio of cross sections.

Calculating the yield ratios requires first calculating the yield for each target. The yields calculated are binned in Bjorken $x$. The bins are 0.03 wide. The bin centers are defined by segmenting the range of 0 to 0.99 by 0.03 (i.e. the bins are centered at $0.015,0.045,0.075$, etc.). The charge-normalized yield, $Y$, of each bin is then calculated using the simple equation

$$
\begin{equation*}
Y=\frac{\text { Counts }}{\text { Scattering Centers } \cdot \text { Charge }} \cdot \text { Corrections. } \tag{5.1}
\end{equation*}
$$

Here, "Counts" are the number of electrons measured that pass the cuts placed on the data and fall into that bin, "Scattering Centers" are the number of nucleons per $\mathrm{cm}^{2}$ in the target (calculated from the target thickness, density, and mass number $A$ ), and "Corrections" are physics or systematic effects that are otherwise unaccounted for in the data. Dividing by the "Charge" in each kinematic gives the "Charge-Normalized Yield", that is the electron count for each unit of charge incident on the target.

As described in Section 5.2, the charge-normalized yield is calculated for each target on a per-kinematic basis. For each kinematic, the target ratios, $R$, are then calculated. This is done simply by dividing the two yields and propagating the associated uncertainties. The equations for evaluating the ratio are:

$$
\begin{equation*}
R=\frac{Y_{3^{3} \mathrm{He}}}{Y_{{ }^{2} \mathrm{H}}} \tag{5.2}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma_{R}=R \sqrt{\left(\frac{\sigma_{3_{3} \mathrm{He}}}{Y_{3^{\mathrm{He}}}}\right)^{2}+\left(\frac{\sigma_{2_{\mathrm{H}}}}{Y_{2_{\mathrm{H}}}}\right)^{2}}, \tag{5.3}
\end{equation*}
$$

where $\sigma_{R}, \sigma^{3} \mathrm{He}$, and $\sigma_{2}{ }_{\mathrm{H}}$ are the uncertainties associated with the ratio, Helium-3, and Deuterium, respectively.

Once the per-kinematic yield ratios are calculated, each kinematic has its edge bins dropped, that is the lowest and highest $x$ bin. This is because these bins are located at the edge of the
acceptance of the spectrometer with low counting rates and large uncertainties. After this is done, all kinematics are combined through a weighted average. The "weight" is the uncertainty on the measurement in that bin. The equations for the weighted average are:

$$
\begin{align*}
& \text { Average }=\frac{\sum_{i} \frac{R}{\sigma_{R}^{2}}}{\sum_{i} \frac{1}{\sigma_{R}^{2}}}  \tag{5.4}\\
& \sigma_{\text {Average }}=\frac{1}{\sum_{i} \frac{1}{\sigma_{R}^{2}}} \tag{5.5}
\end{align*}
$$

Once all corrections have been applied, this calculated weighted average is the final result for the yield ratio.

### 5.4 Cuts

Cuts must be applied to data to provide a set of "good electrons". Cuts are a defined set of conditions that must be met by an event to be classified as "good". A good event is one in which the detected particle is an electron that deep inelastically scattered from the target. In this analysis, the cuts can be classified in two categories: Particle Identification and Acceptance. Particle Identification cuts are used to ensure that all events being studied are electrons. Acceptance cuts are used to ensure that all events passed through areas of the spectrometer that are well constrained by the spectrometer optics.

### 5.4.1 Particle Identification

Particle Identification (PID) cuts are applied to the detectors in the HRSs. The first PID cut is referred to as the "Trigger 2" cut. This cut requires that an event fires Trigger 2 (Trigger 5 for the RHRS) which is the (S0 \&\& S2) \&\& Cherenkov trigger. This cut requires that S0 and S2 fire, which will ensure proper event timing and tracking. Ensuring that the Cherenkov fires is the first step to limiting the events to electrons. As described in Section 4.4.3, the Cherenkov will, in most cases, only fire for electrons due to the $4.8 \mathrm{GeV} / c$ momentum threshold for pion detection.

The next cut is the "1-Track" cut. This cut requires that an event must have only one track through the spectrometer associated with it. A single track assures that the event corresponds to a single electron. When multiple tracks are present, there is a risk that tracking and spectrometer readings will be incorrect.

There is a cut placed on the $W^{2}$ of an event. The goal of MARATHON is to measure the DIS cross section ratios. A cut on $W^{2}$ must be placed in order to ensure that all events are from the DIS region and to reject events that originate from Resonance scattering. This cut is placed for $W^{2}>3.24\left(\mathrm{GeV} / c^{2}\right)^{2}$.

A cut is placed on the Cherenkov signal. This cut is placed on the ADC spectrum of the Cherenkov. For the LHRS, the cut is at ADC channel 1500; the cut is placed at ADC channel 2000 for the RHRS. While the Cherenkov will generally only fire for electrons, there are a small number of high momentum pions that are capable of creating a signal. The high momentum threshold for pions means that any pions that do create a signal will create a very small signal. This can be seen in Figure 28. By placing this cut, the pion peak is completely removed while the electron peak is nearly completely allowed to pass.

The final PID cut is on the ratio of the energy of the particle to the momentum of the track $(E / p)$. For electrons, which have a low mass compared to the energy scale of the experiment, the $E / p$ is expected to be approximately 1 . Any particle of larger mass that passes the "Trigger 2 " cut (and thus created a Cherenkov signal) will have an $E / p$ value significantly smaller than 1 . In order to eliminate these non-electron events, a cut on $E / p$ is placed at 0.7 for both HRS arms. This spectrum can be seen in Figure 29.

The Cherenkov and calorimeter cuts are needed to work in tandem to completely remove nonelectron events. This can be seen clearly by plotting the two spectra on a 2D histogram. As shown in Figure 30, both cuts are needed to isolate the electron signals.

### 5.4.2 Acceptance

Acceptance cuts are placed on the kinematic region that the spectrometer is sensitive to. These cuts are applied to the tracking variables "Target $\phi$ ", "Target $\theta$ ", and " $\delta p$ ". Target $\phi$ is a measure of the rotation in the plane of the hall floor. The value is the angular displacement from the central angle of the spectrometer. Target $\theta$ is the vertical angular displacement from the beam and target plane of the scattered electron. $\delta p$ is the relative deviation of the momentum of the scattered electron from the central momentum of the spectrometer. The RHRS also has a cut placed on the focal plane. These cuts are defined by examining the acceptance of the spectrometer. The cuts are placed around the area where the events are concentrated. The areas where events "fell off" were


Figure 28: Cherenkov Sum ADC spectrums for both the LHRS (top) and RHRS (bottom). All cuts are applied except for the PID cut on the Cherenkov Sum. The LHRS plot shows events from kinematic 0 while the RHRS plot shows events from kinematic 16 . The black lines show where the cuts are applied.


Figure 29: Calorimeter $E / p$ spectrum from the LHRS. All cuts are applied except for the PID cut on the Calorimeter. This plot shows events from kinematic 0 . The black line shows where the cut is applied.
considered to be outside of the acceptance of the spectrometer. These cuts create a well-defined acceptance band in the focal plane. In the RHRS, a very small number of events that passed these cuts were outside of this band. This led to a very loose cut being placed on the focal plane variables to omit these events. The acceptance regions for these variables are shown in Figures 31 and 32. This is also how the focal plane cuts were determined for the RHRS, as shown in Figure 33.

The other acceptance cut that is applied is to the target length. This cut is applied to the " $z$ Target" value of an event, where the event happened in the target along the axis of the beamline. This cut is used in order to minimize the amount of target endcap contamination events that need to be corrected for. The cut is defined on a kinematic-by-kinematic basis, with higher kinematics having more of the target length allowed to pass the cut due to a smaller endcap contribution to the overall event rate. These cuts are defined in Table 4.


Figure 30: 2D Calorimeter $E / p$ versus Cherenkov Sum ADC spectrum for the LHRS. All cuts are applied except for the PID cuts on the Cherenkov Sum and the Calorimeter. This plot shows events from kinematic 0 . The black lines show where the cuts are applied. The events in the upper-right quadrant pass the cuts and are considered to be good electrons.

Beyond these acceptance cuts, the first and last bin in each $x$ kinematic are not used. These bins are, by definition, at the edge of the acceptance. This is done as an additional level of minimizing acceptance effects.

### 5.5 Corrections

When analyzing the data, there are several corrections that need to be made. These are due to various physical or systematic effects that took place during the experiment. These effects have been studied in order to determine a "correction factor" that can be applied to the data.

