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PHYSICS

MEASUREMENT OF THE EMC EFFECT OF THE TRITIUM NUCLEUS AT  
JEFFERSON LAB (139 pp.)

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Experiment E12-10-103 (MARATHON) was conducted in the Hall A Facility of JLab in the winter/spring of 2018 during the initial phase of the 12 GeV energy upgrade of the Lab. One goal of the experiment was the first measurement of the EMC effect of  $^3\text{H}$ . Four sealed gas targets were used during the experiment,  $^1\text{H}$ ,  $^2\text{H}$ ,  $^3\text{H}$ , and  $^3\text{He}$ . All measurements were in the deep inelastic scattering kinematical regime with large  $Q^2$  and  $W^2$ , which are the four-momentum transfer squared and invariant mass squared of the final hadronic state in the inelastic scattering interaction, respectively. The range of  $Q^2$  and  $W^2$  were  $3 < Q^2 < 12 \text{ (GeV}/c)^2$  and  $3.2 < W^2 < 12.3 \text{ GeV}/c^2$ , respectively. The measurements spanned a wide range of the Bjorken variable  $x$ , between 0.19 and 0.83, where  $x = Q^2/2M(E - E')$  is the momentum fraction carried by the struck quark in the interaction, with  $M$  being the nucleon mass. Knowledge of the EMC effect of light nuclei, such as  $^3\text{H}$ , will provide an opportunity to better understand the origin of the EMC effect.

MEASUREMENT OF THE EMC EFFECT OF THE TRITIUM NUCLEUS AT  
JEFFERSON LAB

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to Kent State University  
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by  
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## CHAPTER 1

### **Electron Scattering**

#### 1.1 Introduction

Nuclear physics is the study of the fundamental structure that comprises the nucleus. The origins of nuclear physics can be traced back to the discovery by Ernest Rutherford that atoms contained a localized positive charge which was not evenly distributed throughout the volume of the atom as predicted by J. J. Thomson. The discovery was made by detecting scattered alpha particles off a gold foil target and, surprisingly, detecting several of them at large angles. This can be viewed as the catalyst from which a series of important discoveries have been made. Over time, the boundary was pushed as experiments probed deeper, providing further clues to the nature of the nucleus. Lepton scattering by nuclei has played a pivotal role in the elucidation of nuclear structure. The fundamental point-like nature of leptons makes them well suited for probing the more complicated structure of nucleons. Such experiments have helped to explain a broad range of nuclear structure, across a wide range of energy scales. From mapping out nucleon-nucleon interactions to discovering the underlying fundamental nucleon structure based on quarks and gluons; they have contributed to providing a more complete picture of the nucleon.

### 1.1.1 Overview

Charged lepton scattering is the best method to learn about the internal structure and dynamics of nuclei and the nucleons that comprise them. It is especially well suited due to the point-like nature of leptons such as electrons and muons. The interaction between an electron and a nucleus is well understood in terms of Quantum Electrodynamics (QED). The strength of the electron's coupling to the electromagnetic field is expressed by the fine structure constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0} \simeq \frac{1}{137}, \quad (1.1)$$

where  $\hbar = c = 1$ ,  $e$  is the elementary charge, and  $\epsilon_0$  is the permittivity of free space. The relative weakness of the coupling constant allows for a perturbation expansion to be made in  $\alpha$ . Working in the lowest order expansion in  $\alpha$  corresponds to the exchange of a single virtual photon  $\gamma^*$  which mediates the electron-nucleus interaction in QED. This approximation allows for detailed investigations of nuclear structure. Experimental electron scattering can be divided into 3 regions: elastic, quasi-elastic, and inelastic scattering. A brief overview of the three types of electron scattering will be presented before focusing the remainder of the chapter on Deep Inelastic Scattering (DIS).

### 1.1.2 Electron Scattering

Electron Scattering processes can be described in terms of the following Lorentz invariant quantities defined as:

$$\nu = \frac{pq}{M} = E - E' \quad (1.2)$$

$$Q^2 = -q^2 = 4EE' \sin^2 \left( \frac{\theta}{2} \right) \quad (1.3)$$

$$W^2 = (q + p)^2, \quad (1.4)$$

where  $\nu$  is the energy transfer of the electron,  $Q^2$  is the square of the four-momentum transfer of the virtual photon,  $W^2$  is the square of the invariant mass of the final hadronic state,  $E$  and  $E'$  are the electron's initial and final energy respectively,  $M$  is the mass of the proton,  $\theta$  is the angle at which the electron is scattered in the laboratory frame, and  $p$  is the four-momentum of the proton. Electron scattering can be broadly separated into distinct regions which can be used to describe specific structure of the nucleon. The spatial extent of the nuclear environment to which the electron is sensitive depends on both the energy and momentum transfer of the virtual photon.

Elastic scattering is usually characterized by relatively large  $Q^2$  and small  $\nu$ . At such four-momentum and energy transfer values, the interaction takes place within the nucleus, leaving it in a bound state after the interaction, and is a technique to study nuclear structure. The final hadronic state for elastic scattering, in this case, is defined as,  $W^2 = M^2$ . Quasi-elastic scattering can be considered as elastic scattering off of individual nucleons moving inside the nucleus [1]. With the increasing energy transfer of the electron, the interacting nucleon is knocked out of the nucleus and

provides the opportunity to study such things as the momentum distributions of the nucleons. Finally, in Deep Inelastic Scattering, by increasing the  $Q^2$  of the virtual photon, the internal components of the nucleon become accessible and the electron begins to scatter off of the constituent quarks inside the nucleus. Inclusive DIS processes can be represented by the Feynman diagram in Figure 1, which can likewise be expressed by Equation 1.5:

$$e^-(k) + H(p) \rightarrow e^-(k') + X, \quad (1.5)$$

where  $k = (E, \vec{k})$  and  $k' = (E', \vec{k}')$  are the four-momenta of the initial and scattered electrons respectively,  $p$  is the four-momentum of the target nucleon of mass  $M$  in the laboratory frame, and  $X$  represents the undetected hadronic system after scattering. The virtual photon,  $\gamma^*$ , possesses four-momentum  $q = (\nu, \vec{q})$ , where  $\nu = E - E'$  is the energy transfer, and  $\vec{q} = \vec{k} - \vec{k}'$  is the three momentum transfer.

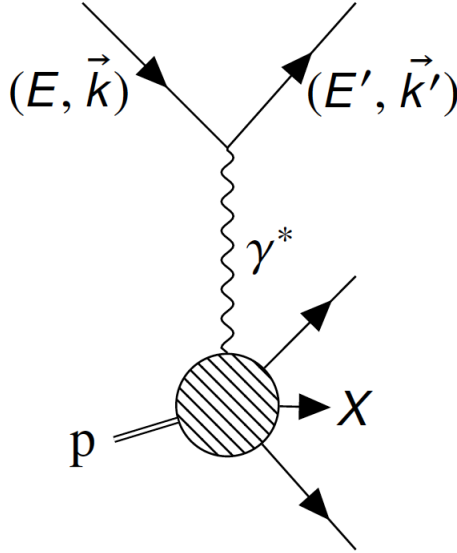


Figure 1: Feynman diagram depiction of electron-proton Deep Inelastic Scattering.

By expanding Equation 1.4 and noting that in the laboratory frame, where the target is at rest, the four-momentum of the target proton is  $p = (M, 0)$ . The invariant mass squared of the final hadronic system in a DIS process can be expressed by

$$W^2 = (q + p)^2 = M^2 + 2M\nu - Q^2. \quad (1.6)$$

Well known resonant states exist in the inelastic region, so to ensure that the process is deep inelastic it is not only important that  $Q^2$  is large but also that the invariant mass is large enough to be outside of the resonance region (such as the  $\Delta$  resonance). It is typically taken that the invariant mass should be  $W^2 > 3.25 \text{ (GeV}/c^2)^2$ . Figure 2 is a plot of the differential cross section versus the invariant mass, for a fixed Energy ( $E$ ) and scattering angle ( $\theta$ ) [2]. The invariant mass spectrum illustrates the different regions of electron scattering as well as the need for DIS data to be outside of prominent resonances. The elastic peak can be seen at  $W \simeq 0.9$ , followed by resonant peaks, and lastly the DIS continuum at  $W > 1.84 \text{ (GeV}/c^2)$ .

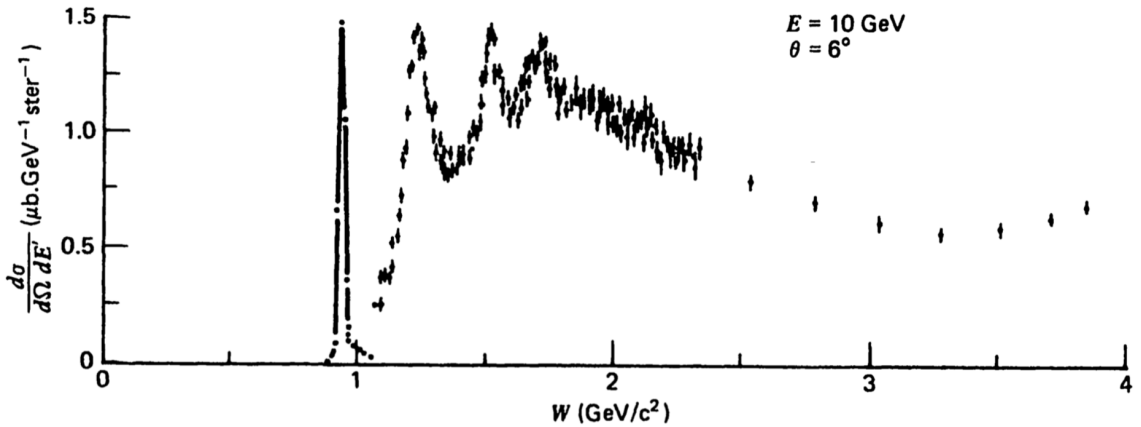


Figure 2: Invariant mass spectrum of electron-proton scattering [2]. The DIS range is defined to have  $W > 1.84 \text{ (GeV}/c^2)$  in order to exclude prominent resonances. Note that the elastic peak has been scaled by a factor of  $1/8.5$  to fit on the plot.

### 1.1.3 QED and Deep Inelastic Scattering

Having outlined the general principles of electron scattering, we will next focus on a theoretical formalism which can be used to interpret experimental observations. Quantum Electrodynamics governs the electromagnetic interaction in electron scattering. The experimentally measured quantity is referred to as the cross section,  $\sigma$ . It can be interpreted as a measure of the probability for the given reaction to occur. In QED, to construct the cross section, one must determine the invariant amplitude squared,  $|A|^2$  of the process [1]. The differential cross section for deep inelastic, electron-proton scattering, can be expressed as:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4} \frac{E'}{E} |A|^2 \quad (1.7)$$

where  $d\Omega$  is the solid angle into which the electron has scattered. The invariant amplitude is formed from the electron and proton transition currents and contains the physics of the interaction. The quantity  $|A|^2$  can be written more explicitly as:

$$|A|^2 = L_{\mu\nu} W^{\mu\nu}, \quad (1.8)$$

where  $L_{\mu\nu}$  and  $W^{\mu\nu}$  are the lepton and hadronic tensors respectively. The lepton tensor is given by [3]:

$$L_{\mu\nu} = \frac{1}{2} \sum_{spins} [\bar{u}(k') \gamma^\mu u(k)] [\bar{u}(k') \gamma^\nu u(k)]^*. \quad (1.9)$$

The quantities  $u(k)$  and  $\bar{u}(k')$  are Dirac spinors which represent the incoming and scattered electron, and  $\bar{u} = u^\dagger \gamma^0$ .

The  $\gamma^\mu$  terms are four  $(\gamma^0, \gamma^1, \gamma^2, \gamma^3)$  4x4 matrices which satisfy the anti-commutation relation [3]:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}, \quad (1.10)$$

where  $g^{\mu\nu}$  is the metric tensor. Equation 1.9 can be further simplified by utilizing the completeness relation along with trace theorems of the  $\gamma$  matrices. The completeness relation is given by:

$$\sum_{s=1}^2 u^{(s)}(k) \bar{u}^{(p)}(k) = \not{k} + m, \quad (1.11)$$

where  $\not{k} = \gamma^\mu k$ . Equation 1.9 reduces to

$$L_{\mu\nu} = 2(k'^\mu k^\nu + k'^\nu k^\mu - (k' \cdot k - m^2)g^{\mu\nu}). \quad (1.12)$$

Similarly, the hadronic component can be expressed in terms of a hadronic tensor,  $W^{\mu\nu}$ . The general form of  $W^{\mu\nu}$  can be expressed as [4]:

$$W^{\mu\nu} = \frac{1}{2} \sum_s \sum_N \langle p, s | J^\mu | X \rangle \langle X | J^\nu | p, s \rangle (2\pi)^4 \delta^4(p + q - p_N), \quad (1.13)$$

where  $J^\mu$  is the proton transition current. The hadronic tensor,  $W^{\mu\nu}$ , has a more complicated form than that of the lepton tensor due to the complex internal

structure of the proton. In this way,  $W^{\mu\nu}$  is the parameterization of this complicated system. By invoking Lorentz invariance, the most general form of the hadronic tensor can be written as [5]:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} (p^\mu q^\nu + q^\mu p^\nu). \quad (1.14)$$

The tensor  $W^{\mu\nu}$  can be further simplified by enforcing current conservation of the hadronic vertex [6]:

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0, \quad (1.15)$$

from which we can deduce the following two equations:

$$-\frac{W_2}{M^2} (q \cdot p) p^\nu + \frac{W_5}{M^2} q^2 p^\nu = 0 \quad \text{and} \quad -W_1 q^\nu + \frac{W_4}{M^2} q^2 q^\nu + \frac{W_5}{M^2} (q \cdot p) q^\nu = 0. \quad (1.16)$$

This reduces  $W^{\mu\nu}$  to an expression containing only  $W_1$  and  $W_2$ . In this form, the lepton and hadronic tensors can now be contracted as:

$$L_{\mu\nu} W^{\mu\nu} = 2 \left[ k'_\mu k_\nu + k'_\nu k_\mu - (k' \cdot k - m^2) \right] \left[ W_1 (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) + \frac{W_2}{M^2} (p^\mu - \frac{(p \cdot q)}{q^2} q^\mu) (p^\nu - \frac{(p \cdot q)}{q^2} q^\nu) \right]. \quad (1.17)$$

By working in a frame where the proton is initially at rest,  $p = (M, 0)$ , and neglecting the mass of the electron, Equation 1.17 can be simplified to

$$L_{\mu\nu} W^{\mu\nu} = W_1 E E' \sin^2 \frac{\theta}{2} + 4 W_2 E E' \cos^2 \frac{\theta}{2}. \quad (1.18)$$

The differential cross section can then be expressed as [7]:

$$\frac{d^2 \sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[ 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} + W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} \right]. \quad (1.19)$$

The final outcome is that the inclusive electron-proton deep inelastic cross section depends on two quantities, referred to as the  $W_1$  and  $W_2$  structure functions, which will be discussed further in the subsequent sections.

#### 1.1.4 Virtual Photoproduction

Further insight can be gained by viewing electron-proton deep inelastic scattering in terms of virtual photoproduction [8, 9]. The connection can be seen by noting that by replacing the electron-virtual photon ( $\gamma^*$ ) vertex in Figure 1, with a real photon ( $\gamma$ ), will not change the form of the hadronic tensor given by Equation 1.13. However, the differences in polarization states between virtual and real photons must be taken into account. Virtual photons ( $\gamma^*$ ) can be both transversely and longitudinally polarized while real photons ( $\gamma$ ) have only transverse polarization [5]. To make the connection to Deep Inelastic Scattering, we must sum over all possible polarization states when constructing the cross section. The differential cross section for electron-proton scattering can be expressed in terms of the transverse and longitudinal total cross sections. The virtual photon polarizations are given by [5]:

$$\begin{aligned}\epsilon_+ &= -\sqrt{\frac{1}{2}}(0; 1, +i, 0) \\ \epsilon_- &= +\sqrt{\frac{1}{2}}(0; 1, -i, 0) \\ \epsilon_0 &= -\sqrt{\frac{1}{Q^2}}\left(\sqrt{\nu^2 + Q^2}; 1, +i, 0\right).\end{aligned}\tag{1.20}$$

The total cross section can be expressed as:

$$\sigma_{tot} = \frac{4\pi\alpha}{K}\epsilon_\mu^*\epsilon_\nu W^{\mu\nu},\tag{1.21}$$

where  $K$  is the laboratory photon energy producing a final state of total mass  $W$  upon absorption of a proton at rest and is defined as [9]:

$$K = \frac{W^2 - M^2}{2M}.\tag{1.22}$$

The connection between the transverse ( $\sigma_t$ ) and longitudinal ( $\sigma_l$ ) cross sections and the  $W_1$  and  $W_2$  structure functions can be shown to be:

$$\sigma_t = \frac{4\pi^2\alpha}{K} W_1 \quad (1.23)$$

$$\sigma_l = \frac{4\pi^2\alpha}{K} \left[ \left( 1 + \frac{\nu^2}{Q^2} \right) W_2 - W_1 \right]. \quad (1.24)$$

The ratio of the longitudinal to transverse cross sections,  $\sigma_l/\sigma_t$ , can be used to express  $W_1$  in terms of  $R$  and  $W_2$  since:

$$R = \frac{\sigma_l}{\sigma_t} = \frac{\left( 1 + \frac{\nu^2}{Q^2} \right) W_2(\nu, Q^2) - W_1(\nu, Q^2)}{W_1(\nu, Q^2)}. \quad (1.25)$$

Rearranging Equation 1.25 gives

$$W_1(\nu, Q^2) = \frac{\left( 1 + \frac{\nu^2}{Q^2} \right) W_2(\nu, Q^2)}{1 + R}. \quad (1.26)$$

This is a key result and to which we will return to in the next section.