### 5.5.1 Energy Loss

Incoming and outgoing electrons lose energy due to passing through target material. Energy loss occurs due to the electrons from the beam striking electrons in the atoms of the target and the


Figure 31: 2D Target $\phi$ and Target $\theta$ acceptance spectrum of the spectrometers. All cuts are applied except for the Target $\phi$, Target $\theta$, and $\delta p$ cuts. The LHRS plot (top) shows events from kinematic 0 while the RHRS plot (bottom) shows events from kinematic 16. The black lines show where the cuts are applied.


Figure 32: Relative momentum $\delta p$ acceptance spectrum of the spectrometers. All cuts are applied except for the Target $\phi$, Target $\theta$, and $\delta p$ cuts. The LHRS plot (top) shows events from kinematic 0 while the RHRS plot (bottom) shows events from kinematic 16. The black lines show where the cuts are applied.


Figure 33: Dispersive Focal Plane variables for RHRS kinematic 16. All cuts are applied. It was noted that after acceptance cuts were applied to RHRS data, there were still a few events that were not in the main region of the focal plane. A loose cut is applied to remove these events. The black lines show where cuts are applied.

| Kinematics | $z$-Target Acceptance (m) |
| :---: | :---: |
| $0-4$ | $-0.08-0.1$ |
| $5-7$ | $-0.09-0.1$ |
| $9-11$ | $-0.095-0.1$ |
| $13-15$ | $-0.1-0.105$ |
| 16 | $-0.105-0.11$ |

Table 4: The range of the target, along the beam axis, that events are accepted from. This is done to minimize contamination from the endcaps of the targets.
surrounding material, known as ionization energy loss. This calculation is done inside of the replay code for the experiment, which assumes that the scattering occurs at the center of the target for all events. The air gap, upstream endcap, and target material through half of the target length is used to calculate the energy loss for the incoming electron. The scattering angle, the target material, cell wall, scattering chamber wall, and outgoing air gap are used to calculate the energy loss for the scattered electron. This calculation is done using the Bethe-Bloch stopping-power formula as described in Reference [46].

### 5.5.2 Computer Deadtime Correction

Our Data Acquisition System (DAQ) is unable to continuously record data. While we can probabilistically determine the mean time spacing between events, in the real world events can deviate greatly from these means. Sometimes events will occur that are too close in time for our DAQ to record, as the computer has not completed recording the previous event. Deadtime is a function of event rate; when events happen more rapidly, there is a higher chance that events will occur too closely spaced in time to be recorded.

When deadtime is low and the number of recorded events is high, it is a reasonable assumption that the events recorded will accurately reflect the distribution of events in the "zero-deadtime limit". In this case, correcting for the deadtime is simply done by scaling the number of events by the "livetime" (LT) of the experiment, defined as ( 1 - Deadtime).

The computer livetime was measured using the Trigger Supervisor (TS) and scalers in each HRS. The trigger signals generated from detector signals are copied and sent to both the Trigger Supervisor and a scaler unit. The Trigger Supervisor is subject to the computer deadtime event loss discussed above. The scaler unit, on the other hand, simply increments a register when a trigger signal is received. The ratio of these the number of events recorded by these two systems gives a measure of the livetime of the measurement. The livetime is defined on a run-by-run basis as:

$$
\begin{equation*}
\mathrm{LT}=\frac{\mathrm{TS} \text { Triggers }}{\text { Scaler Triggers }} \tag{5.6}
\end{equation*}
$$

In an ideal world, the deadtime will be identical for all runs within a kinematic. However, the deadtime is measured and applied on a run-by-run basis in order to account for any deviations from this assumption. The average deadtime for each target in each kinematic is plotted in Figure 34.


Figure 34: Average computer deadtime in each kinematic (plotted at the central $x$ of the kinematic) for the four targets used in the MARATHON experiment.

### 5.5.3 Target Boiling

When the beam is incident on the target, it deposits heat into the gas. This causes a density fluctuation of the gas referred to as "target boiling". The target does not actually boil, however that is the standard nomenclature for a density fluctuation in a gas target. When the density of the target changes, it changes the effective target thickness as seen by the beam. When the target thickness changes, there is a changed number of scattering centers which will lead to a change in the number of electrons scattered. Specifically, heating will decrease the target density which will decrease the number of scattered electrons.

The gas density change is a function of the current on the target. Each run is taken at a single current so that the correction can be applied run by run. The correction is approximately linear for small deviations in current. A study of the effect during beam ramping found that the density settles very quickly. This means that so long as we cut events that occur with the beam off, it is unnecessary to take into account the beam ramping when correcting for this effect.


Figure 35: The multiplicative correction to the target density due to beam heating for the four gas targets. The correction is dependent on the beam current. The uncertainty band is plotted around each curve.

To determine the correction, a dedicated set of runs was taken with the Left HRS spectrometer set at $16.8^{\circ}$ and $3.1 \mathrm{GeV} / c$. At this kinematic, several data runs were recorded with varying current between them. Each of these runs were analyzed with the standard data cuts to determine the yield. This allows for the yield to be determined as a function of beam current.

Now that the yields have been determined as a function of beam current, they can be plotted and fit. The data is fit with a quadratic polynomial. This fit is constrained to require no correction (a correction factor of 1 ) at zero current. This is because the density must be the nominal fill density when there is no beam heating.

During analysis, the average current is calculated for a run (ignoring beam trips). The average current is then used to calculate the target density correction with the fit function. The correction is then applied on a run-by-run basis by multiplying the number of scattering centers by this correction factor (or equivalently dividing the yield by the correction factor). Figure 35 shows the gas density correction factors for each target [47].

The uncertainty from this correction is calculated using the covariance matrix from the fit to the data. This uncertainty, like the correction, is applied on a run-by-run basis using the average current from the run. When all runs in a kinematic are combined, these uncertainties are summed linearly, rather than in quadrature. This is because the uncertainties from the correction are completely correlated to each other. In a given kinematic, the combined uncertainty due to target boiling varies from $0.2 \%$ to $0.3 \%$.

### 5.5.4 Target Endcap Contamination

The gas targets used in this experiment are housed in aluminum cells, as described in Section 4.3.1. The thickness of the aluminum greatly exceeds the thickness of the gas and will contribute background that can survive the cuts placed on the data. By quantifying this contribution, the events that originate from the target cell endcaps can be subtracted from the final results.

To determine this contribution, the empty cell is used. The empty cell, being an exact replica of the gas target cells with a vacuum inside, allows us to approximately isolate the contribution of the cell walls to the data. The empty cell and the target being studied are compared by calculating the yields on a kinematic-by-kinematic basis and normalizing them by charge and endcap thickness.

The normalization to the endcaps for each target must be done in two parts. This is because each endcap is not of the same thickness. When calculating the yield for a target, it is assumed that any contamination upstream (downstream) of the center of the target must originate from the upstream (downstream) endcap. The two halves are then combined to arrive at the endcap thickness normalized yield. This yield calculation only has livetime corrections applied. All cuts are applied except for target length, which is adjusted to only include events upstream (downstream) of the center of the target.

The data for the target being studied and the normalized empty target are then binned in Bjorken $x$. Dividing the empty cell data by the gas target data then gives an approximation of the fractional contribution of the cell walls to the electron data. As this correction is applied to the final results, the contamination corrections for the targets are divided by each other to create the correction to the target ratios. These results are then fit with the functional form $1 \pm e^{A x+B}$. The choice of adding or subtracting the exponential is done by determining if the correction is greater or smaller than 1. The fit function and the covariance matrix of the fit are used to apply the


Figure 36: This plot shows the Endcap Contamination factor applied to each ratio studied MARATHON. The uncertainty band is plotted around each curve.
correction to the final results as well as determine the uncertainty contribution of this correction. Figure 36 shows the correction factors for each target ratio. The uncertainty from this correction varies from $0.05 \%$ to $0.075 \%$.

### 5.5.5 Charge Symmetric Background Subtraction

As an inclusive scattering experiment, we are particularly susceptible to background from charge symmetric processes from the target. That is, events which involved the production of both an electron and a positron, rather than the electron simply scattering. The primary source of this is $\pi^{0}$ decay, which creates photons that decay into electron-positron pairs. To study this, the polarity of the LHRS was reversed so that positively charged particles are directed into the detectors rather than negatively charged particles. With this setting, a number of runs were taken for kinematics 0 through 5. These runs were taken with all targets, just as the electron data was taken. This allows for a measurement of the positron yield which corresponds to a measure of the charge symmetric background.

This measurement allows us to determine the proportion of electrons that originated from pair production. Applying the same cuts and as for the electron data allows us to determine the charge normalized positron yield. Unlike in the electron analysis, it was noted that there was significant pion contamination in the positron data. This pion contamination had to be subtracted in order to get an accurate calculation of the positron yield. This is achieved by fitting the main pion peak and the subtracting the tail of the fit which survives the cuts applied from the positron data.

The charge-normalized positron yields over these kinematics are then combined (using the same methods as the electron yield) and binned in Bjorken $x$, and then divided by the charge-normalized electron yield. This is a fractional measure of the charge normalized background contamination. The ratio is then fit with an exponential of the form $e^{A x+B}$, where $A$ and $B$ are the fit parameters. These fits are shown in in Figure 37.

This correction is applied to the final yield ratio results. Both the numerator and denominator must have the charge symmetric background subtracted. Each target yield in the ratio is scaled by $\left(1-e^{A x+B}\right)$. For each bin, the fit is calculated at the bin center. The covariance matrix of the fit is used to calculate the uncertainty from this correction. The uncertainty varies from approximately $0.06 \%$ at low $x$ to $.002 \%$ at high $x$.

### 5.5.6 Coulomb Correction

A correction must be made for the effect of the Coulomb field of the target on the incident and scattered electron. The Coulomb interaction causes the $Q^{2}$ of the event to shift to an effective $Q^{2}$ value, $Q_{\text {eff }}^{2}$. The shift can be approximately evaluated with the formula:

$$
\begin{equation*}
Q_{\mathrm{eff}}^{2}=Q^{2}\left(1+\frac{3 Z \alpha \hbar c}{2 R E}\right)^{2} \tag{5.7}
\end{equation*}
$$

In this formula, $R$ is the hard-sphere equivalent radius of the nucleus which is defined as $R=$ $\left[(5 / 3)\left\langle r^{2}\right\rangle\right]^{1 / 2}$, where $\left\langle r^{2}\right\rangle$ is the root-mean-square radius of the nucleus [48].

Since $x=Q^{2} / 2 M \nu$, it is clear that a shift in $Q^{2}$ will result in a proportional shift in $x$. Using a model cross-section, the cross section is calculated at both the nominal $x$ and at $x_{\text {eff }}$. As the results have been bin centered, this calculation uses a nominal $x$ at the center of the bin, which will lead to a correction of

$$
\begin{equation*}
\sigma_{\text {Coulomb Corrected }}=\sigma_{\text {data }} \frac{\sigma_{\text {model }}\left(x_{\mathrm{eff}}\right)}{\sigma_{\text {model }}(x)} . \tag{5.8}
\end{equation*}
$$



Figure 37: Charge symmetric background correction to each of the four targets studied in MARATHON. The uncertainty band is plotted around each curve.

The correction is applied to the final ratio and each target yield in the ratio must be corrected. The ratio is multiplied by the correction to the numerator and divided by the correction to the denominator. The covariance matrix of the fit to the model is used to determine the uncertainty of this correction. Figure 38 shows the correction factors for each bin of the Helium-3 EMC ratio. The uncertainty from the Coulomb Correction varies from $0.15 \%$ to $0.2 \%$.

### 5.5.7 Bin Centering Correction

The cross-section over the width of a bin is not constant. This means that the measurement does not correspond to the true cross section at the center of the bin. Rather, because the data are effectively "sampled" over the bin, the measured value corresponds to the expectation value of the cross-section within the bin. If the results are to be reported at the bin center, the data must be corrected to the bin center.


Figure 38: Coulomb Correction values to the ${ }^{3} \mathrm{He} /{ }^{2} \mathrm{H}$ ratio by $x$ bin with the uncertainty on the correction.

Using a model that matches the shape of our data well, the location of the measurement within the bin can be calculated. This is done by calculating the expectation value of the model within that bin and determining the $x$ value corresponding to this value. The expectation value is given by

$$
\begin{equation*}
\left\langle f_{\text {measured }}\right\rangle=\frac{1}{\Delta x} \int_{x_{\text {low }}}^{x_{\text {high }}} f(x) d x \tag{5.9}
\end{equation*}
$$

where $f$ is a function representing the chosen model. In practice, the $x$ value does not need to be calculated if the data will be reported at the bin center. Rather, the correction is simply the ratio of the model at the bin center, which is

$$
\begin{equation*}
\sigma_{\text {Bin Centered }}=\frac{f\left(x_{\text {Bin Center }}\right)}{\left\langle f_{\text {measured }}\right\rangle} \sigma_{\text {measured }} \tag{5.10}
\end{equation*}
$$



Figure 39: Bin Centering Correction values to the ${ }^{3} \mathrm{He} /{ }^{2} \mathrm{H}$ ratio by $x$ bin with the uncertainty on the correction.

The correction must be applied to both targets in the ratio. That is, the ratio must be multiplied by the correction to the numerator and divided by the correction to the denominator [49]. The covariance matrix of the fit to the model is used to determine the uncertainty of this correction. Figure 39 shows the correction factors for the Helium-3 EMC ratio. The uncertainty from the Bin Centering Correction is approximately $0.3 \%$.

### 5.5.8 Radiative Corrections

The Deep Inelastic Cross Sections being studied are the Born approximation of a single-photon exchange. The Born approximation is the simplest interaction that can occur, in which an electron enters, exchanges a photon with the parton, and then subsequently scatters. In this approximation, no other photons are exchanged or radiated. The measurement, however, contains contributions from higher order processes that will increase the measured cross section. These processes include Bremsstrahlung, vertex corrections, and vacuum polarization. Bremsstrahlung is when the incoming or outgoing electrons radiate photons due to deceleration. Vertex corrections are the emission and subsequent reabsorption of a photon by the incoming and outgoing electron. Vacuum polar-
ization is when the virtual photon exchanged between the electron and nucleon annihilates into a particle-antiparticle pair which then reannihilate back into a virtual photon. Using a model, these higher order contributions can be corrected for and removed from the measurement. The Feynman diagrams for these processes are shown in Figure 40.

The experiment used a JLab Hall A and C software package, based on the original SLAC algorithm by Mo and Tsai [50], called T2_EXTERNALS that calculates both the Born cross section and the radiated cross section for a given target at a kinematic set $\left(E, E^{\prime}, \theta\right)$. For this analysis, the values used in the calculation correspond to the center of the bin being corrected. After the calculation is complete, the radiative correction is given by:

$$
\begin{equation*}
\mathrm{RC}=\frac{\sigma_{\text {model }}^{\text {radiated }}}{\sigma_{\text {model }}^{\text {Born }}} \tag{5.11}
\end{equation*}
$$

The uncertainty was determined by using different models of the cross section used in the radiative corrections. There was little difference among the models. Figure 41 shows the correction factors for the Helium-3 EMC ratio. Ultimately, the uncertainty due to radiative corrections was determined to be $0.5 \%$ for all data points.

### 5.5.9 Isoscalar Corrections

When studying an EMC ratio, the strength of the EMC effect is typically defined using isoscalar nuclei, nuclei where the number of protons and the number of neutrons are equal. Helium-3 is not an isoscalar nucleus. To study the EMC effect, we "correct" for this by creating a fictitious $A=3, Z=3 / 2$ isoscalar nucleus. This correction uses $F_{2}^{n} / F_{2}^{p}$ as extracted from the MARATHON ${ }^{3} \mathrm{H} /{ }^{3} \mathrm{He}$ ratio to transform the proton excess into equal quantities of protons and neutrons [51]. This correction is defined as:

$$
\begin{equation*}
\text { Isoscalar Correction }=\frac{\frac{1}{2}\left(1+\left(F_{2}^{n} / F_{2}^{p}\right)\right)}{\frac{1}{A}\left(Z+(A-Z)\left(F_{2}^{n} / F_{2}^{p}\right)\right)} \tag{5.12}
\end{equation*}
$$

The covariance matrix of the fit to the $F_{2}^{n} / F_{2}^{p}$ data was used to determine the uncertainty of this correction. Figure 42 shows the correction factors for the Helium-3 EMC ratio. After the uncertainty is propagated, the uncertainty is between $0.36 \%$ and $0.67 \%$.


Figure 40: Feynman diagrams of the processes associated with radiative corrections. (a) is the Born process that describes Deep Inelastic Scattering. (b)-(d) are higher-order radiative processes that are measured and must be corrected for.