## 1.2 Scaling and The Quark-Parton Model

Elastic electron scattering experiments were performed throughout the 1950's and culminated in the discovery that the proton was not a point-like object [10]. By the 1960's, construction of the Stanford Linear Accelerator Center (SLAC) was complete, providing higher energies that afforded new opportunities to study nucleon structure [11]. In 1967, a collaboration between Massachusetts Institute of Technology (MIT) and SLAC began a program of studying deep inelastic electron scattering. The elastic scattering experiments reported a cross section which had a strong  $Q^2$  dependence [12]. Surprisingly, the results from the DIS experiments showed an unexpected feature. The DIS cross section exhibited a weak  $Q^2$  dependence, in contrast with the elastic results. Figure 3 illustrates this difference.

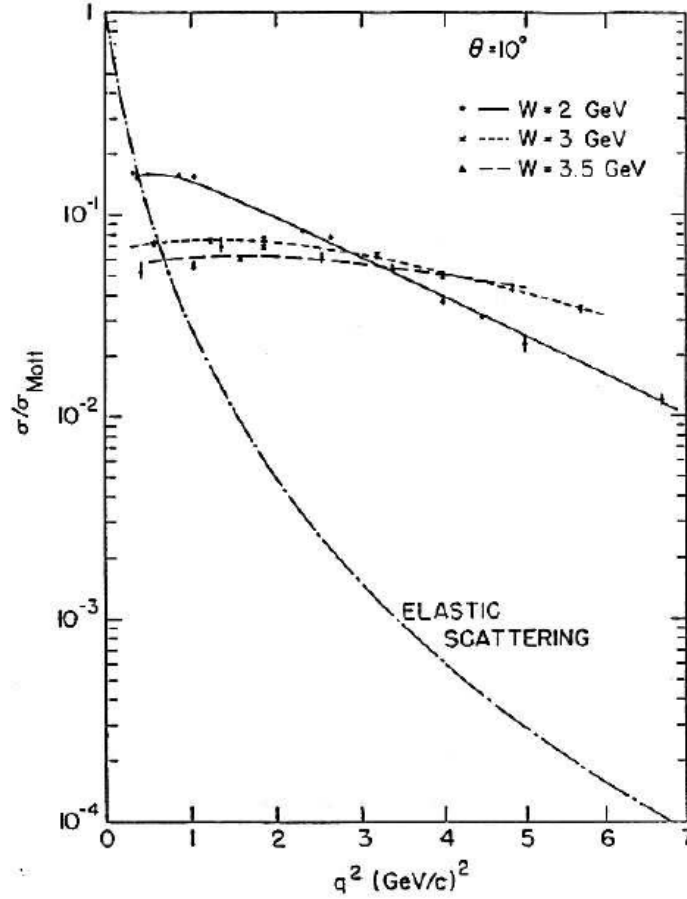


Figure 3: Comparison of elastic and deep inelastic electron-proton cross sections [13]. An apparent difference in the  $Q^2$  dependence for each cross section is observed.

An explanation of the experimental DIS results came in the form of the Quark-Parton Model (QPM). In 1969, Bjorken [14] proposed that in a frame of reference where a nucleon has infinite momentum, inelastic electron scattering can be interpreted as the incoherent sum of elastic scattering of the electron off of a parton inside of the nucleon, where the partons are the constituent particles which comprise the nucleon. In this Lorentz frame, the partons appear to be effectively free (non-interacting) from each other [4]. Also, the partons have no transverse momentum in

this frame and lie along the same axis before and after the interaction. Finally, it is assumed that the electron-parton interaction is not affected by final-state interactions from the subsequent confinement of the partons into color singlet particles. The QPM can be approximated in the limiting case where

$$\begin{aligned} Q^2 &\rightarrow \infty \\ \nu &\rightarrow \infty . \end{aligned} \tag{1.27}$$

In this “Bjorken” limit, the structure functions  $W_1$  and  $W_2$ , which depend on two kinematic variables,  $\nu$  and  $Q^2$ , become functions of a single variable,  $x$ :

$$x = \frac{Q^2}{2M\nu} , \tag{1.28}$$

which is the fraction of the momentum carried by the struck quark. To further explain the scaling behavior, we can first imagine electron scattering off of a point-like, spin-1/2 particle. It can be shown that the cross section for such a reaction is given by [3]

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 E'^2}{Q^4} \left( \cos^2 \frac{\theta}{2} + \frac{Q^2}{2m^2} \sin^2 \frac{\theta}{2} \right) \delta \left( \nu - \frac{Q^2}{2m} \right) . \tag{1.29}$$

Treating the partons as spin-1/2 particles and comparing Equation 1.19 with Equation 1.29, we can see that for elastic scattering off of point-like particles in the nucleons implies:

$$\begin{aligned} 2mW_1(\nu, Q^2) &= \frac{Q^2}{2m\nu} \delta \left( 1 - \frac{Q^2}{2m\nu} \right) \\ \nu W_2(\nu, Q^2) &= \delta \left( 1 - \frac{Q^2}{2m\nu} \right) , \end{aligned} \tag{1.30}$$

which indicate that the structure functions  $W_1$  and  $W_2$  are indeed a function of a single quantity,  $x$ , as defined in Equation 1.28.

In the Bjorken limit, the structure functions are often written as:

$$MW_1(\nu, Q^2) \rightarrow F_1(x) \quad (1.31)$$

$$\nu W_2(\nu, Q^2) \rightarrow F_2(x).$$

The differential cross section of Equation 1.19 can then be expressed as:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[ \frac{2F_1(x)}{M} \sin^2 \frac{\theta}{2} + \frac{F_2(x)}{\nu} \cos^2 \frac{\theta}{2} \right]. \quad (1.32)$$

By recalling that  $F_1$  and  $F_2$  are related through  $R$  via Equation 1.26, Equation 1.32 can be expressed in terms of  $F_2$  and  $R$  as:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \cos^2 \frac{\theta}{2} F_2(x) \left[ \frac{1}{\nu} + \frac{1 + \frac{Q^2}{\nu^2}}{xM(1+R)} \tan^2 \frac{\theta}{2} \right]. \quad (1.33)$$

Thus, experimentally,  $F_2$  can be extracted if the quantity  $R$  is known. The validity of the Bjorken approximation is illustrated in Figure 4. One can see that at intermediate values of  $x$ , the  $F_2$  structure function is approximately independent of  $Q^2$ . Logarithmic scaling violations which can be observed at low and high  $x$  can be explained by Quantum Chromodynamics (QCD) [15]. Additional details about scaling violations will be discussed in Section 1.3.

Finally, in the Bjorken limit, given by Equation 1.27, and with  $x$  constant, the structure functions can be expressed as [2]:

$$F_1(x) = \frac{1}{2} \sum_i e_i f_i(x) \quad (1.34)$$

$$F_2(x) = x \sum_i e_i f_i(x),$$

where  $e_i$  are the fractional charges of the quarks (partons) and  $f_i(x)$  is the probability density function of a quark of flavor (type)  $i$ .

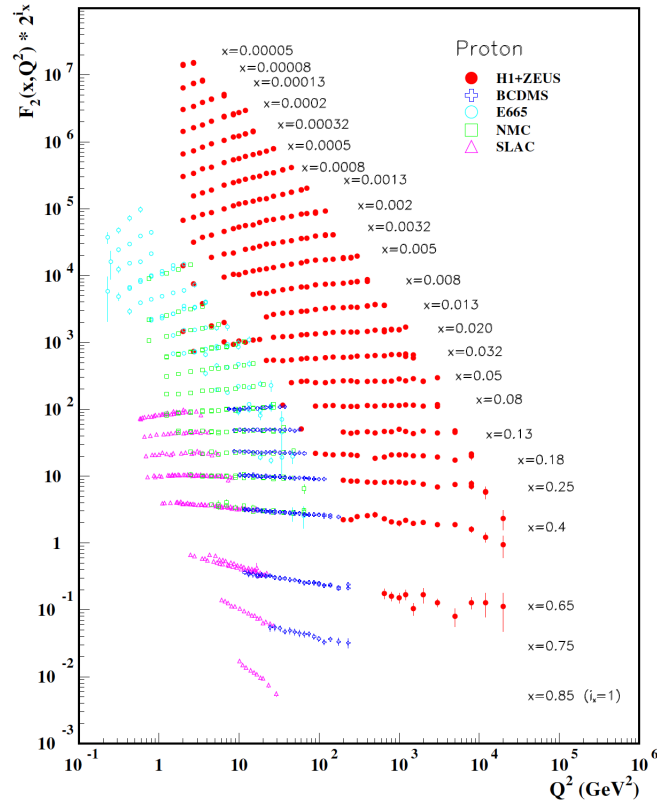


Figure 4: Compilation of world data illustrating the approximate scaling of  $F_2$  [15].

The independence of  $F_2$  with  $Q^2$  appears most strongly at medium values of  $x$  with scaling violations at low and high  $x$ .

In the Bjorken limit, the structure function of the proton,  $F_2^p(x)$  can be written explicitly in terms of the probability distributions of the constituent quarks and anti-quarks as [6]

$$F_2^p(x) = x \left[ \left(\frac{2}{3}\right)^2 [u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2 [s^p(x) + \bar{s}^p(x)] \right]. \quad (1.35)$$

Exploiting the fact that the proton and neutron form an isospin doublet, the up (down) quark distribution in the proton (neutron) can be related to the down (up) quark distribution in the neutron (proton)

$$\begin{aligned} u^p &= d^n \\ d^p &= u^n \\ s^p &= s^n. \end{aligned} \quad (1.36)$$

In a similar fashion, the structure function of the neutron can be expressed as

$$F_2^n(x) = x \left[ \left(\frac{2}{3}\right)^2 [d^n(x) + \bar{d}^n(x)] + \left(\frac{1}{3}\right)^2 [u^n(x) + \bar{u}^n(x)] + \left(\frac{1}{3}\right)^2 [s^n(x) + \bar{s}^n(x)] \right]. \quad (1.37)$$

By exploiting this symmetry, the QPM allows one to examine several features, such as the limiting behavior of the  $F_2^n/F_2^p$  ratio. The first observation is

$$\frac{1}{4} \leq \frac{F_2^n}{F_2^p} \leq 4. \quad (1.38)$$

Equation 1.38 is known as the Natchmann Inequality [16], which places upper and lower bounds on the  $F_2^n/F_2^p$  ratio. Further insight can be gained by looking at extreme limiting cases.

For instance, assuming a flavor symmetric virtual quark and anti-quark sea, one would expect to find at low  $x$

$$\lim_{x \rightarrow 0} \frac{F_2^n}{F_2^p} \rightarrow 1. \quad (1.39)$$

One can also look at the high Bjorken- $x$  limit, where in the valence region there is little influence from the sea or contribution from strange quark distributions. The limiting behavior of  $F_2^n/F_2^p$  can be shown to be [17]:

$$\lim_{x \rightarrow 1} \frac{F_2^n}{F_2^p} \rightarrow \frac{U + 4D}{4U + D}, \quad (1.40)$$

where  $U$  and  $D$  are the sum of the quark and anti-quark distribution functions and are given (after neglecting strange quark distributions) by

$$U = u + \bar{u} \quad (1.41)$$

$$D = d + \bar{d}.$$

From Equation 1.40, theoretical predictions have been made which can be compared to experimental measurements of  $F_2^n/F_2^p$  to discern the quark dynamics in the valence region of medium and high Bjorken- $x$ .

### 1.3 QCD Scaling Violations

The Quark-Parton Model, discussed above, is an extremely useful framework to both conceptualize and calculate Deep Inelastic Scattering processes. This can be seen in Figure 4, which shows that at medium values of  $x$ , and over a larger range in  $Q^2$ , the Quark-Parton Model approximation provides an accurate description of the structure functions of the proton. Nevertheless, a small but non-negligible  $Q^2$  dependence exists in the structure function. A complete description requires moving

beyond the QPM to a field theory description in terms of Quantum Chromodynamics. QCD is a non-abelian field theory describing the strong force between quarks and gluons [3]. The coupling constant for QCD,  $\alpha_s$ , can be expressed in a similar fashion as in QED by solving the renormalization group equations. The unique feature of the non-abelian nature of QCD results in the coupling constant of QCD to become smaller with increasing energy. In QCD, it can be shown that the violations seen in the QPM are in fact due to the interactions between quarks and gluons. An example of such interaction is shown in Figure 5, in which a quark has radiated a gluon before the interaction with the virtual photon,  $\gamma^*$ , resulting in a reduction of the fractional momentum carried by the struck quark.

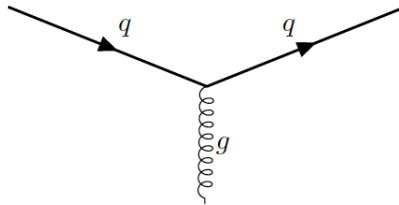


Figure 5: Feynman diagram of a gluon radiating off of a quark. Such interactions result in the observed DIS scaling violations.

This implies that a fraction of the momentum of the interacting quark has been carried away by a related parton, resulting in the violations of scaling noted above. Recalling from Equation 1.34, the structure functions in QPM were written in terms of the quark distributions, which are functions of the scaling variable  $x$ . In evaluating the quark distributions in QCD, it can be shown that they can be expressed in terms of the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equations [18]. The distribution functions are no longer a function of simply  $x$  but also  $Q^2$ . Including

the gluon distribution, the parton distribution functions in terms of the DGLAP equations can be written as [19, 20]:

$$\begin{aligned}\frac{\partial q_i(x, \mu^2)}{\partial \ln \mu^2} &= \frac{\alpha}{2\pi} \int_x^1 \frac{dz}{z} \left[ P_{q \leftarrow q}(z) q_i\left(\frac{x}{z}, \mu^2\right) + P_{q \leftarrow g}(z) g\left(\frac{x}{z}, \mu^2\right) \right] \\ \frac{\partial g_i(x, \mu^2)}{\partial \ln \mu^2} &= \frac{\alpha}{2\pi} \int_x^1 \frac{dz}{z} \left[ P_{g \leftarrow q} \sum_i q_i(z) \left(\frac{x}{z}, \mu^2\right) + P_{g \leftarrow g}(z) g\left(\frac{x}{z}, \mu^2\right) \right],\end{aligned}\tag{1.42}$$

where the  $P_{x \leftarrow y}$  terms are splitting functions, which describe the probability that a parton (e.g. quark) has lost a portion of its momentum through radiation of a parton (e.g. gluon) [21],  $z$  is the fraction of the momentum carried by the radiated gluon or anti-quark, and  $\mu$  is a factorization scale parameter. The DGLAP equations do not solve the parton distribution function at a certain scale,  $\mu_1$ , but with a known value for an input at  $\mu_1$ , will evolve the distributions at the new desired scale  $\mu_2$ . The  $F_2$  structure function can then be expressed as:

$$F_2(x, Q^2) = x \sum_{n=0} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z, Q^2, \mu_R^2, \mu_F^2) \phi_p^i\left(\frac{x}{z}, \mu_F^2\right), \tag{1.43}$$

where the sum is over parton distribution functions, labeled  $\phi_p^i$  in Equation 1.43. The terms,  $\mu_F$  and  $\mu_R$ , are the factorization and renormalization scale respectively. Commonly, a single scale is used,  $\mu_F = \mu_R$ . The procedure of factorization, from which the factorization scale ( $\mu_F^2$ ) comes, acts to explicitly separate perturbative and non-perturbative processes [17]. Processes below some threshold ( $< \mu_F^2$ ) are grouped into the parton distribution functions  $\phi_p^i$  and those above the threshold in the  $C_{2,i}$  coefficients. Equation 1.43 is an evolved  $F_2$  structure function. An important feature to note is the lowest order contribution to Equation 1.43. In the case when  $n = 0$ , Equation 1.43 simplifies to Equation 1.34, returning the QPM picture of Deep Inelastic Scattering.