Figure 41: Radiative Correction values to the ${ }^{3} \mathrm{He} /{ }^{2} \mathrm{H}$ ratio by $x$ bin with the $0.5 \%$ uncertainty on each value.


Figure 42: Isoscalar Correction values to the ${ }^{3} \mathrm{He} /{ }^{2} \mathrm{H}$ ratio by $x$ bin with the uncertainty on the correction.

## CHAPTER 6

## Results

### 6.1 Helium-3/Deuterium Yield Ratio

The Helium-3/Deuterium yield ratio was calculated using the analysis method outlined in Section 5.2. The data are shown in Figure 43 and listed in Table 5. The uncertainties plotted consist of statistical and all systematic uncertainties added in quadrature, except for the normalization uncertainty due to target density which is not shown.

### 6.2 Isoscalar Helium-3 EMC Yield Ratio

The Helium-3 EMC ratio is the Helium-3/Deuterium ratio with Helium-3 corrected for protonexcess. That is, Helium-3 is transformed into a hypothetical isoscalar $\mathrm{A}=3 / 2$ nucleus. The isoscalar correction is done using the method described in Section 5.5.9. This analysis uses an input of an $F_{2}^{n} / F_{2}^{p}$ fit extracted from the ${ }^{3} \mathrm{H} /{ }^{3} \mathrm{He}$ MARATHON data [51]. These data are shown in Figure 44 and listed in Table 6. The uncertainties plotted consist of statistical and all systematic uncertainties added in quadrature, except for the normalization uncertainty due to target density which is not shown.

### 6.3 Normalizing the data

A feature of previous EMC data is a unity crossing near $x=0.3$. This leads to the understanding that there are minimal nuclear effects in the region near $x=0.3$. A look at Figure 44 indicates that this feature is not present in this set of data. An interpretation of this, as this work will examine, is that the Helium-3 data are in need of normalization. This is further justified when reading Reference [51] in which it is noted that the MARATHON ${ }^{3} \mathrm{H} /{ }^{3} \mathrm{He}$ yield data required a 2.8\% normalization in order for the extraction of $F_{2}^{n} / F_{2}^{p}$ to agree with the extraction from ${ }^{2} \mathrm{H} /{ }^{1} \mathrm{H}$. The ${ }^{2} \mathrm{H} /{ }^{1} \mathrm{H}$ data are in agreement with world data. This method is derived from and described in Reference [52].


Figure 43: The yield ratio of Helium-3 and Deuterium.


Figure 44: The isoscalar EMC yield ratio of Helium-3.

| $x$ | $F_{2}^{3} \mathrm{He} / F_{2}^{2} \mathrm{H}$ | Statistical Uncertainty | Systematic Uncertainty | Radiative Correction |
| :---: | :---: | :---: | :---: | :---: |
| 0.195 | 1.041 | 0.003 | 0.007 | 0.999 |
| 0.225 | 1.042 | 0.003 | 0.007 | 0.998 |
| 0.255 | 1.042 | 0.004 | 0.007 | 0.998 |
| 0.285 | 1.050 | 0.004 | 0.007 | 0.997 |
| 0.315 | 1.037 | 0.005 | 0.007 | 0.997 |
| 0.345 | 1.050 | 0.005 | 0.007 | 0.997 |
| 0.375 | 1.063 | 0.007 | 0.007 | 0.997 |
| 0.405 | 1.053 | 0.009 | 0.007 | 0.997 |
| 0.435 | 1.037 | 0.009 | 0.007 | 0.997 |
| 0.465 | 1.053 | 0.009 | 0.007 | 0.997 |
| 0.495 | 1.074 | 0.009 | 0.007 | 0.997 |
| 0.525 | 1.078 | 0.010 | 0.007 | 0.997 |
| 0.555 | 1.061 | 0.010 | 0.007 | 0.997 |
| 0.585 | 1.056 | 0.010 | 0.007 | 0.997 |
| 0.615 | 1.055 | 0.010 | 0.007 | 0.998 |
| 0.645 | 1.079 | 0.011 | 0.007 | 0.998 |
| 0.675 | 1.053 | 0.012 | 0.007 | 0.998 |
| 0.705 | 1.094 | 0.011 | 0.007 | 0.998 |
| 0.735 | 1.090 | 0.011 | 0.007 | 0.999 |
| 0.765 | 1.102 | 1.103 | 0.0073 | 0.999 |
| 0.795 |  |  |  |  |
|  |  | 0.007 |  |  |

Table 5: Helium-3/Deuterium yield ratio. The associated systematic uncertainties are discussed in Chapter 5.

| $x$ | Isoscalar Helium-3 EMC Ratio | Isoscalar Correction |
| :---: | :---: | :---: |
| 0.195 | $0.988 \pm 0.008$ | $0.949 \pm 0.004$ |
| 0.225 | $0.984 \pm 0.008$ | $0.944 \pm 0.004$ |
| 0.255 | $0.978 \pm 0.008$ | $0.939 \pm 0.004$ |
| 0.285 | $0.981 \pm 0.009$ | $0.934 \pm 0.004$ |
| 0.315 | $0.964 \pm 0.009$ | $0.930 \pm 0.004$ |
| 0.345 | $0.971 \pm 0.009$ | $0.925 \pm 0.004$ |
| 0.375 | $0.979 \pm 0.010$ | $0.921 \pm 0.005$ |
| 0.405 | $0.965 \pm 0.011$ | $0.917 \pm 0.005$ |
| 0.435 | $0.947 \pm 0.011$ | $0.913 \pm 0.005$ |
| 0.465 | $0.957 \pm 0.011$ | $0.909 \pm 0.005$ |
| 0.495 | $0.973 \pm 0.011$ | $0.906 \pm 0.005$ |
| 0.525 | $0.973 \pm 0.012$ | $0.902 \pm 0.005$ |
| 0.555 | $0.955 \pm 0.012$ | $0.899 \pm 0.005$ |
| 0.585 | $0.947 \pm 0.012$ | $0.896 \pm 0.005$ |
| 0.615 | $0.943 \pm 0.012$ | $0.894 \pm 0.005$ |
| 0.645 | $0.962 \pm 0.013$ | $0.891 \pm 0.005$ |
| 0.675 | $0.936 \pm 0.013$ | $0.889 \pm 0.006$ |
| 0.705 | $0.971 \pm 0.013$ | $0.887 \pm 0.006$ |
| 0.735 | $0.966 \pm 0.013$ | $0.886 \pm 0.006$ |
| 0.765 | $0.975 \pm 0.013$ | $0.885 \pm 0.006$ |
| 0.795 | $0.975 \pm 0.014$ | $0.884 \pm 0.006$ |

Table 6: Isoscalar Helium-3 EMC yield ratio. The listed uncertainty includes all systematic errors discussed in Chapter 5, including the isoscalar correction uncertainty. The fractional contribution of the statistical uncertainty to this ratio is equivalent to the fractional contribution of the statistical uncertainty listed in Table 5.

The normalization of the ${ }^{3} \mathrm{He} /{ }^{2} \mathrm{H}$ yield data is determined in the same way that it was determined for the ${ }^{3} \mathrm{H} /{ }^{3} \mathrm{He}$ data. This is done by first extracting $F_{2}^{n} / F_{2}^{p}$ from the Helium-3/Deuterium Yield Ratio using the method described in Section 2.6. Since nuclear effects are minimal near $x=0.3$, it is expected that all $F_{2}^{n} / F_{2}^{p}$ extractions should agree in this region. Figure 45 shows that, without normalization, this is not the case.

This extraction requires a model input. For this analysis, the Kulagin-Petti (KP) model was used [52,53]. This model was chosen by comparing it to the non-isoscalar yield ratio. Of the models examined, this one best matched the overall shape of the data. A comparison of the Helium-3/Deuterium yield ratio with the KP model prediction is shown in Figure 46.

The normalization was determined by calculating the reduced $\chi^{2}$ of the four data points symmetric around $x=0.3(x=0.255,0.285,0.315$, and 0.345$)$ when compared to the extraction of $F_{2}^{n} / F_{2}^{p}$ from ${ }^{2} \mathrm{H} /{ }^{1} \mathrm{H}$. This process takes into account the variance, $\sigma^{2}$, of the ${ }^{3} \mathrm{He} /{ }^{2} \mathrm{H}$ data in order to minimize the effect of statistical fluctuations. Different normalizations of the yield ratio were iterated over in steps of $0.1 \%$. This is continued until the extractions were clearly deviating again and the normalization with the minimum reduced $\chi^{2}$ is chosen. For these data, a normalization of $2.8 \%$ was found to be necessary for the $F_{2}^{n} / F_{2}^{p}$ extraction to match that of ${ }^{2} \mathrm{H} /{ }^{1} \mathrm{H}$. Figure 47 shows the $F_{2}^{n} / F_{2}^{p}$ extraction with this normalization applied. Figure 48 shows the Helium-3/Deuterium yield ratio and isoscalar Helium-3 EMC ratio with this normalization applied. A comparison of the normalized Helium-3/Deuterium yield ratio to the KP model used in this extraction can be seen in Figure 49. Table 7 lists the data with the normalization applied.