## CHAPTER 2

### EMC Effect

Prior to 1983, the general assumption was that the nucleons inside of a nucleus could be imagined as being weakly bound to one another with their internal properties largely unaffected by the specific nuclear environment in which they were in. Thus, the structure functions were assumed to simply scale with nuclei according to [22]:

$$AF_2^A = ZF_2^p + (A - Z)F_2^n, \quad (2.1)$$

where  $A$  is the atomic mass and  $Z$  is the atomic number of each nucleus. In this view,  $F_2^p$  and  $F_2^n$  are the structure functions of a free proton and neutron respectively. But, in 1983, The European Muon Collaboration (EMC) at CERN measured the per nucleon ratio  $F_2^{56\text{Fe}}/F_2^{2\text{H}}$ , and surprisingly found that the isoscalar ratio was not unity as expected. Figure 6 shows the results of their findings. It should be mentioned that the low  $x$  behavior has been found to be inconsistent with subsequent experiments and further reanalysis showed agreement among data sets.

Recalling from Equation 1.34, the  $F_2$  structure functions are the sum of the quark probability distribution functions, weighted by the fractional charge of the quarks. Combining the definition of the structure function with the experimental results indicates that the fraction of the quark distributions are modified in bound nucleons [23], in direct contradiction to the prevailing opinions of the day. While there has been intense theoretical and experimental activity to explain the mechanism causing the effect, the EMC effect has so far, persisted without a definitive explanation.

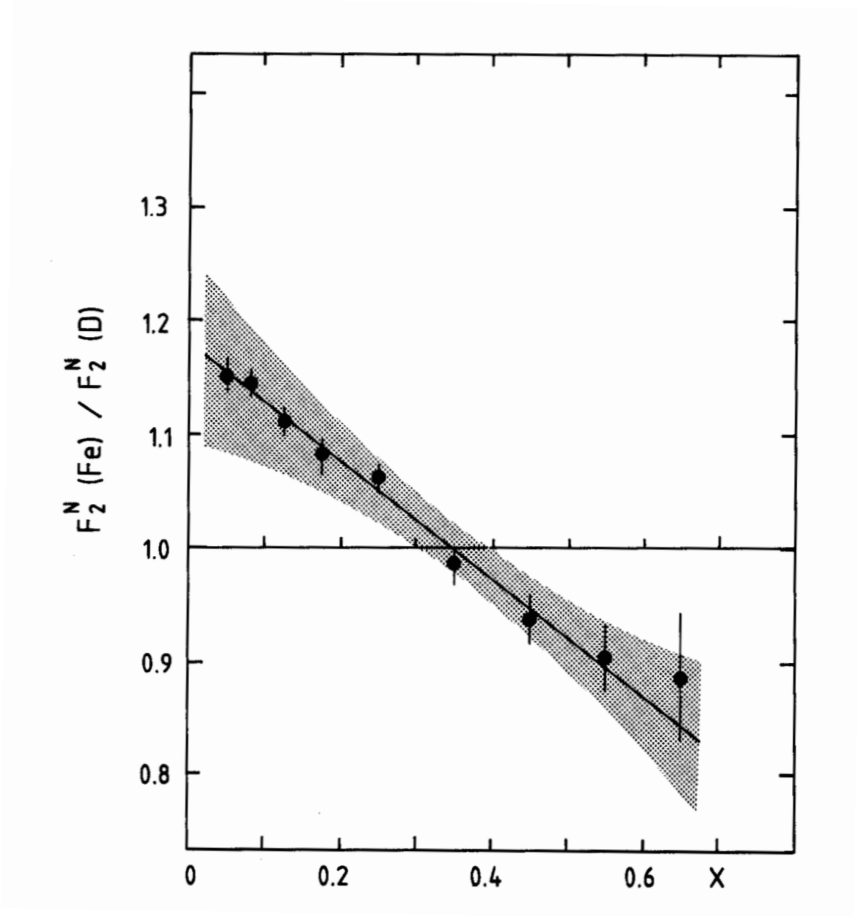


Figure 6: Original results from the European Muon Collaboration showing the first indication of the EMC effect [22]. Note that low  $x$  data have subsequently been revised.

## 2.1 Cross Section Ratios

As discussed in the previous section, the EMC effect is expressed as the per nucleon ratio of  $F_2$  structure functions for a given nucleus ( $A$ ) and Deuterium ( $^2\text{H}$ ):

$$\frac{F_2^A/A}{F_2^{^2\text{H}}/2}, \quad (2.2)$$

where  $A$  represents the number of nucleons in the nucleus. In order to arrive at such an expression, we must first recall the form of the cross section, as shown in Equation

1.33. The cross section has been written in terms of the  $F_2$  structure function as well as  $R$ . The ratio of cross sections for any two nuclei, a and b, can be expressed as

$$\frac{\sigma_a}{\sigma_b} = \frac{\frac{4\alpha^2 E'^2}{Q^4} \cos^2 \frac{\theta}{2} F_2^a(x) \left[ \frac{1}{\nu} + \frac{1 + \frac{Q^2}{\nu^2}}{xM(1+R_a)} \tan^2 \frac{\theta}{2} \right]}{\frac{4\alpha^2 E'^2}{Q^4} \cos^2 \frac{\theta}{2} F_2^b(x) \left[ \frac{1}{\nu} + \frac{1 + \frac{Q^2}{\nu^2}}{xM(1+R_b)} \tan^2 \frac{\theta}{2} \right]}. \quad (2.3)$$

The kinematic quantities,  $Q^2$ ,  $E$ ,  $E'$ , and  $\theta$  will cancel if the cross section measurements are made at the same set of values. Furthermore, the cross section ratio depends on the four quantities  $F_2^a$ ,  $F_2^b$ ,  $R_a$  and  $R_b$ . The difference in  $R$  for different nuclei,  $\Delta R = R_a - R_b$ , has been measured in high precision experiments at both SLAC [24] and Jefferson Lab (JLab) [25] in an attempt to determine the universality of this quantity. To date, these experiments have found that within errors,  $\Delta R=0$ . If  $R$  is indeed the same for all nuclei, then the ratio of cross sections given by Equation 2.3 would result in a ratio of structure functions [26]

$$\frac{\sigma_a}{\sigma_b} = \frac{F_2^a}{F_2^b}. \quad (2.4)$$

Hence, from the measured cross section ratio, we can directly extract the  $F_2$  structure function ratio. Throughout the remainder of this thesis, the terms cross section ratio and structure function ratio, may both be used but should be understood as being interchangeable as implied by Equation 2.4.

## 2.2 EMC Effect in Nuclei

Deep Inelastic Scattering data, such as the structure function ratio of the EMC effect, plotted as a function  $x$ , fall between the extreme values of  $0 < x < 1$ . Historically, the term “EMC effect”, has described the structure function ratio between

$0 < x < 1$ , slicing it into regions along the  $x$ -axis in an attempt to explain the physical processes affecting each region. This division of the effect can be seen in Figure 7, which is a compilation of carbon data from SLAC [27] and NMC [28] experiments, where 4 distinct regions are highlighted. More recently, the term has been used to describe specifically the region between  $0.3 < x < 0.7$ . I will take the historical approach in describing the effect while keeping in mind the current formal definition.

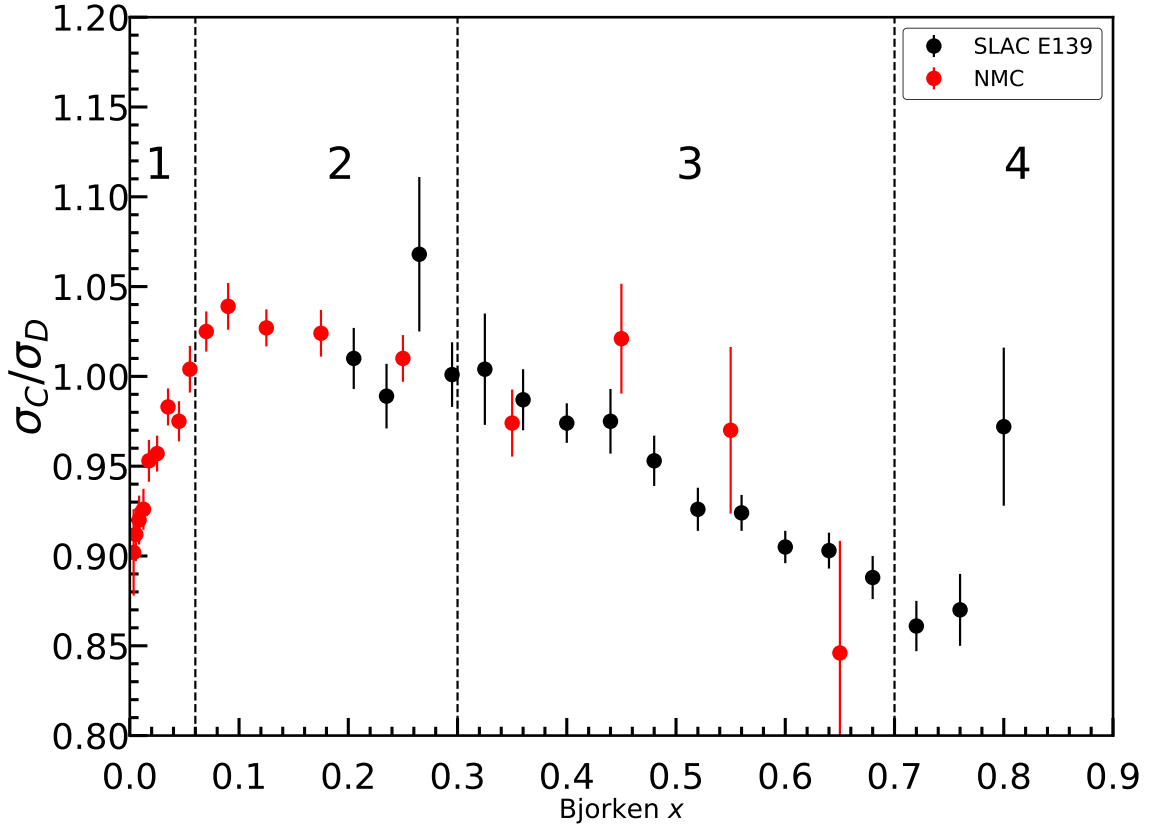


Figure 7: Compilation of carbon data showing the standard regions of the EMC effect. See text for a complete description.

Starting at  $x = 0$  and moving to the right of Figure 7, region 1 is commonly referred to as the shadowing region. The shadowing region range is typically defined as  $0 < x < 0.06$ . In this region, a reduction in the cross section ratio is observed. It

is believed that shadowing is due to fluctuations of the virtual photon into  $q\bar{q}$  pairs (mesons) which interact with the target by means of the strong force. This process occurs at low  $x$ , and thus is not sensitive to valence quarks but instead dominated by interactions with the sea quarks. Between  $0.06 < x < 0.3$ , region 2 is known as the anti-shadowing region. A clear enhancement in the cross section ratio can be seen. There appears to be no conclusive agreement on the cause, but suggestions have related it to constructive interference among the partons in the nucleus [29]. Region 3 is found between  $0.3 < x < 0.7$  and is referred to as the “EMC” or depletion region. In this region, there is a pronounced reduction in the cross section ratio. The amount of reduction appears to be  $A$  dependent with the amount of reduction increasing with  $A$ . An important feature of this region is the universal unity crossing at  $x \simeq 0.3$ . This crossing occurs for all measured nuclei. Lastly, region 4,  $0.7 < x < 1.0$ , in which a dramatic increase in the cross section ratio occurs, is attributed to the Fermi motion of the nucleons inside the nucleus [1].

### 2.2.1 EMC Effect Experiments

In the three decades since the original discovery, considerable focus has been focused on mapping out the effect in a variety of nuclei at a number of experimental facilities, in order to pin down the possible mechanism(s) responsible for the effect. A brief highlight of a selected number of experiments and their contributions will follow.

#### European Muon Collaboration

As mentioned earlier, the EMC effect was first discovered in 1983 by the European Muon Collaboration at CERN. The goal of the collaboration was to measure DIS cross sections at high  $Q^2$ . They could achieve higher  $Q^2$  by using muon beams that

reached energies of 280 GeV [22]. The  $F_2$  structure function ratio of iron to deuterium  $F_2^{56\text{Fe}}/F_2^{2\text{H}}$  was measured and showed the first clear indication of unknown effects in bound nucleons for  $x > 0.3$ , as can be seen in Figure 6.

### SLAC E139

Soon after the publication from the EMC collaboration, experiment E139 at SLAC began a detailed investigation, studying both the  $A$  and  $Q^2$  dependence of the effect. They did so by measuring the EMC effect of a wide range of light, medium, and heavy nuclei, including  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{108}\text{Ag}$ , and  $^{197}\text{Au}$ , for  $2 < Q^2 < 15 \text{ (GeV}/c)^2$  [27]. A noticeable  $A$  dependence of the effect was observed, with increased suppression of the ratio between  $0.3 < x < 0.7$ . Also, no obvious  $Q^2$  dependence was observed. Figure 8 illustrates the EMC effect of the measured nuclei, along with the apparent  $A$  dependence of the effect [27].

### NMC Collaboration

After the initial discovery by the European Muon Collaboration, the New Muon Collaboration (NMC) began a program specifically aimed at studying the EMC effect, with the goal to greatly reduce the uncertainty in the final ratios, as well as to determine the  $A$  and  $Q^2$  dependence. To do so, they included additional targets of  $^4\text{He}$ ,  $^{12}\text{C}$ , and  $^{40}\text{Ca}$  and covered a range in  $Q^2$  between 0.5 and 90  $(\text{GeV}/c)^2$  [28].

### Jefferson Lab

Experiment E03-103 was performed at Jefferson Lab and focused on measuring the EMC effect of light nuclei, including  $^3\text{He}$ ,  $^4\text{He}$ ,  $^9\text{Be}$ , and  $^{12}\text{C}$ . Portions of the data, specifically at high  $x$ , did not reach the typical  $W^2$  threshold considered to be DIS. Instead, they employed Bloom-Gilman duality, which states that the  $Q^2$  averaged

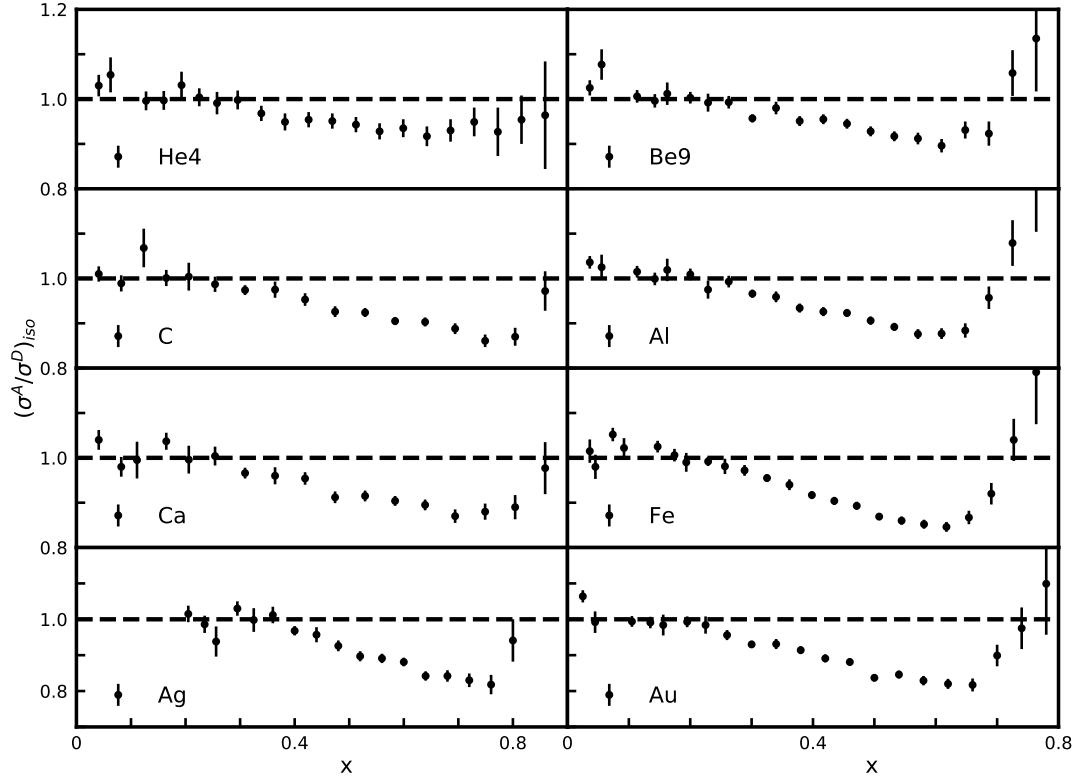


Figure 8: The EMC effect for different nuclei. An increase of the effect with  $A$  can be observed. Reproduced from Reference [27].

structure functions in the resonance region approximates the structures functions in the DIS region [30].