### 6.4 Comparison to Previous Helium-3 EMC Data

There are two previous measurements of the Helium-3 EMC ratio: the HERMES experiment at HERA [21, 22] and JLab Hall C E03-103 [23]. Figure 50 shows a comparison of the measured MARATHON yield ratio, without isoscalar corrections or normalization, to the non-isoscalar E03103 and HERMES data. The HERMES data are only published corrected for proton-excess. NMC data in Reference [54] were used to approximate the isoscalar correction applied in order to arrive at the non-isoscalar data. The HERMES data are published with a $0.9 \%$ normalization after comparison to SLAC and NMC ${ }^{4} \mathrm{He}$ data, which is removed for the following comparisons, except where explicitly stated. These three sets of data agree well within error bars, with the HERMES


Figure 45: $F_{2}^{n} / F_{2}^{p}$ extracted from the Helium-3/Deuterium yield ratio compared to the extraction from the MARATHON Deuterium/Hydrogen data. The error bars include statistical and systematic uncertainties.


Figure 46: Helium-3/Deuterium yield ratio compared to the Kulagin-Petti (KP) model. While the data and model do not match in value, they do match in shape. The error bars include statistical and systematic uncertainties.


Figure 47: $F_{2}^{n} / F_{2}^{p}$ extracted from the Helium-3/Deuterium yield ratio, with a $2.8 \%$ normalization, compared to the extraction from the MARATHON Deuterium/Hydrogen data. The error bars include statistical and systematic uncertainties.


Figure 48: The yield ratio of Helium-3/Deuterium and the isoscalar EMC yield ratio of Helium-3 with a $2.8 \%$ normalization applied. The error bars include statistical and systematic uncertainties.

| $x$ | Normalized $F_{2}^{3} \mathrm{He} / F_{2}^{2} \mathrm{H}$ | Normalized Isoscalar Helium-3 EMC Ratio |
| :---: | :---: | :---: |
| 0.195 | $1.041 \pm 0.008$ | $0.988 \pm 0.008$ |
| 0.225 | $1.042 \pm 0.007$ | $0.984 \pm 0.008$ |
| 0.255 | $1.042 \pm 0.008$ | $0.978 \pm 0.008$ |
| 0.285 | $1.050 \pm 0.008$ | $0.981 \pm 0.009$ |
| 0.315 | $1.037 \pm 0.008$ | $0.964 \pm 0.009$ |
| 0.345 | $1.050 \pm 0.009$ | $0.971 \pm 0.009$ |
| 0.375 | $1.063 \pm 0.010$ | $0.979 \pm 0.010$ |
| 0.405 | $1.053 \pm 0.011$ | $0.965 \pm 0.011$ |
| 0.435 | $1.037 \pm 0.011$ | $0.947 \pm 0.011$ |
| 0.465 | $1.053 \pm 0.011$ | $0.957 \pm 0.011$ |
| 0.495 | $1.074 \pm 0.011$ | $0.973 \pm 0.011$ |
| 0.525 | $1.078 \pm 0.012$ | $0.973 \pm 0.012$ |
| 0.555 | $1.061 \pm 0.012$ | $0.955 \pm 0.012$ |
| 0.585 | $1.056 \pm 0.012$ | $0.947 \pm 0.012$ |
| 0.615 | $1.055 \pm 0.012$ | $0.943 \pm 0.012$ |
| 0.645 | $1.079 \pm 0.013$ | $0.962 \pm 0.013$ |
| 0.675 | $1.053 \pm 0.013$ | $0.936 \pm 0.013$ |
| 0.705 | $1.094 \pm 0.014$ | $0.971 \pm 0.013$ |
| 0.735 | $1.090 \pm 0.013$ | $0.966 \pm 0.013$ |
| 0.765 | $1.102 \pm 0.013$ | $0.975 \pm 0.013$ |
| 0.795 | $1.103 \pm 0.015$ | $0.975 \pm 0.014$ |
|  |  |  |

Table 7: Helium-3/Deuterium yield ratio and Isoscalar Helium-3 EMC yield ratio with $2.8 \%$ normalization applied. The listed uncertainties include all statistical and systematic uncertainties.


Figure 49: Helium-3/Deuterium yield ratio with a $2.8 \%$ normalization compared to the KulaginPetti (KP) model. The implementation of a normalization brings the data and model into good agreement. The error bars include statistical and systematic uncertainties.
data having a slightly higher offset. The offset of the HERMES data is within their normalization uncertainty.

Each of these three measurements has had a different isoscalar correction applied. Figure 51 shows these three sets of data, each with the isoscalar correction from their original analysis applied. This figure does not have any normalizations applied. This figure again shows good data agreement and a slight offset of the HERMES data, which is within their normalization uncertainty.

For a more direct comparison, it is necessary to apply the same isoscalar correction to all data. Figure 52 shows these data sets with the MARATHON isoscalar correction applied. Given the clear agreement between raw MARATHON and E03-103 data, any normalization applied to one must be applied to the other. Figure 53 shows the normalized isoscalar data with the MARATHON isoscalar correction and normalization also applied to the E03-103 data. In this figure, the HERMES normalization of $0.9 \%$ is also applied. The culmination of these normalizations yields good agreement between all data sets and a disappearance of the offset in the HERMES data. Figure 54 also shows that using these normalizations and the MARATHON isoscalar correction yields good agreement between the data sets and the KP model.


Figure 50: A comparison of non-isoscalar, unnormalized data from MARATHON, E03-103, and HERMES. The error bars include statistical and systematic uncertainties.


Figure 51: A comparison of isoscalar, unnormalized data from MARATHON, E03-103, and HERMES. The isoscalar corrections applied are the corrections determined by each experiment. The error bars include statistical and systematic uncertainties.


Figure 52: A comparison of isoscalar, unnormalized data from MARATHON, E03-103, and HERMES. The MARATHON isoscalar correction is applied to all data. The error bars include statistical and systematic uncertainties.


Figure 53: A comparison of isoscalar, normalized data from MARATHON, E03-103, and HERMES. The MARATHON isoscalar correction is applied to all data. The MARATHON and E03-103 are normalized up by $2.8 \%$ and the HERMES data are normalized up by $0.9 \%$. The error bars include statistical and systematic uncertainties.


Figure 54: A comparison of isoscalar, normalized data from MARATHON, E03-103, and HERMES with the Kulagin-Petti (KP) model. The MARATHON isoscalar correction is applied to all data. The MARATHON and E03-103 are normalized up by $2.8 \%$ and the HERMES data are normalized up by $0.9 \%$ (see text). The error bars include statistical and systematic uncertainties.

### 6.5 Analyzing the EMC slope

Studies of the EMC effect often strive to look for correlations between the strength of the EMC effect and other nuclear quantities. To do this, a measure of the EMC strength must be defined. The typical definition of this is the absolute value of the slope of the isoscalar EMC ratio in the range $0.35 \leq x \leq 0.7$, referred to as $\left|d R_{\mathrm{EMC}} / d x\right|$. One benefit of this definition is that it is largely free of normalization uncertainties.

Figure 55 shows this fit for the Helium-3 EMC ratio. In selecting the data for the fit, the cut was placed on the range of $0.35 \leq x \leq 0.7$, as defined in the previous paragraph. This has the effect of omitting the bins centered at $x=0.345$ and $x=0.705$. As these bins are close to the fit range and can be considered part of the EMC region, their inclusion was studied. Ultimately, the decision was made to exclude these points in order to maintain consistency with the traditional extraction that only uses data in the defined range. The inclusion of the $x=0.345$ bin resulted in an approximately $5 \%$ lowering of the slope. This is a small effect because the bins value is nearly centered within the statistical fluctuations in the data and the EMC slopes are often nearly linear
down to approximately $x=0.25$. Inclusion of the $x=0.705$ bin resulted in a nearly $50 \%$ decrease in the slope, which is a significantly larger effect. This is because the bin at $x=0.705$ not only appears to have a very large fluctuation from the trend of the data, but this is also the region where Fermi motion begins to flatten out the EMC slope until the data makes a sharp turn upward. The exclusion of these points means that the fit data is in the range of $0.36 \leq x \leq 0.69$.