### 2.2.2 EMC Models

Over the past 30 plus years, many models have attempted to explain this phenomenon, with varying success and at present, no one single model has been able to give a full accounting of the effect. A short summary of these different models follows, covering the physics motivation as well as highlighting the strengths and weaknesses of each.

### Dynamical Rescaling

Comparing the structure functions of iron and deuterium, there is an apparent similarity of the  $F_2^{56\text{Fe}}$  and  $F_2^{2\text{H}}$  structure functions, but with  $F_2^{56\text{Fe}}$  at a higher  $Q^2$  value. It has been suggested that this difference is related to the confinement size of the quarks in the nucleus. As the confinement size changes so does the  $Q^2$ . The comparison of  $F_2^{56\text{Fe}}$  (or of any other nucleus) with  $F_2^{2\text{H}}$  should not be made at the same  $Q^2$  but at a shifted  $Q^2$ , such that  $F_2^{56\text{Fe}}$  has been “re-scaled” [31]. The relationship between nuclear structure functions is given by:

$$F_2^A = F_2^{2\text{H}}(x, \xi Q^2), \quad (2.5)$$

where  $\xi$  is given by

$$\xi(Q^2) = \left[ \frac{\mu_N^2}{\mu_A^2} \right]^{-\frac{\alpha(\mu_N^2)}{\alpha(Q^2)}}, \quad (2.6)$$

where  $\mu_N$  and  $\mu_A$  are the renormalization scale factors of QCD and  $\xi$  is the “rescaling” value that is determined through the  $Q^2$  evolution of the DGLAP equations. Dynamical rescaling models have been capable of matching EMC data in the range  $0.20 < x < 0.70$  but show less promise at low and high  $x$ .

### Pion Enhancement

Considering the effect in terms of more standard nuclear physics, a possible explanation focused on an increase in additional hadronic components in the nucleus, such as due to that of the pion field. It is understood that the binding of nucleons in a nucleus occurs through meson exchange, most notably due to pions. Not all the momentum in the nucleus can be attributed to the nucleons but a portion of the contribution is shared by these virtual pions. In this way, the structure function of

the nucleus,  $F_2^A$ , is a convolution of the sum of nucleon structure functions as well as a pion structure function and can be expressed as [32]:

$$F_2^A(x) = \int_y^A f_N(y) F_2^N\left(\frac{x}{y}\right) dy + \int_y^A f_\pi(y) F_2^\pi\left(\frac{x}{y}\right) dy, \quad (2.7)$$

where  $y$  is the fraction of the nuclear momentum carried by a nucleon ( $N$ ) or pion ( $\pi$ ) and  $F_2^N$  and  $F_2^\pi$  are the structure functions of the nucleon and pion, respectively. Models including an enhancement of the pion field have been able to reproduce some features of the EMC effect but they also predict an increase of sea quarks, which appears to disagree with recent Drell-Yan experiments [33].

### Multi-Quark Cluster

In a similar fashion to the pion enhancement model, a more exotic explanation considers the possibility of multi-quark clusters that may form inside a nucleus. If such color-singlet multi-quark clusters form, the nuclear structure functions would take a form similar to the convolution formulation of Pion enhancement [33]:

$$F_2^A(x) = \int_y^A f_N(y) F_2^N\left(\frac{x}{y}\right) dy + \int_y^A f_B(z) F_2^B\left(\frac{x}{y}\right) dy, \quad (2.8)$$

where  $F_2^B$  the structure of any color singlet quark cluster. The formation of multi-quark clusters can be explained as the overlapping of the nucleon wave functions when two or more nucleons come close together. This overlap can create color-singlet quark “bags” or clusters of 6, 9, or more quarks.

### Short Range Correlations

The theories and models discussed above were put forward around the time of the discovery to explain the effect. The most recent model is due largely to the

findings from the experiment E03-103 at Jefferson Lab. Prior to experiment E03-103, the compilation of EMC results from world data seemed to indicate the effect could be largely explained by the average nuclear density of the nuclei. Further investigations discovered that the slope of the cross section ratio between  $x$  of 0.3 and 0.7 was a meaningful way to characterize the effect since the slope is not sensitive to normalization uncertainties, which may introduce a constant shift in the data [34]. Thus, this region is defined colloquially as the “EMC region”.

As mentioned in Section 2.2.1, experiment E03-103 measured the EMC effect of a series of lighter nuclei including  $^9\text{Be}$ . Once including  $^9\text{Be}$  into the picture, it was apparent that  $^9\text{Be}$  did not fit nicely in the correlation between the EMC slope and the average nuclear density. A new or a more detailed explanation would be needed that could encompass  $^9\text{Be}$  as well. Recognizing that  $^9\text{Be}$  could be thought of as two alpha particles plus a neutron, perhaps it was instead a local density effect [35]. A local density effect, such as the overlapping of nucleon pairs in the nucleus, known as Short Range Correlations (SRCs) [36], appears as a possible explanation. SRCs are highly correlated nucleon pairs that are believed to be the source of the high momentum tails of nucleon distributions [36]. SRCs have been probed through quasi-elastic scattering in the kinematic region  $x > 1$  and have been explored in experiments at both SLAC and JLab. In a similar way to the EMC effect, SRCs manifest themselves in cross section ratios of a given nucleus and deuterium. The cross section ratio is given by [37]:

$$\frac{\sigma_A/A}{\sigma_D/2} = a_2(A), \quad (2.9)$$

where the ratio of cross sections corresponds to the probability to find high momentum nucleons [34]. Direct evidence of SRCs can be seen by noting that if SRCs are the

origin of the high momentum tail for each nucleus, the ratio of Equation 2.9 should be constant, up to some proportionality factor. This region of scaling (or plateau), where SRCs are dominant is described by  $a_2$  in Equation 2.9. Figure 9 shows the slope of the EMC effect versus the  $a_2$  scaling variable. A strong correlation can be observed for all nuclei, including  ${}^9\text{Be}$ , suggesting that these overlapping nucleon pairs may be the mechanism of the EMC effect. More experimental results are still needed to further validate this correlation.

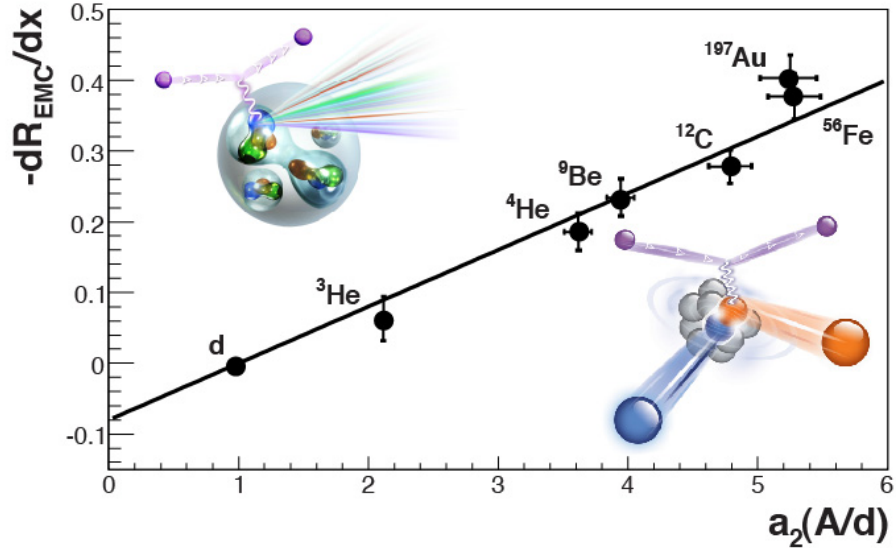


Figure 9: The slope of the EMC effect versus the scaling plateaus [35]. A strong correlation can be seen between these two phenomenon.

### 2.3 MARATHON Experiment

This thesis presents work related to the MARATHON experiment (E12-010-103) at Jefferson Lab in Hall A. The goals of the MARATHON experiment were two-fold. First, the measurement of  $F_2^n/F_2^p$  and  $D/U$  at large  $x$ , for which model uncertainty due to nuclear effects has plagued previous measurements. The unique nature of the

$A = 3$  system makes this possible by looking directly at the DIS cross section ratio of  ${}^3\text{H}$  and  ${}^3\text{He}$ , in which nearly all nuclear effects cancel. Theoretical predictions exist to describe the behavior of both  $F_2^n/F_2^p$  and  $D/U$  in the limit as  $x \rightarrow 1$ . A measurement which is not hampered by the model uncertainties of previous measurements offers a direct way to discern between these various predictions. The second goal of the MARATHON experiment was the measurement of the EMC effect of  ${}^3\text{H}$  and  ${}^3\text{He}$ . The MARATHON experiment will provide the first measurement of the EMC effect of the  ${}^3\text{H}$  nucleus. Also, it provide strictly DIS result for the  ${}^3\text{He}$  nucleus.

The focus of this thesis will be on the measurement of the EMC effect of  ${}^3\text{H}$ . A key to understanding the underlying mechanism of the EMC effect is precise measurements of the effect in light nuclei. More specifically, the  $A=3$  mirror nuclei can provide deeper insight into the effect due to their isospin symmetry. This provides a test for the plausible models and theories put forward over the past three decades. This thesis will present the results of the experiment.

## CHAPTER 3

### Experimental Setup

#### 3.1 CEBAF

The Thomas Jefferson National Accelerator Facility (Jefferson Lab) has a Continuous Electron Beam Accelerator Facility (CEBAF), which delivers high energy electrons to 4 different experimental halls. Jefferson Lab has recently undergone an upgrade making it capable of delivering electrons with energies up to 11 GeV (12 GeV for Hall D) [38]. The accelerator is designed in a “racetrack” shape, consisting of 2 superconducting linacs (North and South), used to accelerate electrons, and recirculation arcs, as can be seen in Figure 10. The electrons can be accelerated a total of 5 times around both linacs to reach the highest energy available of 11 GeV.

A polarized photogun in the injector produces the electrons via photoemission from a GaAs cathode using a 1497 MHz gain-switched diode [39]. Electrons are transmitted to the North Linac, where they are accelerated through a series of cryomodules and subsequently extracted to each of the 4 experimental halls. In the original configuration used during the 6 GeV era, both linacs consisted of 20 cryomodules. An upgrade to the existing infrastructure added 5 newly designed niobium cryomodules, with higher field gradients, to each linac [40]. These additions allowed the accelerator to reach an energy of 11 GeV. An additional factor in the CEBAF operation is the ability to operate efficiently while also delivering a range of currents simultaneously to each of the 4 experimental halls. The MARATHON experiment required the highest energy deliverable, which during the experiment was 10.6 GeV. Administrative

procedures limited the maximum current that could be delivered to Hall A during the MARATHON experiment to  $22.5 \mu\text{A}$ . This was due to the use of gaseous tritium, one of the targets used by the experiment, which is highly radioactive.

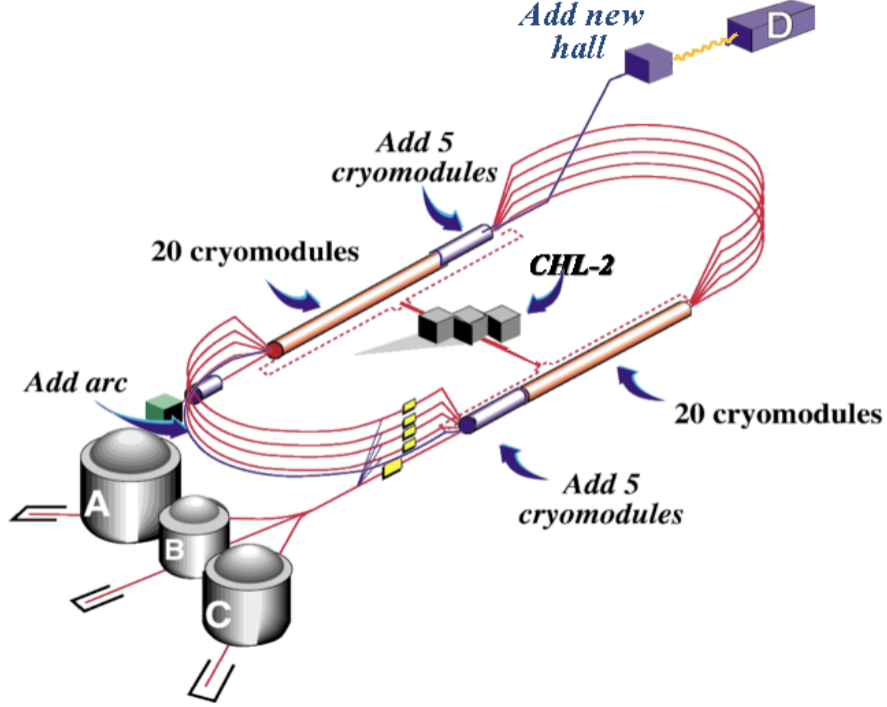


Figure 10: Jefferson Lab CEBAF Accelerator. Shown are the experimental halls A, B, C, and D, along with the superconducting linacs and recirculation arcs. Also shown are the new cryomodules added for the 12 GeV upgrade.

### 3.2 Hall A

Hall A is one of the four experimental Halls at Jefferson Lab, focusing on the studying of electro- and photo-induced reactions [39]. The standard Hall A configuration can be seen in Figure 11, which is a pair of identical 4 GeV/ $c$  High Resolution

Spectrometers. They are commonly referred to as the Left High Resolution Spectrometer (LHRS) and Right High Resolution Spectrometer (RHRS). Both spectrometers are capable of being rotated around the center of the hall, where the target system is located. Scattered particles traverse a series of magnets and bend upwards by  $45^\circ$  into a detector stack. The magnet configuration for both spectrometers consists of three quadrupoles used for focusing and a bending dipole for momentum analyzing the scattered particles. The arrangement of the magnet system during the MARATHON experiment consisted of a resistive quadrupole, a superconducting quadrupole, a superconducting dipole, and lastly a superconducting quadrupole. Additional information about the spectrometers will be given in Section 3.5.

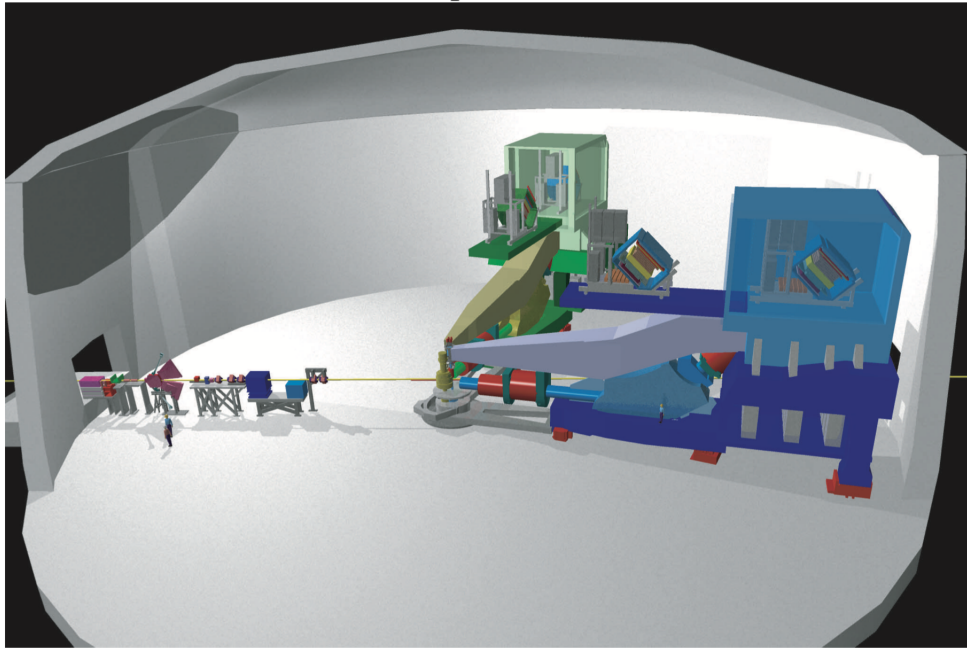


Figure 11: Side view illustration of Hall A, showing the two High Resolution Spectrometers.

### 3.3 Beam Line

#### 3.3.1 Beam Energy

The beam energy used for the MARATHON experiment was the highest available beam energy the accelerator was able to provide to Hall A, which during the running period for MARATHON was approximately 10.6 GeV. A precise knowledge of the incident energy is required to extract meaningful physics from the data. The beam energy was measured using the arc-energy Method [41], which measures the deflection of the beam in the arc section of the accelerator. Both series of arcs can be seen in Figure 10. The beam is steered using a series of 8 dipole magnets in the arc. In the arc near Halls A, B, and C, a 9<sup>th</sup> dipole is connected in series with the other 8 in the arc and is accessible outside of the accelerator. A field mapper equipped with two coils mounted on a table is moved through the 9<sup>th</sup> dipole at a constant velocity to measure the magnetic field integral. Also, the bending angle of the arc is determined using a series of harps, located at the beginning and end of the arc. The bending angle of the beam is 34.3°. With the integrated field of the dipoles, the energy of the beam can be calculated by

$$p = k \frac{\int \vec{B} \cdot d\vec{l}}{\theta}, \quad (3.1)$$

where  $k = 0.299792 \text{ GeV rad T}^{-1} \text{ m}^{-1}/c$ ,  $\int \vec{B} \cdot d\vec{l}$  is the field integral, and  $\theta$  is the bending angle. The uncertainty of the beam energy has been determined to be  $\pm 5 \times 10^{-4}$  [42].

#### 3.3.2 Beam Current Monitor

Located inside of the experimental hall is a beam current monitor, which is used to determine the beam current and subsequently, the amount of charge that is delivered

to the target. The monitoring system is located 25 meters upstream of the target system and consists of a Parametric Current Transformer (PCT), also known as an Unser monitor, and two radio-frequency (RF) cavities [43]. The Unser Monitor is positioned along the beam line and in between the two RF cavities. Each RF cavity is tuned to match the 1497 MHz frequency of the beam. Figure 12 indicates the layout of each component along the beamline.

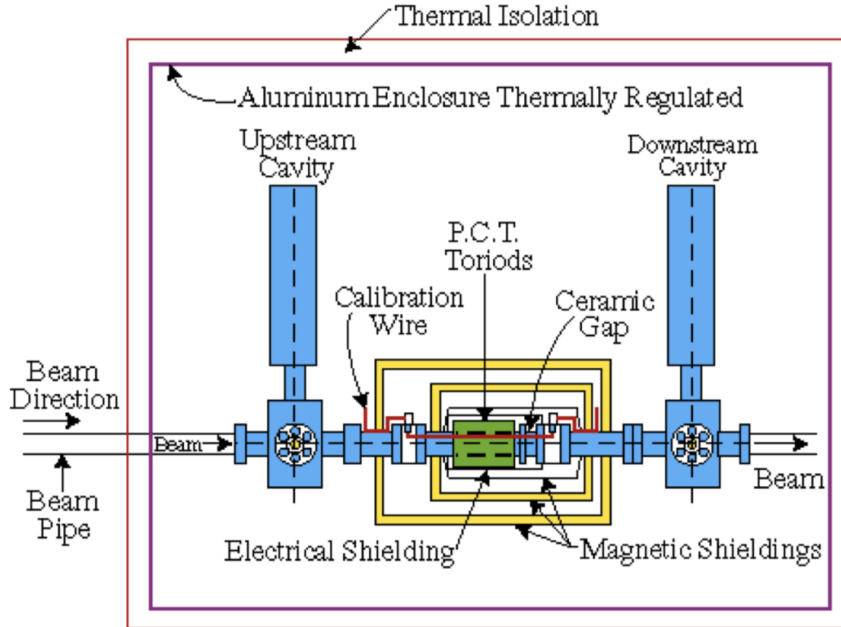


Figure 12: Beam current monitoring system layout. The beam current monitors are located upstream and downstream of the PCT monitor (see text).

The Unser monitor is used as an absolute reference to which the RF cavities can be calibrated. This is due to a drift in the output signal from the Unser monitor, which is unpredictable and can fluctuate during the timescale of individual physics runs. This precludes it from being used independently for beam current monitoring. Instead, it is used as an absolute calibration reference for the RF cavities, which are

used during beam operations to measure the beam current. The Unser monitor is calibrated periodically, when no beam is being delivered to Hall A. A known current is passed through a wire along the beam line and the gain of the Unser, which is a constant value that converts the Unser frequency to current, can be determined.

After calibration of the Unser monitor, the RF cavities can then be calibrated to it. The RF cavity calibration is performed in a similar fashion as the Unser calibration, except using the electron beam. In order to accommodate the large range of currents that could possibly be delivered to Hall A, as well as accounting for the linearity of the intermediate electronics, amplifiers with gains of x3 and x10 are used to improve the linearity of lower currents while sacrificing that of the higher currents. After the 12 GeV upgrade, new digital receivers were added, which have a larger dynamic range over which their signals are linear, no longer requiring the need for amplifiers. The signals are sent to scalers in both the LHRS and RHRS and inserted into the data stream. A diagram of the relevant signals sent to scaler can be seen in Figure 13.

Table 1 lists the RF signals that were available during the MARATHON experiment. Physics analyses utilized the downstream digital receiver (dnew), while the remaining signals were used to check the stability of the system.

| Amplification    | Upstream Cavity | Downstream Cavity |
|------------------|-----------------|-------------------|
| x 1              | u1              | d1                |
| x 3              | -               | d3                |
| x 10             | -               | d10               |
| No Amplification | unew            | dnew              |

Table 1: A list of the BCM signals and amplification factors for both upstream and downstream cavities.

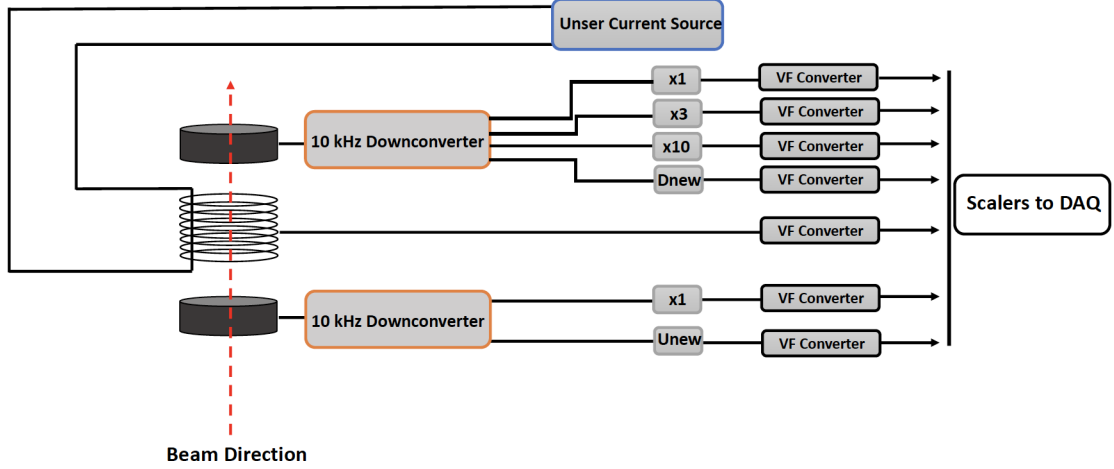


Figure 13: Schematic of the signal pathway for the beam current monitoring system. See Table 1 for list of BCM signals.

### 3.3.3 Beam Position Monitors

The position of the beam delivered to Hall A is determined by a pair of Beam Position Monitors (BPMs), which are located at 7.34 and 2.22 meters upstream of the target. The BPMs provide a non-invasive method to determine the relative position of the beam to  $100 \mu\text{m}$  [44]. The calibration of the BPMs requires first determining the absolute beam position from an invasive measurement using a harp fork (wire scanners), which is located adjacent to the BPMs along the beam line. It is to this absolute position that the BPMs can be calibrated to. Figure 14 shows a drawing of the harp fork. The calibration of the BPMs to the harps requires dedicated runs, during which the harp fork is passed through the beam, allowing for a precise determination of the beam position. The BPMs can then be calibrated to the harps allowing for the beam position to be known on a run-by-run basis.

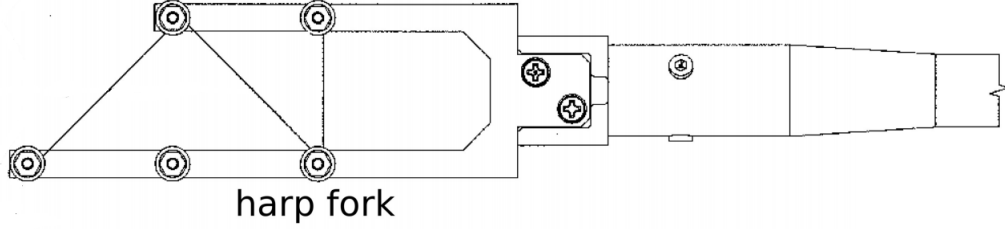


Figure 14: Drawing of the harp which is used to determine the absolute beam position during the BPM calibration.

### 3.3.4 Raster

In order to reduce density fluctuations of the target due to local heating from the beam as well as to minimize any possible damage to the target cell from the small beam profile, the size of the beam is increased by four dipoles, called the raster system, which can change the size of the beam in both the vertical and horizontal directions. The rastering system is positioned 23 meters upstream of the target [39] and consists of a pair of vertical and a pair of horizontal dipoles, which deflect the incoming beam at 25 kHz. Both vertical and horizontal dipoles are synchronized together respectively. The raster current signals are sent to and recorded by an analog-to-digital-converter (ADC) and are used offline to more accurately determine the beam position at the target.

Prior to the MARATHON experiment, several upgrades were made to the electronics and readout system used for the raster signals. Previous to the MARATHON experiment, the raster signals were sent to an offset and attenuator board, before going to the Fastbus ADCs to be recorded. This was due to the limited signal size the ADCs could receive. After the 12 GeV upgrade, the normally flat raster signal distribution exhibited a distinct fluctuation. Attempts were made to reduce the fluctuations but

with limited success. To help rectify this, before the beginning of MARATHON, the offset and attenuator board was replaced with a 20 dB inline attenuator. Also, the signals were sent directly to Flash ADCs, which are JLab designed ADCs and were recently installed on both spectrometers. An overview of the Flash ADC will be presented later in Section 3.6.1. An important feature of the Flash ADC is that it has a larger dynamic range than the Fastbus ADC, which can be manually adjusted. This feature allows the Flash ADCs to receive larger amplitude signals. The new setup resulted in a more uniform raster signal which produced similar results to those from during the 6 GeV era.

### 3.4 Target System

The target system was specifically designed and fabricated for the unique requirements of the MARATHON experiment. Driving the need for a new target system was the inclusion of  $^3\text{H}$ , a radioactive isotope of hydrogen. The target system needed to meet both the safety standards as well as the physics goals of the experiment. This process took many years and designs in order to reach the final design. The final design chosen was a sealed 25 cm long cell with a 1.25 cm radius, made from 7075 Aluminum Alloy.

Figure 15 shows the final assembly of the target ladder used during MARATHON, which included 4 gas cells, hydrogen, deuterium, helium-3, and tritium as well as an empty cell. The gas target and empty target cells were fabricated to be identical. Along with these gas cells were a series of solid targets that were used for calibration and other functions during the experiment. The target ladder was contained inside of the scattering chamber, which was under constant vacuum during the experiment. A complete list of the targets used is given in Table 2.



Figure 15: Target Ladder Assembly. Gas targets are located at the top and solid targets at the bottom. A heat sink is attached to the gas targets for conductive cooling.

The  $^3\text{H}$  gas cell was filled by The Tritium Facility of Savannah River National Laboratory (SRNL) and shipped to Jefferson Lab for installation. The  $^1\text{H}$ ,  $^2\text{H}$ , and  $^3\text{He}$  gas targets were all filled on site at Jefferson Lab. The temperature and pressure conditions during the filling process are summarized in Table 3.

| Target Position | Target Material |
|-----------------|-----------------|
| 1               | Tritium         |
| 2               | Deuterium       |
| 3               | Hydrogen        |
| 4               | Helium-3        |
| 5               | Empty           |
| 6               | Dummy           |
| 7               | Optics          |
| 8               | Raster Target   |
| 9               | Carbon          |
| 10              | Titanium        |
| 11              | BeO             |

Table 2: MARATHON Target Assembly.

| Target    | Temperature<br>(K) | Pressure<br>(atm) |
|-----------|--------------------|-------------------|
| Tritium   | 293.80             | 13.81             |
| Deuterium | 296.25             | 33.02             |
| Helium-3  | 294.45             | 17.19             |
| Hydrogen  | 297.55             | 33.02             |

Table 3: The fill Temperature and Pressure of each gas target.

The gas targets were sealed after being filled, thus a single value for the density and target thickness were used for each target throughout the MARATHON experiment. The densities of  $^1\text{H}$  and  $^2\text{H}$  were determined from a NIST (National Institute of Standards and Technology) database [45] while the densities of  $^3\text{H}$  and  $^3\text{He}$  were calculated. Table 4 is a list summarizing these parameters for each gas target. A complete description of the target system can be found in Reference [46].

A target operator located in the counting house was able to access each target via a target GUI (Graphical User Interface) system, designed by the JLab Target group. This allowed the operator to quickly switch between targets with little downtime to

| Target    | $\rho$<br>(mg/cm <sup>3</sup> ) | Length<br>(cm) | Thickness<br>(mg/cm <sup>2</sup> ) |
|-----------|---------------------------------|----------------|------------------------------------|
| Tritium   | 3.404                           | 25             | 85.1 $\pm$ 0.8                     |
| Deuterium | 5.686                           | 25             | 142.2 $\pm$ 0.8                    |
| Helium-3  | 2.135                           | 25             | 53.4 $\pm$ 0.6                     |
| Hydrogen  | 2.832                           | 25             | 70.8 $\pm$ 0.4                     |

Table 4: Physical characteristics for each gas target. From Reference [46].

the experiment. This meant the experiment could continuously cycle through the target cells, eliminating systematic errors on the cross section ratios, which would arise from spending long periods on a single target.

Temperature stabilization of the gas targets was achieved via conductive cooling from a copper heat sink attached to the gas target cells. The JLab End Station Refrigerator (ESR) supplied 15 K liquid helium to Hall A, which was then heated to a temperature of 40 K, and subsequently used to cool the heat sink and stabilize the target temperature of the target system.

### 3.5 High Resolution Spectrometers

The Hall A facility is comprised of two High Resolution Spectrometers, the Left High Resolution Spectrometer (LHRS) and the Right High Resolution Spectrometer (RHRS). The layout of the spectrometers is shown in Figure 11. As their names imply, the HRSs were designed to provide high resolution for the momentum of the scattered particle, on the order of  $\pm 2 \times 10^{-4}$  [39]. Both HRS systems are nearly identical, with each arm consisting of two focusing quadrupole magnets, a bending dipole magnet, and a focusing quadrupole, such that the order is: QQDQ. All the magnets are superconducting, with the exception of the first quadrupoles (Q1) on

both LHRS and RHRS, which are resistive quadrupoles. This is due to the failure and replacement of the original superconducting Q1 magnets with resistive quadrupoles on both spectrometers. The current magnet configuration has been in place since Fall 2016. Detailed optics studies were performed by experiments proceeding the MARATHON experiment [47].

### 3.5.1 Detector packages

The MARATHON experiment utilized the standard detector configuration of the two High Resolution Spectrometers. Each spectrometer is equipped with a series of detectors designed to detect and characterize the scattered particles originating from the target cell. The detector packages in both spectrometers are essentially identical, each consisting of a pair of vertical drift chambers, two scintillator planes, a threshold gas Cherenkov counter, and a segmented lead-glass calorimeter. The two main differences between the detector packages are due to the differences in the total radiation length of the segmented calorimeters and also the radiator lengths of the Cherenkov counter. Figure 16 shows the layout of each detector in the detector stack. Signals from the detectors are sent to analog-to-digital-converter (ADC) and time-to-digital-converter (TDC) modules. These and other electronic modules are stationed above the detectors and shielded from radiation in the detector stack.