In this section the data are shown along with other EMC data to study correlations with nuclear quantities. These data are from the SLAC E139, JLab Hall C E03-103, and JLab Hall B CLAS experiments. Extractions of the EMC slopes from these data have been completed by both Arrington et. al. [55] and Malace et. al. [56]. Each of these experiments and extractions use a different isoscalar correction, which has an effect on the extracted slope. In order to ensure that correlations are properly examined, isoscalar corrections should be applied in a consistent way to all data. For the following plots (Figures 56-61), the data from each of these experiments has the MARATHON $F_{2}^{n} / F_{2}^{p}$ based isoscalar correction applied. The EMC slopes are then extracted by a linear fit of the data in the $0.35 \leq x \leq 0.7$ region. Due to binning, the range of data included for E139 is $0.36 \leq x \leq 0.68$. Due to binning and available data, the range of data included for CLAS is $0.353 \leq x \leq 0.58$.

The two most commonly studied correlations are with respect to mass number $A$ and scaled nuclear density. Nuclear density, measured in nucleons $/ \mathrm{fm}^{3}$, is the number of nucleons per unit volume of the nucleus. In this analysis, the nuclear density is calculated using the hard-sphere approximation, $\rho(A)=3 A / 4 \pi R^{3}$. In this equation, $R^{2}=5\left\langle r^{2}\right\rangle / 3$, where $\left\langle r^{2}\right\rangle$ is the root-meansquare (rms) charge radius of the nucleus being studied. The rms charge radii used come from Reference [57] (Atomic Data and Nuclear Data Tables), which were extracted from electron scattering experiments. In cases where multiple values are listed, the average of the values are used. The exception to this is Silver-108 (Ag108), as charge radius data for this nucleus is unavailable. Nuclear charge radius is correlated with $A$, so to approximate the charge radius of $\operatorname{Ag} 108$, the charge radii of Ag107 and Ag109 from Reference [58] were averaged. Scaled nuclear density is the nuclear density scaled by a factor of $(\mathrm{A}-1) / \mathrm{A}$. This scaling removes the density contribution of the struck nucleon from the overall value. The correlation of the EMC effect with nuclear density has been noticed in many past experiments, as described in Chapter 3. Figures 56 and 57 show the correlation with $A$ and scaled nuclear density, respectively.


Figure 55: A fit of the EMC slope of the MARATHON Helium-3 EMC data. The error bars include statistical and systematic uncertainties.

EMC Slope vs. Mass Number


Figure 56: EMC slope versus mass number $A$ for a selection of nuclei.

Reference [59] examines the correlation of the EMC slope with nuclear binding energy per nucleon and the residual strong interaction energy (RSIE) per nucleon. Nuclear binding is typically understood to play a part in the EMC effect, but is considered insufficient to completely explain the effect. RSIE is calculated by removing the Coulomb contribution from the binding energy of the nucleus. That is, $\operatorname{RSIE}(A, Z)=B(A, Z)+(0.71 \mathrm{MeV}) Z(Z-1) A^{-1 / 3}$ where $B(A, Z)$ is the binding energy of the nucleus. This calculation assumes that nuclear binding is only comprised of strong and electro-magnetic interactions. For all Figures and calculations, the binding energies from Reference [60] are used. Figures 58 and 59 show the correlation with binding energy per nucleon and RSIE per nucleon, respectively.

Another correlation often studied is with respect to the average nucleon separation energy, $\langle\epsilon\rangle$. This quantity provides another means of examining the nuclear binding model in the context of the EMC effect. In this scheme, nuclear binding causes the nucleons to have a level of "offshellness". The separation energy of a nucleon is a measure of how off-shell the nucleon is. The separation energy needed causes a rescaling of $x$ by approximately $\langle\epsilon\rangle / M$, where $M$ is the mass of the nucleon. Nucleon separation energy is at the heart of models that include off-shell corrections. Off-shell corrections have been shown to well-approximate the shape of the EMC effect, though they cannot describe the effect on their own. The $\langle\epsilon\rangle$ values used here were obtained from Reference [55]. Figure 60 shows the correlation of the EMC slope with $\langle\epsilon\rangle$. Nuclear separation energy data for Lead-208 is unavailable and is thus excluded from this figure.

The final correlation considered is with the Short-Range Correlation Scaling Coefficient, $a_{2}$. In the SRC model, correlated pairs of nucleons are greatly modified giving rise to the EMC effect. A measure of the probability that a nucleon belongs to a nucleon-nucleon SRC pair can be obtained by measuring $a_{2}$, which is the height of a plateau observed when studying the per-nucleon cross section ratio of a nuclear target to that of deuterium in the $Q^{2}>1.4(\mathrm{GeV} / c)^{2}$ and $1.5 \leq x \leq 1.9$ range. The $a_{2}$ values used were obtained from Reference [55], as it has the most complete set. This ensures that the extractions were treated accordingly. Data for $a_{2}$ for Lead-208 are only available in Reference [24], so that measurement is used here. Figure 61 shows this correlation. There have been no measurements of $a_{2}$ for Calcium-40 and Silver-108, so they are excluded from the plot.

EMC Slope vs. Scaled Nuclear Density


Figure 57: EMC slope versus scaled nuclear density for a selection of nuclei.

EMC Slope vs. Binding Energy per Nucleon


Figure 58: EMC slope versus binding energy per nucleon for a selection of nuclei.

EMC Slope vs. Residual Strong Interaction Energy per Nucleon


Figure 59: EMC slope versus residual strong interaction per nucleon energy for a selection of nuclei.


Figure 60: EMC slope versus nuclear separation energy for a selection of nuclei.

EMC Slope vs. SRC Scaling Coefficient


Figure 61: EMC slope versus Short-Range Correlation Scaling Coefficient $a_{2}$ for a selection of nuclei.

## CHAPTER 7

## Summary and Outlook

The MARATHON experiment measured the Helium-3/Deuterium yield ratio in the kinematic region of $0.195 \leq x \leq 0.825,2.5(\mathrm{GeV} / c)^{2} \leq Q^{2} \leq 13(\mathrm{GeV} / c)^{2}$, and $3.5\left(\mathrm{GeV} / c^{2}\right)^{2} \leq W^{2} \leq$ $13\left(\mathrm{GeV} / c^{2}\right)^{2}$. The data were collected in the Hall A facility of the Thomas Jefferson National Accelerator Facility (JLab) in Virginia utilizing both standard equipment High Resolution Spectrometers (HRS). The Left HRS had a momentum setting of $3.1 \mathrm{GeV} / c$ and measured scattered electrons over an angular range of $16.8^{\circ}-33.5^{\circ}$. The Right HRS had a momentum setting of $2.9 \mathrm{GeV} / c$ and measured scattered electrons at an angle of $36.1^{\circ}$. The CEBAF accelerator delivered 10.59 GeV electrons incident on the target at $22.5 \mu \mathrm{~A}$. This is the first measurement of the Helium-3 EMC effect purely in the Deep Inelastic Scattering region.

The Helium-3 EMC ratio has been measured twice before, which is useful for checking our data against. The HERMES experiment at the HERA collider in Hamburg, Germany reports Helium-3/Deuterium data in the region of $0.013 \leq x \leq 0.35$. This data is useful for studying the shadowing and anti-shadowing region of nuclear structure data, but it lacks visibility of the EMC slope. The E03-103 experiment at JLab in Hall C reports Helium-3/Deuterium data in the region of $0.3 \leq x \leq 0.9$. The data collected was not purely in the DIS region. However, the experiment did extensively verify that the data were independent of $Q^{2}$, thus avoiding any resonance contributions. Section 6.4 shows that the MARATHON Helium-3 EMC data is in very good agreement with these previous measurements.

A study was done comparing the $F_{2}^{n} / F_{2}^{p}$ extraction from the Helium-3/Deuterium yield ratio to the extraction from MARATHON ${ }^{2} \mathrm{H} /{ }^{1} \mathrm{H}$ data. The $F_{2}^{n} / F_{2}^{p}$ extraction from ${ }^{2} \mathrm{H} /{ }^{1} \mathrm{H}$ agrees well with world data. An agreement between these extractions was expected in the vicinity of $x=0.3$, which was absent in our data. This analysis of the data finds that a $2.8 \%$ normalization to the ${ }^{3} \mathrm{He}$ cross section data is necessary in order to bring the extractions into agreement.