#### Vertical Drift Chambers

Both spectrometers contain a pair of vertical drift chambers (VDCs), which provide tracking information for the scattered particles. Each chamber consists of two planes of sense wires, U & V, which are oriented at  $90^\circ$  to each other [48]. Each plane is comprised of 368 sense wires, which are spaced 4.24 mm apart. The planes are filled

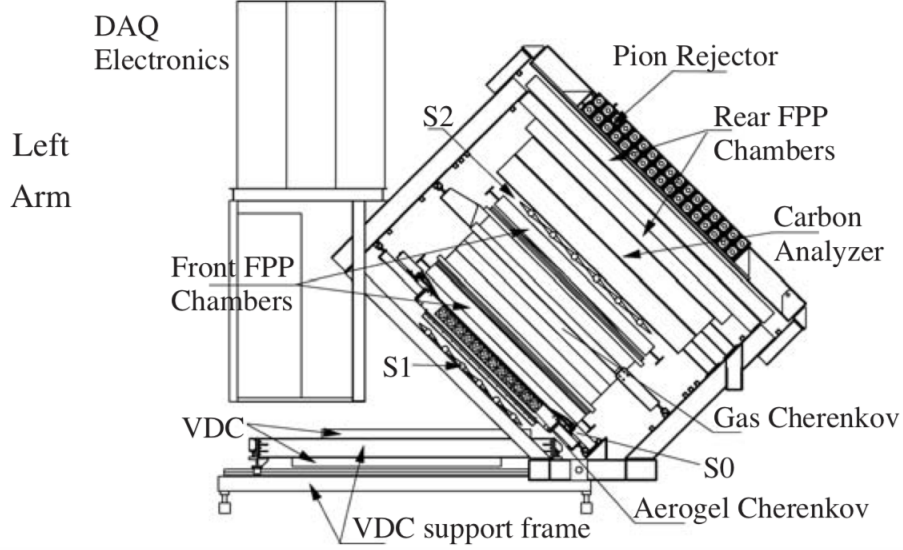


Figure 16: Schematic layout of the LHRS detector Stack showing the relative position of each detector.

with a gas mixture of argon (62%) and ethane (38%). An electric field is formed from mylar cathode planes which bound the top and bottom of each plane. The mylar planes are kept at an operating voltage of  $\sim -4$  kV. When a charged particle passes through a plane, it ionizes the gas mixture, producing ions and electrons along its path. The electrons will drift towards the closest sense wire, and produce additional ions and electrons. This avalanche of electrons will fire the sense wire, which is sent to a TDC module. The TDCs, which record the signals from the VDCs, are operated in Common Stop Mode. TDCs measure the time difference between 2 signals, the first which starts the clock and the second which will stop it. In Common Stop Mode, a common (reference) signal is used to stop all channels in the TDC, which during MARATHON was a trigger signal. Details of the trigger system and setup used during MARATHON will be discussed in Section 3.6.2. Thus the TDCs measure

the time difference between when the sense wire fired and when a trigger signal is formed and sent to the TDC. This time difference is referred to as the drift time. From the drift time, the drift distance can be determined, which when combined with information from both drift chambers, ultimately allows for the determination of the scattered particle's track.

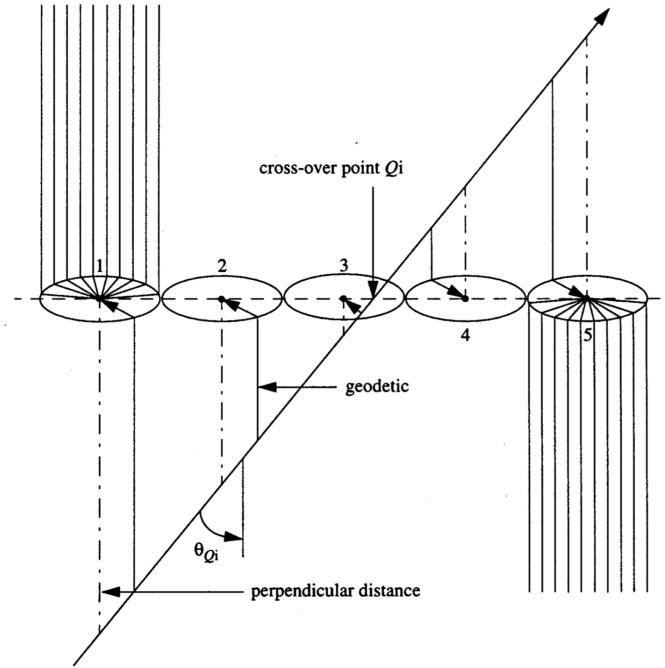


Figure 17: A track of a particle traversing a VDC plane. A typical track will fire 5 sense wires in a plane.

### Scintillator Planes

Two scintillator planes are located in both the LHRS and RHRS, referred to as S0 and S2m, and are separated by approximately 1.6 meters. Each one is made of a plastic material which has a photomultiplier tube (PMT) attached on both ends. When a charged particle passes through the scintillator, the luminescence which is

created can be detected by each PMT attached to the scintillator paddle. S0 is a single paddle, oriented vertically in front of the gas Cherenkov detector, with a 3-inch PMT on both the top and bottom. S2m is made up of 16 individual paddles, stacked behind the Cherenkov, with each paddles' long axis perpendicular to S0 and a PMT on each side. Due to their fast response time they are used for timing as well as are included in the formation of triggers.

#### Gas Cherenkov Detector

The gas Cherenkov detector is located between the S0 and S2m scintillator planes and is used for particle identification. The length of the gas Cherenkov detector is 120 cm and 130 cm in the LHRS and RHRS, respectively.

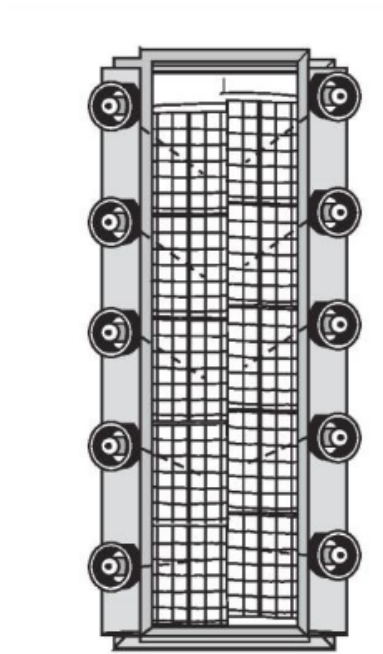


Figure 18: Arrangement of mirrors and PMTs in the gas Cherenkov detector. The nominal particle trajectory is into the page. The orientation is the same for both the LHRS and RHRS.

Both gas Cherenkov detectors contain 10 spherical mirrors and 10 PMTs, divided into two columns of five, with a PMT associated with each mirror. The spherical mirrors are used to focus Cherenkov radiation to the corresponding PMT. Figure 18 illustrates the design and orientation of the gas Cherenkov detector used in both High Resolution Spectrometers. The Cherenkov tank is filled with CO<sub>2</sub> gas, which has a refractive index of 1.0004. Cherenkov radiation is produced when a charged particle travels faster than the speed of light through a medium [49]. The angle between the Cherenkov radiation and charged particle is given by:

$$\cos(\theta) = \frac{1}{\beta n}, \quad (3.2)$$

where

$$\beta = \frac{v}{c}, \quad (3.3)$$

with  $v$  being the speed of the particle and  $n$  the refractive index of the medium. The gas Cherenkov operates as a threshold detector, with only particles having energy greater or equal to a threshold energy,  $E_{thresh}$ , producing Cherenkov radiation. The kinetic energy of the charged particle required to produce Cherenkov radiation is

$$E_{thresh} = mc^2 \left[ \sqrt{\frac{n^2}{n^2 - 1}} - 1 \right]. \quad (3.4)$$

Using the refractive index of CO<sub>2</sub>, one can determine  $E_{thresh}$  for both electrons and pions, with pions being the dominant background of the experiment. The energy threshold is approximately 0.017 GeV/ $c$  and 4.62 GeV/ $c$  for electrons and pions, respectively, providing a clear way to discriminate between electrons and background particles.

## Electromagnetic Calorimeter

The final detector in both the LHRS and RHRS is the segmented electromagnetic calorimeter, which provides an additional method for performing particle identification. The calorimeter in the LHRS is comprised of two separate layers, often referred to as Pion rejector 1 and Pion rejector 2 respectively. Both layers contain two columns of lead-glass blocks. Each column has 17 lead-glass blocks, which are oriented such that their long axis is perpendicular to the trajectory of the particle passing through the detector stack [39]. The top left image of Figure 19 shows a side view of the LHRS calorimeter detector, with the particle trajectory from the bottom to the top of the page.

The calorimeter in the RHRS is also comprised of two layers, a pre-shower and shower layer, but has a total radiation length larger than that of the LHRS. The pre-shower is oriented in a similar fashion as both layers in the LHRS, made up of two columns, but with 24 lead-glass blocks in each column. The long axis of the lead-glass blocks are perpendicular to the particle's trajectory. Unlike the pre-shower, the shower has 5 columns of lead-glass blocks, with the long axis of the lead-glass blocks parallel to the particle trajectory. The shower has a total of 75 lead-glass blocks [39]. The bottom left image of Figure 19 shows a side view of the RHRS calorimeter detector, with the particle trajectory from the bottom to the top of the page.

The electromagnetic calorimeter in both the LHRS and RHRS can differentiate between electrons and hadronic background based on the amount of energy each deposits. The radiation length of the calorimeter is such that the electron will deposit approximately all of its energy into the calorimeter while hadrons need to travel a much longer distance in order to deposit their full energy. Hadrons will thus deposit

less energy per unit length, allowing for the discrimination between electron events and hadronic background.

As an electron travels through the lead-glass blocks of the calorimeter, it loses energy via bremsstrahlung radiation [50]. Photons produced can create secondary particles via pair production in the field of the block's nucleus ( $\gamma \rightarrow e^+ + e^-$ ). This shower process will continue until the particle's energy falls below some critical energy ( $E_c$ ) at which time other mechanisms become the dominant form of energy loss. The cascade produced from bremsstrahlung photons decaying into an  $e^+ e^-$  pair is highly collimated in the direction of travel of the incident electron.

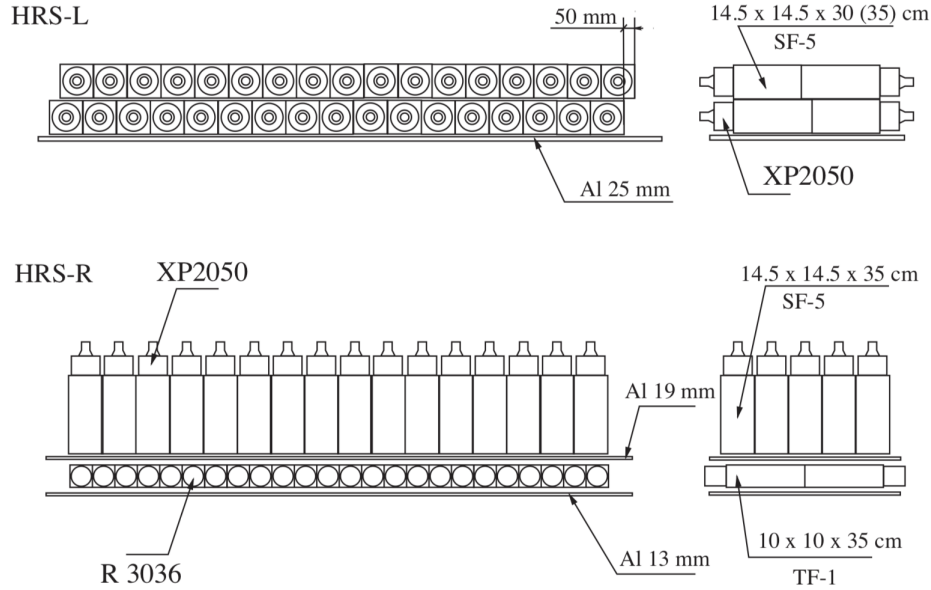


Figure 19: The arrangement of shower detectors in HRS-L (top left, top right) and HRS-R (bottom left, bottom right) from 2 different orientations. The left images of both the top and bottom show a side view of both calorimeters. The right images of both the top and bottom show a top view of the calorimeters. For both orientations, particles traverse from the bottom to the top of the page.

### 3.6 Data Acquisition

Data collection and storage during an experiment occurs through a system designed by the Data Acquisition (DAQ) group at Jefferson Lab called CODA (CEBAF Online Data Acquisition) [51], which works at both the hardware and software level. Front-end crates, such as Fastbus or VME crates, contain ADC and TDC modules. Each front -end crate has a Readout Controller (ROC) and each ROC is connected to the central Trigger Supervisor (TS), which acts to instruct the various modules when to record data as well as ensuring event synchronization between the different ROCs. When a trigger is formed by meeting all the requirements at the hardware level, it is sent to the TS, which checks the state of each ROC to determine if it is able to process a new trigger. The DAQ can be run in two different modes, unbuffered (standard) or buffered mode. In the unbuffered mode, the TS will not accept any new triggers until each of the ROCs has finished recording and writing the event to tape. During MARATHON, the DAQ was run in the second mode, buffered mode. The buffered mode allows for new triggers to be accepted by the TS even while the ROCs are still reading out the previous event. This is accomplished by storing the unprocessed events in a buffer and reading them out sequentially. The advantage of the buffered mode is that it can reduce the dead time of the DAQ. The dead time of the DAQ corresponds to the amount of time in which the DAQ cannot receive new triggers, resulting in missing valid events that should otherwise be recorded.

The dead time associated with each ROC can be separated into two parts, the front-end busy and readout busy. The front-end busy, which is on the order of 10  $\mu$ s, occurs for each new trigger that is accepted, regardless of the operating mode. Second, is the readout busy, which is the time it takes to read the data and write

it to tape. For Fastbus ADC and TDC modules, in the current configuration, the readout busy is approximately  $110\ \mu\text{s}$ . Thus, in the standard unbuffered mode, the time to acquire and readout out a single event is approximately  $120\ \mu\text{s}$ . Buffering decouples the front-end busy from the readout busy as long as the buffer is not full, meaning that the TS can accept new triggers even while the ROCs are reading out an event. Once the buffer is full, the TS will not accept a new trigger until the ROC has readout all events in their buffer. Only when the buffer is full is the dead time due to the readout busy associated with the reading and writing of the event tape.

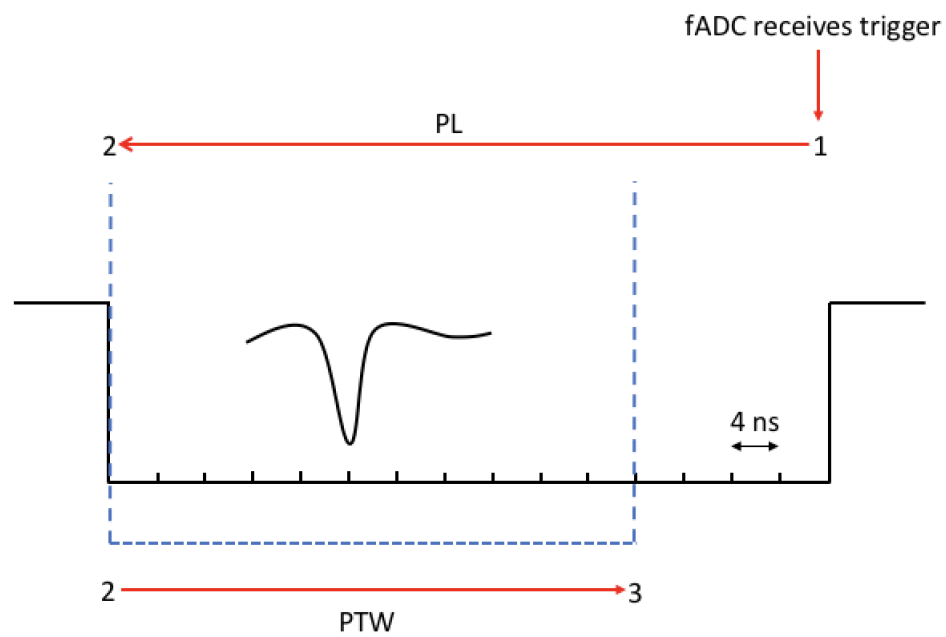
### 3.6.1 Flash ADCs

An upgrade to the existing Data Acquisition system used in Hall A made prior to MARATHON was the implementation of JLab built and designed Flash ADCs (fADCs) to record signals from selected detectors in both the LHRS and RHRS. Flash ADCs are advantageous due to their ability to be used in high rate environments, do not require external delay cables, and have the capability to provide complementary TDC information [52]. Both spectrometers were outfitted with 4 Flash ADC boards apiece. Each Flash ADC board is equipped with 16 channel inputs. The number of inputs limited the number of detectors that could be sent to the Flash ADCs. For this reason, the S0, S2m, and Cherenkov detectors were read out by the Flash ADCs while the calorimeters continued to be readout by Fastbus ADCs. MARATHON was the first experiment in Hall A to use Flash ADCs for full analysis purposes, so as a redundancy, signals from S0, S2m, and the Cherenkov detectors were split and sent to both the Flash ADCs as well as the Fastbus ADCs. Each channel on the Flash ADC board has a series of programmable functions, which it uses to record the appropriate information from each triggered event. These functions include the latency (PL),

trigger window width [ns] (PTW), the number of samples before (NSB) to record, the number of samples after (NSA) to record, and the number of samples (NPED) to use to determine the pedestal. The latency is the amount of time it will look back in its buffer to find the corresponding event. The trigger window width is the size of the window it will look in to for the signal. The general operating principle of the Flash ADC is visualized in Figure 20. First, the Trigger Supervisor (TS) sends a Level 1 Accept (L1A) signal to the Flash ADC. Once it finds the signal, it counts the number of samples before and after to include in what will be the recorded signal. Finally, it performs event-by-event pedestal subtraction for each channel using the predefined NPED before each event which it subtracts from the integrated signal.

### 3.6.2 Triggers

The trigger system used for the MARATHON experiment was formed using the electronic hardware in the front-end to take signals from the detectors to create logic signals. Figures 21 and 22 demonstrate how the coincidences for each detector are created. A trigger is formed when there is a coincidence between certain detectors. For instance, the MARATHON experiment utilized three separate triggers, S0 & S2m, S0 & S2m & GC, and  $(S0 \parallel S2m) \& GC$ , where S0 and S2m are the forward and rear scintillator detectors, and GC is the gas Cherenkov detector. The production trigger for MARATHON was S0 & S2m & GC, requiring a coincidence between the two scintillator paddles as well as the gas Cherenkov. Trigger signals are sent to the Trigger Supervisor. If all ROCs are not busy, the Trigger Supervisor will issue a Level One Accept to each ROC, to record the current event.



- Look back window begins at 1 and ends at 2
- Integration window opens at 2 and ends at 3

Figure 20: Visual description of parameters of the fADC for an event. Once receiving a trigger, the fADC uses predefined values to find and integrate the appropriate signal.

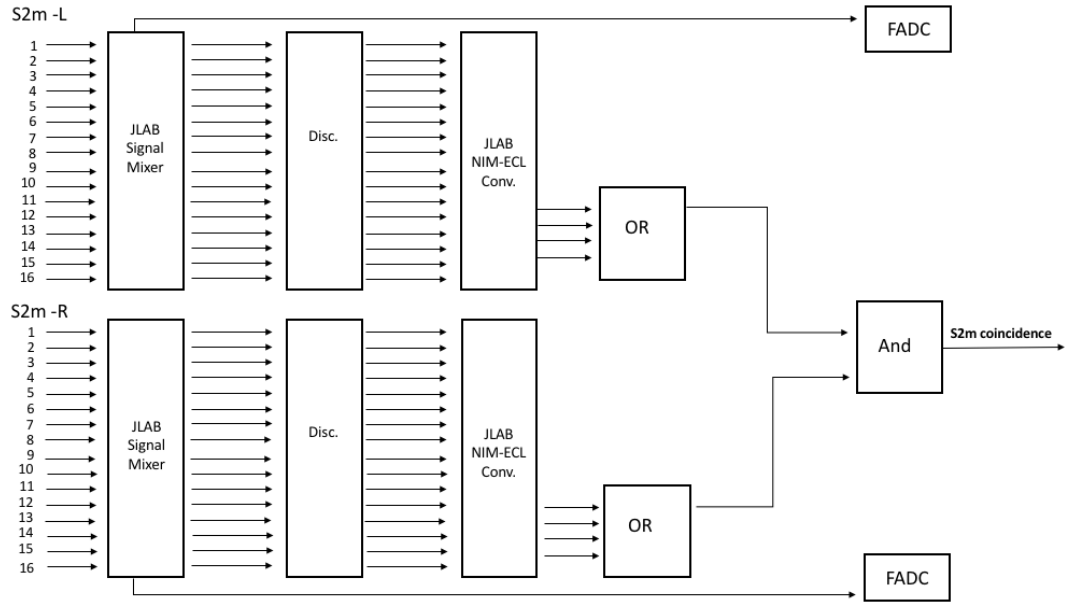


Figure 21: S2m counter coincidence signal logic.

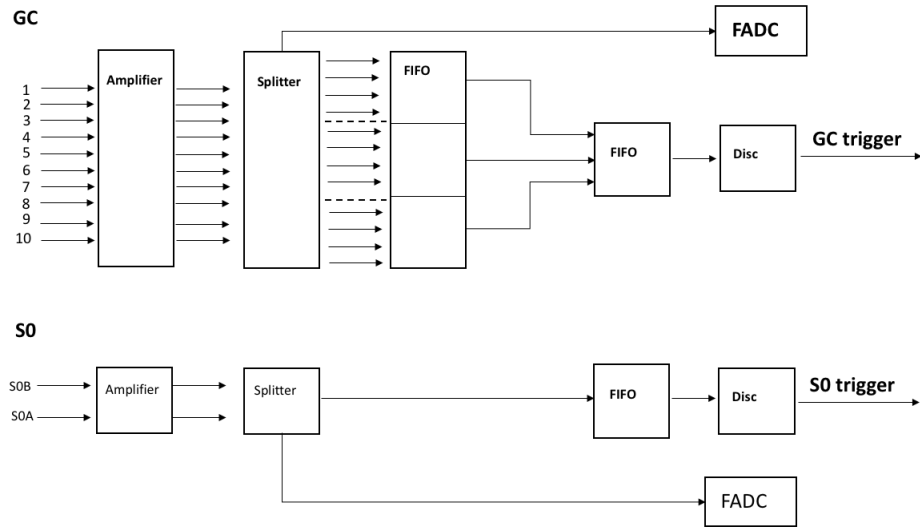


Figure 22: gas Cherenkov detector coincidence signal logic.

## CHAPTER 4

### Calibrations

#### 4.1 Calibrations

As the beam enters Hall A, there are individual components along the beam line used to ensure the beam quality. Likewise, after interacting with the target, a series of detectors are used to measure the scattered electrons. Both the beamline components and the detectors need to be properly calibrated in order to ensure the reported results are accurate. Both during and after the MARATHON experiment, calibrations were performed on each component and the following section will discuss several of the key calibrations along with the specific methods applied.

##### 4.1.1 Unser and Beam Current Monitors

To determine the total charge that was delivered to the Hall and seen by the target, Hall A utilizes two separate monitoring systems in tandem. This combined monitoring system provides an accurate and precise measurement of the beam current, from which the total charge can be determined. As mentioned in Section 3.3.2, this monitoring system is comprised of a Parametric Current Transformer (Unser monitor) and the Beam Current Monitors (BCMs). Both the Unser and the BCM must be calibrated several times during an experiment, to ensure that the resulting measurement is accurate. The basic calibration procedure follows a two-step process, in which the Unser is first calibrated, followed by the BCMs. The necessity of both systems is due to a drift in the offset of the Unser which precludes it for direct use

during the experiment. The gain of the Unser, on the other hand, is extremely stable. A calibrated Unser can then be used as an absolute reference to which the BCMs are calibrated to. The Unser is calibrated during periods when the beam is not being delivered to the Hall. A wire is passed through the cavity with a predetermined set of currents at 90 seconds intervals followed by 90 seconds without current. To accurately calibrate the Unser, a large dynamic range of currents is chosen, 2.5 - 100  $\mu\text{A}$ . A linear fit of wire current versus the Unser response (frequency), determines an overall gain factor for the Unser. The Unser was calibrated 4 separate times during the MARATHON experiment. This calibration was performed before each BCM calibration procedure, to benchmark the Unser's stability. The Unser exhibited high stability. Table 5 lists the gain factor found from the calibration of the Unser during MARATHON.

| Date  | Gain      |
|-------|-----------|
| 03/05 | 0.0002526 |
| 03/28 | 0.0002524 |
| 04/03 | 0.0002529 |
| 04/06 | 0.0002527 |

Table 5: Unser monitor gain values from each calibration. The gain of the Unser was quite stable during the MARATHON experiment.

With a calibrated Unser, the BCMs can then be accurately calibrated. The calibration procedure for the BCMs is similar to that of the Unser, the only difference being the use of the electron beam. Similarly, MCC delivers beam at a set of predetermined currents for 90 second intervals followed by 90 seconds without beam. The range of currents used during the BCM calibration was 2.5 - 22.5  $\mu\text{A}$ , which covers the current range used in the MARATHON experiment. Figures 23 and 24 show

the response of the BCM and Unser respectively during the BCM calibration. Also shown in both Figures 23 and 24 are the cuts made to select stable regions which were used in the calibration. These cuts must be made to exclude regions where the current may fluctuate, which often occurs at the start and end of each interval.

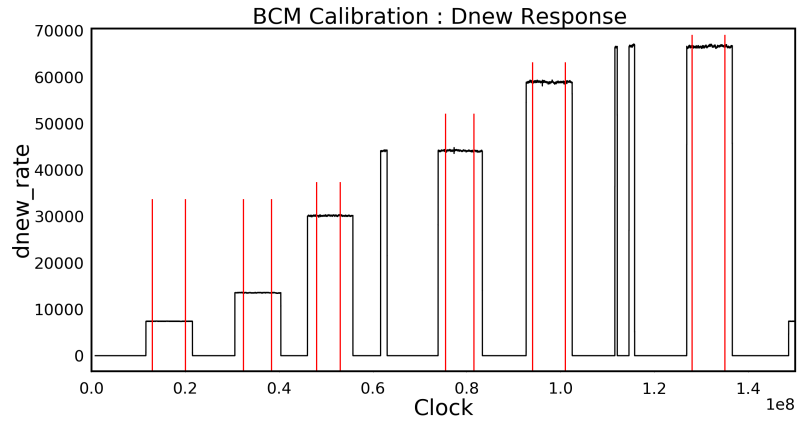


Figure 23: The response of the digital receiver (dnew) during one of the BCM calibration. The lines in red are the cuts applied to determine calibration coefficients.

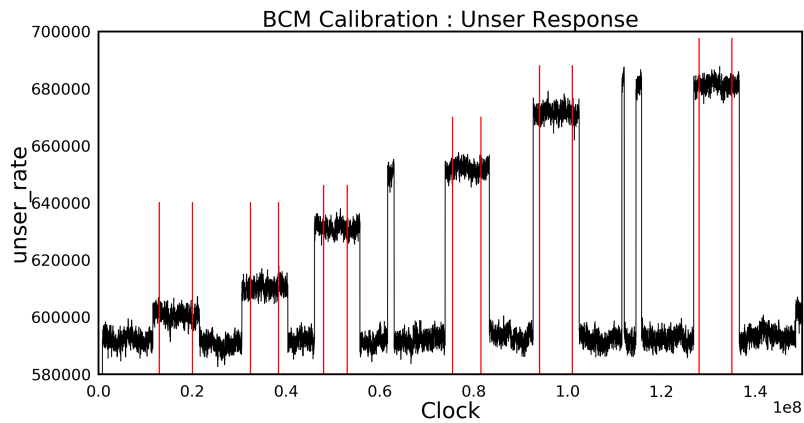


Figure 24: Unser response during BCM calibration. The lines in red are the cuts applied to determine calibration coefficients.

A linear fit to the calibrated Unser current and BCM frequency is performed from which the BCM gain and offset coefficients are determined. The linear fits can be seen in Figures 25 and 26, which are from two separate calibrations.

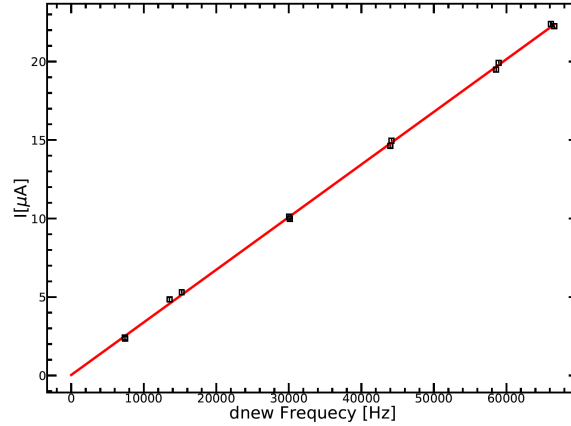


Figure 25: Current versus dneq frequency for the first BCM Calibration. The slope and intercept from the linear fit correspond to the gain and offset of the dneq receiver.

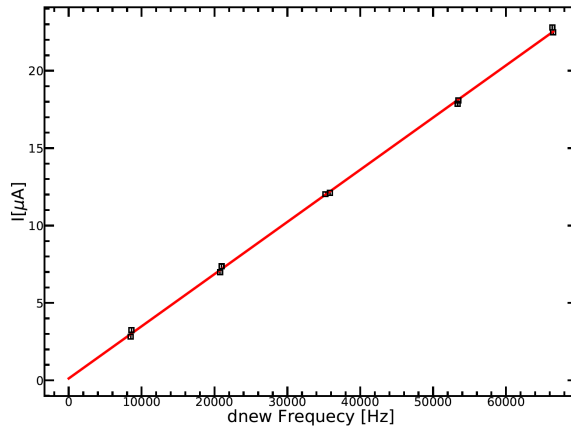


Figure 26: Current versus dneq frequency for the second BCM Calibration. The slope and intercept from the linear fit correspond to the gain and offset of the dneq receiver.

A total of three BCM calibrations were performed during MARATHON and the results of the dnew digital receiver can be seen in Figure 27, which shows the gain and offset of the dnew receiver during the experiment. The figure indicates the BCMs were highly stable throughout the entirety of the experiments with the gain drifting by less than 0.03%.

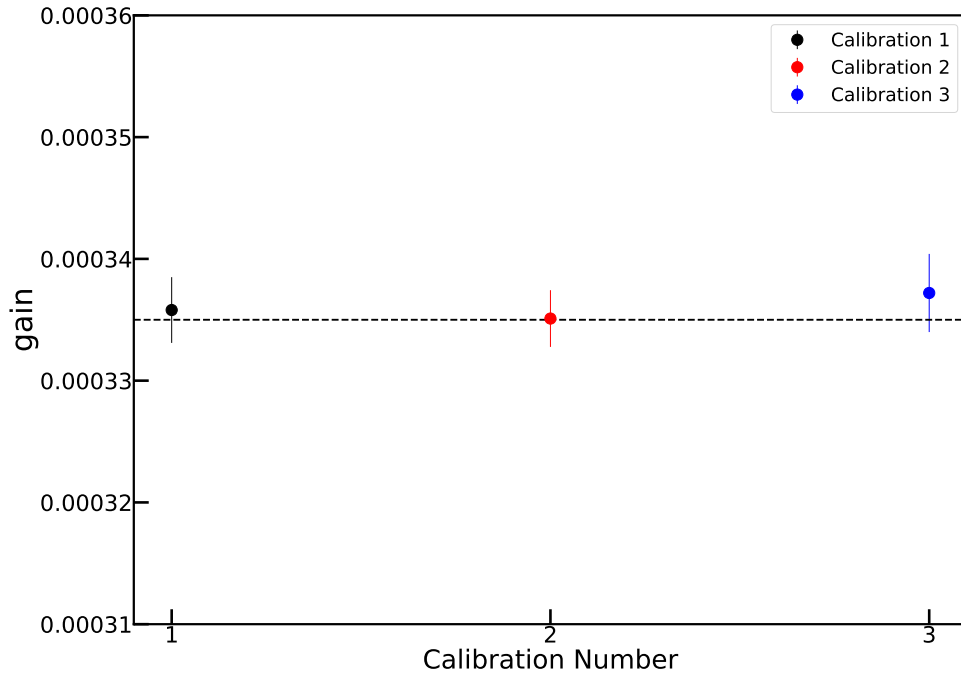


Figure 27: The gain stability of the digital dnew receiver. The vertical axis is the gain factor from each calibration, and the horizontal axis indicates the calibration number. The dashed line is meant to assist in comparison among coefficients.

#### 4.1.2 Beam Position Monitor

Dedicated Beam Position Monitor calibrations were performed at the beginning of the experiment. The calibration procedure involves moving the beam to different

positions at which point the Machine Control Center (MCC) passes the harp through the beam profile to determine the beam positions. An image of the harp is seen in Figure 14.

This process is repeated at several different positions. Afterwards, the BPMs can then be calibrated to the absolute position reported from harp. Figure 28 shows the bull's-eye scan calibration performed during MARATHON [53]. The grey points correspond to the absolute beam position as reported by the harp scan for each of the five measurements. The colored points are the reported positions from the calibrated BPMs for each measurement. After calibration, the BPMs show a great agreement with the harp positions. Further details about the BPM calibration can be found in [53].