The Helium-3 EMC Ratio was corrected for proton excess utilizing the $F_{2}^{n} / F_{2}^{p}$ extracted from MARATHON ${ }^{3} \mathrm{H} /{ }^{3} \mathrm{He}$ data. This yields EMC data for a hypothetical $A=3, Z=3 / 2$ nucleus. As expected, the isoscalar data show a clear EMC depletion, the hallmark of the EMC effect. Section 6.5 details these results in the context of previous studies of the EMC effect. The strength of the EMC effect is commonly defined as the absolute value of the slope of the isoscalar EMC ratio, $\left|d R_{\text {EMC }} / d x\right|$, in the region of $0.35 \leq x \leq 0.7$. The strength of the EMC effect is plotted against various nuclear quantities in order to establish correlations that can help illuminate the effect's origin. The use of the MARATHON isoscalar correction and the inclusion of the MARATHON Helium-3 results agree well with previous searches for correlations. The strength of the EMC effect shows a clear correlation with mass number $A$, the per-nucleon binding energy, per nucleon residual strong interaction energy, nuclear separation energy, and Short-Range Correlation scaling coefficient $a_{2}$. Seen also is a correlation with scaled nuclear density, however ${ }^{9} \mathrm{Be}$ is an outlier suggesting that local density may play a factor in the EMC effect explanation.

The MARATHON measurement provides critical data for the establishment of a complete picture of the EMC effect. While we certainly have a better understanding of the EMC effect, the puzzle is far from solved. Two leading explanations for the EMC effect are Mean Field Enhancement and Short-Range Correlations (SRC). These two theories have proven to have quite accurate predictive power. The next frontier in studying the EMC effect is to make a measurement where the predictions by these models diverge. Mean field enhancement predicts that polarization will enhance the EMC effect while SRCs predict polarization will minimize the effect. CLAS in Hall B at JLab will measure the spin structure functions of ${ }^{7} \mathrm{Li}$ in order to determine its polarized EMC effect.

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## APPENDIX A

## Calibrating the Hall A Raster

This Appendix is to describe the raster calibration that was performed by the author for the tritium family of experiments. When performing the calibration, the traditional method did not work due to unforeseen problems with the Beam Position Monitors inaccurately reporting the full beam spread. This issue led to the development a new way to calibrate the raster. The author has written a technical note for Hall A documenting this procedure [61], should this issue arise for a future experiment. The rest of this Appendix is a subset of the referenced technical note that focuses on the important details for performing the calibration.

## A. 1 What is a Raster Calibration

A well calibrated raster will map a raster current to the instantaneous beam position at both Beam Position Monitors (BPMs) and the target. Accurate beam position data is critical for proper reconstruction of the reaction vertex of physics events. A poorly calibrated horizontal raster will cause poor $z$-vertex resolution, making it more difficult to subtract the background from target endcaps. A poorly calibrated vertical raster will incorrectly reconstruct event momentum, causing errors in physics analysis.

The raster calibration is completed by defining a function to map the raster current (measured in ADC bins) to a position. The default raster class in the Hall A analyzer uses a linear function for the calibration. Each coil will have two calibration values per position: the slope and intercept of this line. When running in single-arm mode, each raster needs to be calibrated for each arm independently. This is because of differences in performance of the ADCs on each arm. This give a total of twenty-four calibration values as there are three positions (two BPMs and the target) and four coils (two per dimension) calibrated for two arms. When running in coincidence mode, the calibration only needs to be done once since only one arm records the beam data that is used. In this case, there are twelve calibration values. The slope of the line determines the size calibration
of the raster coil, the value converts the raster current (in ADC units) to a beam position displaced about the mean beam position. The intercept of the line, in conjunction with the slope, sets this mean beam position. The slope should have units of meters(m)/ADC bin and the intercept should have units of meters.

The Hall A Analyzer software has the capability to include correlation between the $x(y)$ position and the $y(x)$ raster. Historically, this has not been used. There is no evidence that such a correlation exists. This would only come into play if the raster magnets were to shift off-axis (i.e. they rotate so that an electron moving in the $z$-direction is deflected in a direction that is not purely horizontal or vertical).

The calibration is applied through matrix arithmetic. The raster currents (measured in ADC Bin) are represented as a $1 \times 2$ row matrix. This is then multiplied by the ADC Bin to $m$ conversion factor matrix (the slopes of the calibration). This is a $2 \times 2$ matrix that would only have non-zero off-diagonal elements in the case of an off-axis raster correlation. Finally, the $1 \times 2$ offset matrix is added to get the final $1 \times 2$ position matrix.

Equation A. 1 shows how the Hall A Analyzer software applies the calibrations. In this equation $R$ represents the raster current, $S$ represents the conversion factor, and $O$ represents the offset. The $l$ indices represent the location of the calibration (i.e. "A" for BPMA, "B" for BPMB, and " T " for target).

$$
\left[\begin{array}{ll}
{ }^{l} x & { }^{l} y
\end{array}\right]=\left[\begin{array}{ll}
{ }^{l} R_{x} & { }^{l} R_{y}
\end{array}\right] \times\left[\begin{array}{cc}
{ }^{l} S_{x x} & { }^{l} S_{x y}  \tag{A.1}\\
{ }^{l} S_{y x} & { }^{l} S_{y y}
\end{array}\right]+\left[\begin{array}{ll}
{ }^{l} O_{x} & { }^{l} O_{y}
\end{array}\right]
$$

The position at the target is used to determine the beam location when intersecting the $z=0$ plane (the target center). The positions at the BPMs are used to determine the trajectory of the beam (i.e. at what angle is the beam intersecting the $z=0$ plane). When combined with the interaction plane defined by HRS tracking, the true reaction point can be found. This increases the accuracy of physics reconstruction.

Since only one set of coils is used for physics analysis, that is the only set that is required to be calibrated. The method for calibrating the beam position at the target described in this document can only be applied to that particular set of coils. For the Tritium experiments, we used the "traditional" method to calibrate the unused set of coils. This was an unnecessary step, as they
were never used in analysis; this was done to give us a good starting point in case some unforeseen error required their use.

The second set of coils could be calibrated in this same way by modifying the Rastered Beam class (THaRasteredBeam). The data used for physics reconstruction is defined by the THaDetector type class that is added first. By recompiling the analyzer after changing the first raster added, the following calibration methods can be employed for the second set of coils [62].

## A. 2 Determining the Sign of the Calibration

When determining the beam position, it is obvious to first look at the BPM. These are sets of sensing wires in the beamline that can measure the beam position. While the position of the beam reported by the BPMs is highly accurate when averaged over time, the measurement is a slow process and cannot be used on an event-by-event basis. This slowness can cause misleading interpretations of beam movement if we are not careful.

The raster current measures displacement about the central beam position. This measurement says nothing about the direction of the displacement. To determine this relation, which is critical to calibrating the rasters, we must use the BPMs and a fixed position feature that we can see in both the BPM and raster spectra: The Carbon Hole target.

To begin, the Carbon Hole must be visible to the beam. Machine Control Center (MCC) is then asked to steer the beam to another BPM position, changing only one dimension. Once there, a short run is taken using the clock trigger. By observing how the carbon hole has moved in the BPM spectrum, the direction that the beam moved can be determined. Note that the MCC BPM coordinate system is not always the same as the hall coordinate system; the BPM data recorded by the Data Acquisition System (DAQ) is in the hall coordinate system. By then observing how the carbon hole moves in the raster spectrum, it can be determined if the coordinate system of the raster in that dimension is the same as that of the BPMs. If the direction of the movement is the same, the sign of the calibration in that dimension is 1 ; if the movement is opposite, the sign is -1 .

This procedure must be completed for both the horizontal and vertical dimensions. The results of this procedure should not change unless the raster power supplies are worked on and wired differently. Under typical circumstances, an experiment only needs to do this procedure once. For the Tritium experiments, we found the sign of the horizontal calibration to be 1 and the sign of the vertical calibration to be -1 [63].

## A. 3 Calibrating the Beam Position on Target

In the past, the calibration values have been found by assuming that the BPMs accurately reflect the mean position and overall magnitude of the rastered beam. By mapping the mean and RMS of raster current to the mean and RMS of BPM positions, a 1:1 mapping could be quickly achieved. Unfortunately, a bandpass filter on the BPM signal prevents the BPMs from accurately reproducing the full size of the rastered beam. We aim to improve on this method to provide the most accurate picture of the beam spread that we can [64].

While the BPMs can (and still are) used to determine the mean beam position, we must look for other quantities to calibrate the raster size. The obvious choice is the carbon hole target. The carbon hole is used to set the size during data taking because it is known to have a diameter of 2 mm . The hole is visible in the raster spectrum, so we can fit it to determine a size calibration for the raster.

The resolution of reconstructed events does not allow for a hard line between the events originating from the foil and the absence of events in the "hole" region. Events smear across this border suggesting that we ought to use a smooth function to define the edge. We decided to use a radial sigmoid function with a floating "hardness" constant. A sigmoid, mathematically defined as:

$$
\begin{equation*}
a=\frac{1}{1+e^{-h \cdot b}}, \tag{A.2}
\end{equation*}
$$

is a smoothed step function with a constant that determines how hard of a step it is. In this general form of the equation, $a$ is the vertical axis value and $b$ is the horizontal axis value. As shown in Figure 62, a sigmoid approaches a step function as the "hardness" constant approaches infinity. This was found to converge and visually appears to fit the hole well, as seen in Figure 63. When doing these fits the Log Likelihood option is used in the software which ensures that fits are not skewed by empty bins. This fitting procedure determines the edge of the hole to be at the position where the value is halfway between the minimum and maximum value of the function [65].

Sigmoids with varying hardness


Figure 62: A 1-Dimensional Sigmoid Function with Varying Hardness Parameter

Converting the above generalized sigmoid to a 2D sigmoid that describes the carbon hold yields:

$$
\begin{equation*}
\text { Counts }=\frac{p 0}{1+e^{-p 5 *\left((p 1 *(p 2-x))^{2}+(p 3 *(p 4-y))^{2}-1\right)}}+p 6 \tag{A.3}
\end{equation*}
$$

In the above equation, "Counts" is the counts in a given bin, $x$ is the horizontal raster value (in ADC bins), and $y$ is the vertical raster value (in ADC bins). For definitions of the fit parameters $p 0$ through $p 6$, see Table 8. For this fit, the conversion factors are restricted to positive numbers in a small region around an "educated guess" of the approximate conversion factors. These "educated guesses" can be determined by roughly measuring, by eye, the approximate width of the carbon hole in ADC bins and then dividing by 2 mm . The center values are restricted to the approximate range of the raster data. These restrictions ensure that the fit is not fooled by any outlying data, which can be readily determined by eye when the fit is drawn. The sign of the calibration needs to be applied (multiplicatively) to ensure the calibration is done correctly.

Note that in Equation A.3, the fit conversion factor will have units of mm/ADC bin. The resulting conversion factors must be divided by 1000 before further use. This was done for clarity

## Parameter Definitions

```
p0 Approximate signal level outside the carbon hole (measured in ADC bins)
p1 Current (ADC bins) to mm conversion factor for Horizontal Raster
p2 Horizontal center of the carbon hole in current (ADC) units
p3 Current (ADC bins) to mm conversion factor for Vertical Raster
p4 Vertical center of the carbon hole in current (ADC) units
p5 "Hardness" factor for the sigmoid (approaches a step function as this increases)
p6 Approximate signal level inside the carbon hole (measured in ADC bins)
```

Table 8: Definitions of the parameters used by equation A. 3 to fit the carbon hole with a 2 D sigmoid function.
as the size of the Carbon Hole is typically discussed in mm. To avoid the need for further unit conversion, the following equation can be used instead:

$$
\begin{equation*}
\text { Counts }=\frac{p 0}{1+e^{-p 5 *\left((p 1 *(p 2-x))^{2}+(p 3 *(p 4-y))^{2}-10^{-6}\right)}}+p 6 . \tag{A.4}
\end{equation*}
$$

Upon further analysis, it was determined that this calibration method yielded little improvement over the "traditional" method. The fit places the edge of the hole at the horizontal zero-crossing of the sigmoid function. Due to smearing from the spectrometer, this is not the "true edge" of the hole. This was determined by looking at the physics values that the raster calibration affects. For the horizontal rasters, this is the reconstructed $z$ position at the target. For the vertical rasters, this is the square of the invariant mass of the final hadronic state, $W^{2}$. When the rasters are properly calibrated, there should be no correlations between the raster current and the physics variables that are affected by the calibration.

Proceeding further, we determined that we ought to look at these physics values for improving the calibration. Utilizing two "bad" calibrations, we looked at the strength of the correlation as it is related to the calibration. If we assume that the relation is linear, we can interpolate (or extrapolate if they were bad in the same direction) to determine the correct calibration [44].

For the horizontal raster, we used the single carbon foil target and plot the horizontal raster current versus the reconstructed $z$ position. We can then take slices of this plot in horizontal


Figure 63: Using the radial sigmoid function to fit the hole in the Carbon Hole target. In this plot, the density of red rings is directly correlated to the slope of the function at that point. Where the rings are densest is the $50 \%$ position, corresponding to where the fit locates the edge of the hole.
raster current bins and fit the resulting plots with a Gaussian and plot the peaks. With a properly calibrated raster, there should be no correlation between horizontal raster current and reconstructed $z$. Using two improper horizontal calibrations, we measured the slopes of the correlations. We then interpolated to determine the calibration that would yield a slope of 0 . This procedure forces the sign of the calibration to be built into the answer, there is no need to apply it.

In practice, this does not yield a calibration with a slope of precisely zero. This is due to statistical fluctuations in the data around the true correlation line. We attempted to iterate this procedure, but it did not yield significant improvement on the results. Figures 64, 65, and 66 document this process.


Figure 64: Slices in horizontal raster current of the reconstructed $z$ versus raster current plot are fit with a Gaussian. The peaks of the Gaussian are then plotted to see the correlation. In the plot of average $z$, it can be seen that the peak position shifts by over 2 mm over the movement of the raster. The carbon foil is only 0.25 mm thick.


Figure 65: Two "bad" calibrations are fit to find the correlation between horizontal raster current and the average $x$ position. Both of these calibrations show a displacement larger than the thickness of the target.


Figure 66: Using a linear fit of the correlation between average $z$ and horizontal raster current for two "bad" calibrations, we can interpolate to the correct calibration. Here we can see that the shift is approximately 0.1 mm .

For the vertical raster, we look for a momentum feature that the experiment can see. In the case of many of the Tritium era experiments, the Hydrogen elastic peak was measured. Plotting the vertical raster current versus the $W^{2}$ of Hydrogen elastic events, we followed a similar procedure to that of the horizontal raster. As with the horizontal calibration, there is no need to apply the sign of the calibration in this method.

For experiments that do not have an identifiable momentum feature, an approximation of the correct vertical calibration can be found using the horizontal calibration and the sigmoid fit of the carbon hole. Since it is known that the true calibration lies somewhere on the sigmoid above the zero-crossing, we can use the horizontal calibration to determine where that point is. Using the calibration determined using the $z$ vertex reconstruction, we determine how many horizontal ADC bins the "true edge" lies away from center (the "true edge" is 1 mm away from the center). We then evaluate Equation A. 3 using this value. Then, Equation A. 3 is reversed to determine the vertical ADC bin displacement to yield the same value. Once we have the displacement, a new vertical
calibration can be determined. This is an imperfect solution, but ultimately should provide the best calibration possible in the absence of a momentum feature.

Once these methods are used to determine the slope of the raster calibration, we turn our attention back to the BPMs. The BPMs, after calibration with the harp, provide a very accurate reading of the mean beam position at their location in the beamline. This information can then be used to determine the mean position of the beam at the target. To do this, we plot each BPMs' position spectrums and determine their mean value. Each spectrum must have its mean determined independently because the BPM readings lag behind events, the position is only accurate when averaged over time. Once we know the mean positions at each BPM, a track through these positions can be projected to the mean position at the target.

With the slope of the raster calibrations and a point on the calibration lines (the mean position at the target), we have all the information that we need to determine the raster calibration lines. Using a simple point-slope form, we input the information for each raster and solve for the intercept.

## A. 4 Calibrating the Beam Position at the BPMs

The beam position must also be calibrated at the BPMs. To do this, we use the "traditional" method of raster calibration. As in determining the mean position at the target, the raster and BPM spectrums are plotted. The mean and RMS of each spectrum are then calculated. In these cases, since we are determining the calibrations at the BPMs, no projections are needed.

To determine the size calibration, the slope of the calibration line, divide the RMS of the BPM spectrum by the RMS of the raster spectrum and then multiply by the sign of the calibration. To then determine the calibration offset, point-slope form of a line can be implemented using the mean positions and the aforementioned slope calculation [66].