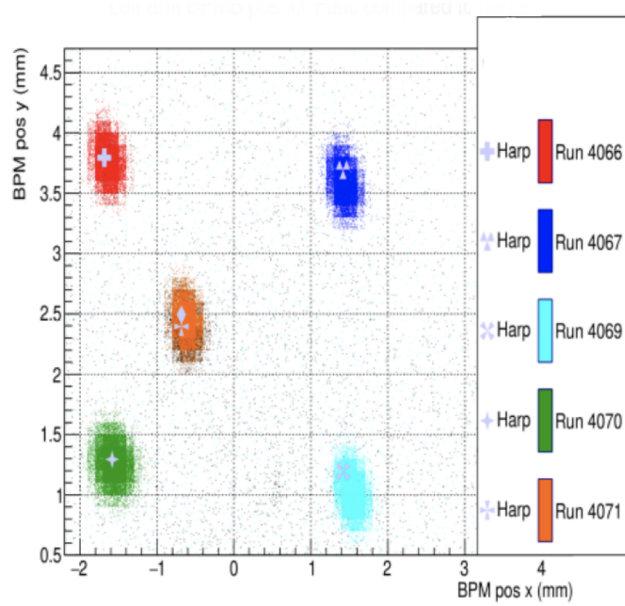


Figure 28: Bulls eye scan performed for BPM calibration [53]. The grey points are the beam positions reported from the harps while the colored points are the same beam positions from the calibrated BPMs.

#### 4.1.3 Raster

The raster ensures a uniform spread of the beam profile before interacting with the target to decrease localized heating of the target as well as to avoid damage to the target cell. Also, an offline calibration of the raster is performed to improve the target  $z$  reconstruction. Due to a time lag in the BPM signals, they cannot be used for an event-by-event beam position determination. Instead, the raster is capable of providing position information using the raster current, which is sent to ADCs in each spectrometer. Translating the ADC values of the raster current to a position requires a raster calibration. For MARATHON, the mapping was achieved by fitting the carbon hole of the target assembly, which has a well-defined 2 mm diameter [54]. This fit could then be used to make the conversion of the raster ADC to a position.

#### 4.1.4 Vertical Drift Chambers

As discussed in Section 3.5.1, the two Vertical Drift Chambers (VDCs) provide tracking information for the scattered electron at the focal plane of each spectrometer. The VDCs measure the drift time for each wire and an algorithm utilizes the drift time to determine the drift distance for each wire. A typical event will fire multiple ( $\sim 5$ ) wires in each plane. A linear fit to these drift distances is performed which determines the cross over point and angle for each of the two planes [48]. Combining each plane from both chambers, one can determine the trajectory of each event. Due to possible time differences introduced along the path of the signal, such as the differences in path length as well as additional offsets from various electronics, a reference time for each wire is determined. The reference time,  $t_0$ , can be determined for each wire by looking at each wire's raw TDC time. The calibration procedure looks for the maximum slope

in the region of the leading edge of the TDC spectrum. The procedure calculates the slope (derivative) between adjacent bins. After determining the maximum slope, a line from the leading edge bin is drawn to cross the time (TDC) axis. Figures 29 and 30 show the calibration for a group of wires in the u1 and u2 planes on the LHRS. This time value corresponds to the  $t_0$  for this wire.

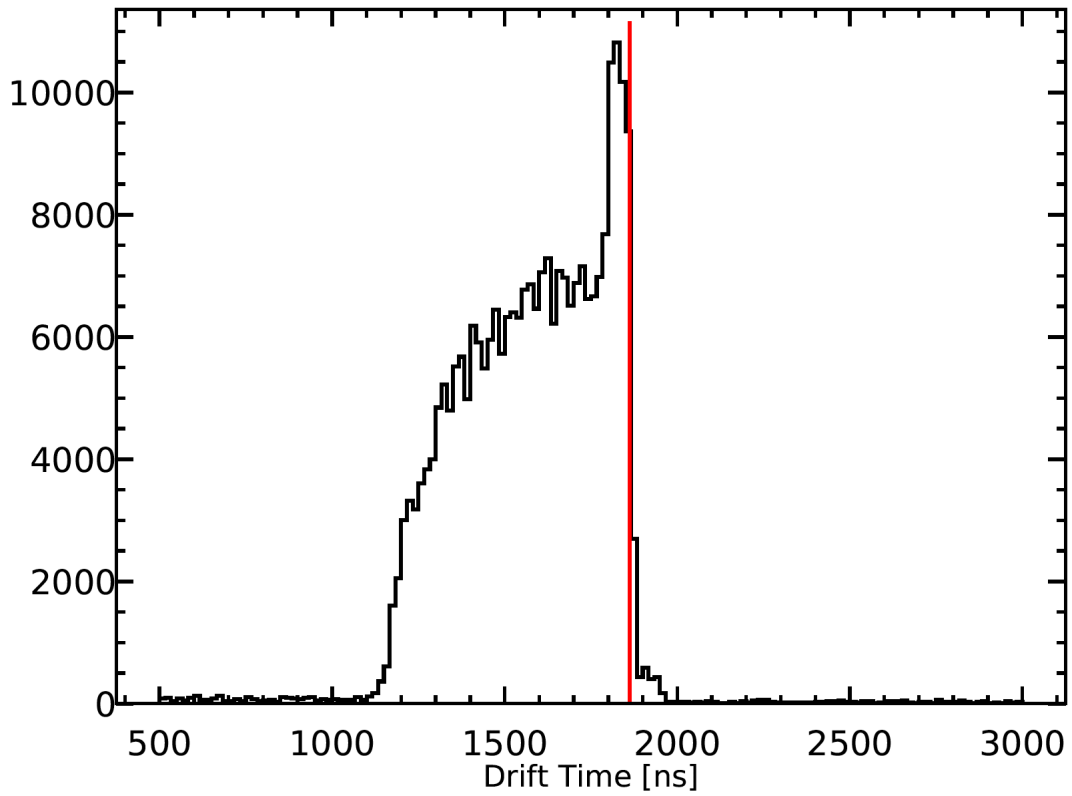


Figure 29: The  $t_0$  reference time calibration for wires 1-15 in wire plane u1 of the LHRS (see text).

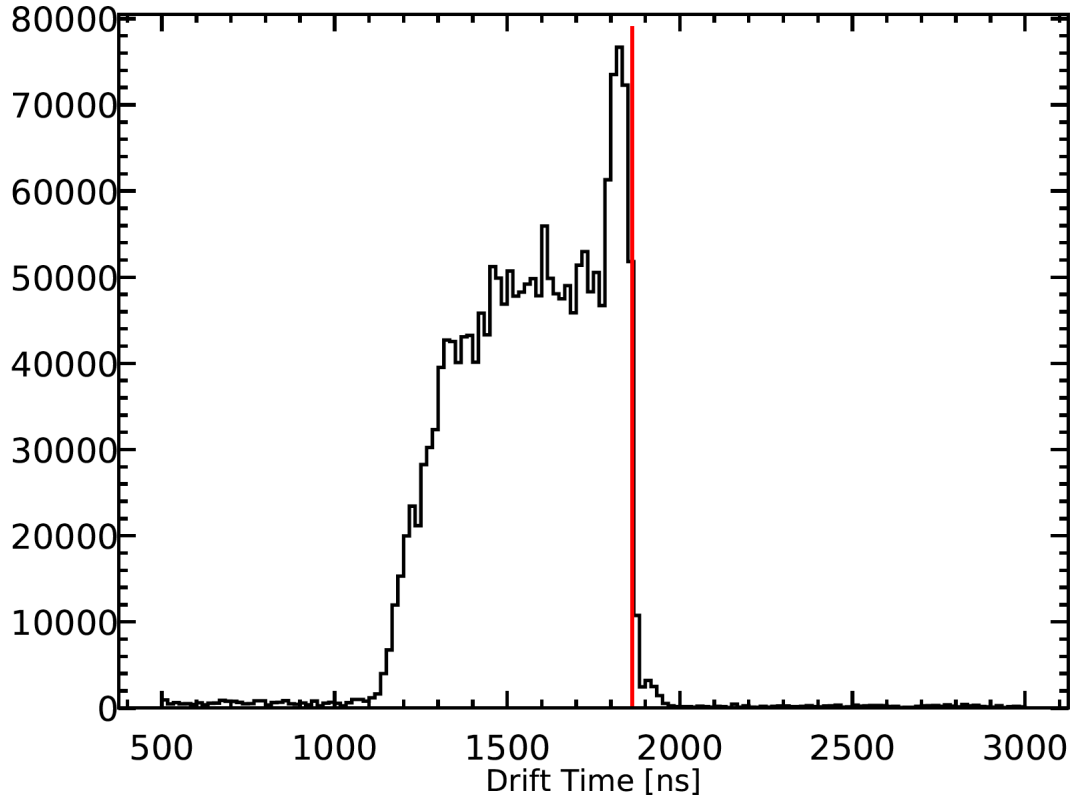


Figure 30: The  $t_0$  reference time calibration for wires 1-15 in wire plane u2 of the LHRS (see text).

#### 4.1.5 Gas Cherenkov Detectors

Each PMT in the Cherenkov detector collects light produced from Cherenkov radiation due to incident particles traversing the detector. The sum of all Cherenkov PMTs for an event is used for particle identification, so it is important that all PMTs produce a uniform response. Since each PMT has a different high voltage applied to provide the best response, the calibration procedure determines unique gain factors for each PMT, which produces a uniform response of the detector. The signals from each PMT are readout by Flash ADCs and the number of ADC channels is proportional to

the size of the signal. A reference value is needed to which all PMTs can be matched to. Traditionally, the reference used is the position of the single photoelectron peak (SPE). The SPE spectrum is the response of the PMT when a photon strikes the PMT's cathode and only a single electron is emitted [55]. For the MARATHON experiment, the mean of the single photoelectron peak spectrum was set to 300 ADC channels. The calibration procedure determines the position of the SPE for each PMT and with that determines the gain factor of each PMT. The gain factor for the  $i_{th}$  PMT can be expressed as:

$$C_i = \frac{300}{M_i}, \quad (4.1)$$

where  $C_i$  is the gain coefficient and  $M_i$  is the uncalibrated mean value of the SPE spectrum for the  $i^{th}$  PMT. The gain coefficient,  $C_i$ , shifts each Cherenkov PMT spectrum such that the mean of the SPE spectrum is at ADC channel 300. With a common reference to which all PMTs have been adjusted to, a sum of Cherenkov ADC signals for each event can be made to form the Cherenkov sum. The Cherenkov sum before and after calibration can be seen in Figure 31.

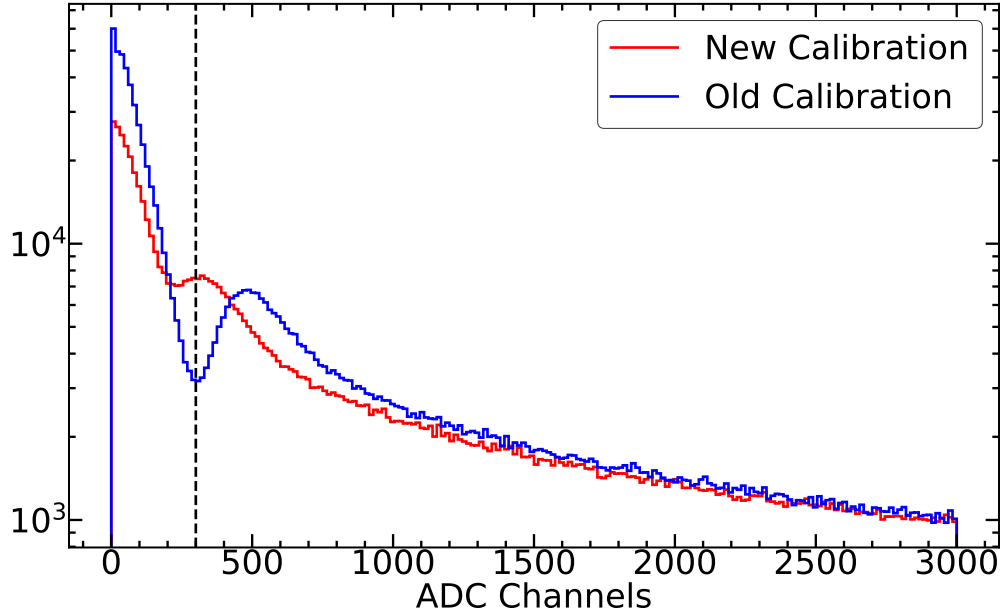


Figure 31: Uncalibrated versus Calibrated RHRs Cherenkov ADC sum. The dashed line is indicated ADC channel 300 (see text).

#### 4.1.6 Electromagnetic Calorimeter

The Calorimeters in both spectrometers are designed to measure the energy deposited from scattered particles entering the detector stack. Due to the differences in the shower process of hadrons and leptons, they can be used to discriminate between hadronic background and electrons. Tracking information from the VDCs can be used to project the track to each segment of the calorimeter. For each event, the analyzer can determine a cluster of blocks involved in the shower process. The sum of each cluster represents the amount of energy deposited, in ADC units. The conversion between ADC channels to energy requires a calibration coefficient that is unique for each PMT. The calibration procedure aims to determine this coefficient for

each block. The high voltage applied to each PMT can change over time, in order to maximize the response of each PMT. Thus, it is natural for the coefficients to change and require periodic updating. The calorimeter calibration for both HRSs involves selecting high statistics runs which will provide full coverage of the entire calorimeter geometry. A minimization routine is performed which minimizes the function [56]:

$$\chi^2 = \sum_{i=1}^N \left[ \sum_{j \in M_{ps}^j} C_j \cdot A_j^i + \sum_{k \in M_{sh}^k} C_k \cdot A_k^i - p_i \right]^2, \quad (4.2)$$

where the sum is over all events  $N$ ,  $j$  and  $k$  are the number of preshower and shower blocks in the reconstructed cluster,  $M_{ps}$  and  $M_{sh}$  respectively, and  $C_i$  and  $C_k$  are the calibration coefficients that are determined from the minimization routine which translate the ADC value to energy. As mentioned previously, an electron will deposit nearly all of its energy in the calorimeter. A reasonable check of the calibration is to plot the  $E/p$  spectrum, where  $E$  is the total amount of energy deposited in the calorimeter, given by

$$E = E_{layer1} + E_{layer2}, \quad (4.3)$$

and  $p$  is the reconstructed momentum from the particle track.  $E/p$  should follow a Gaussian distribution, with a mean value of 1. Figures 32 and 33 are plots of  $E/p$  for the LHRS and RHRS calorimeters respectively. Both show the spectrum before and after the calibration with a noticeable improvement for each. The resolution of the RHRS calorimeter is better than that of the LHRS due to the difference in total radiation lengths of each.

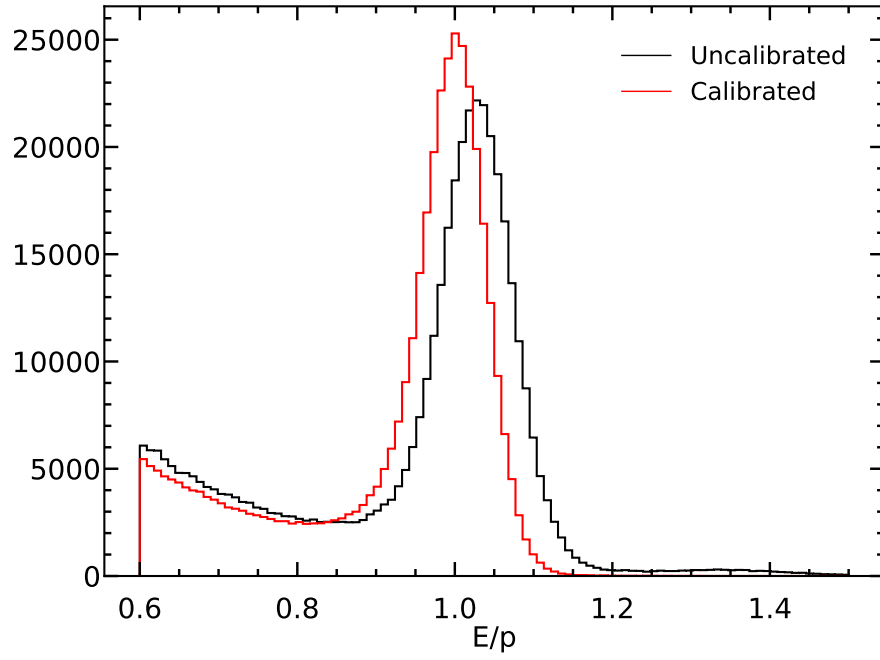


Figure 32: Comparison of  $E/p$  spectrum for LHRS before and after calibration.

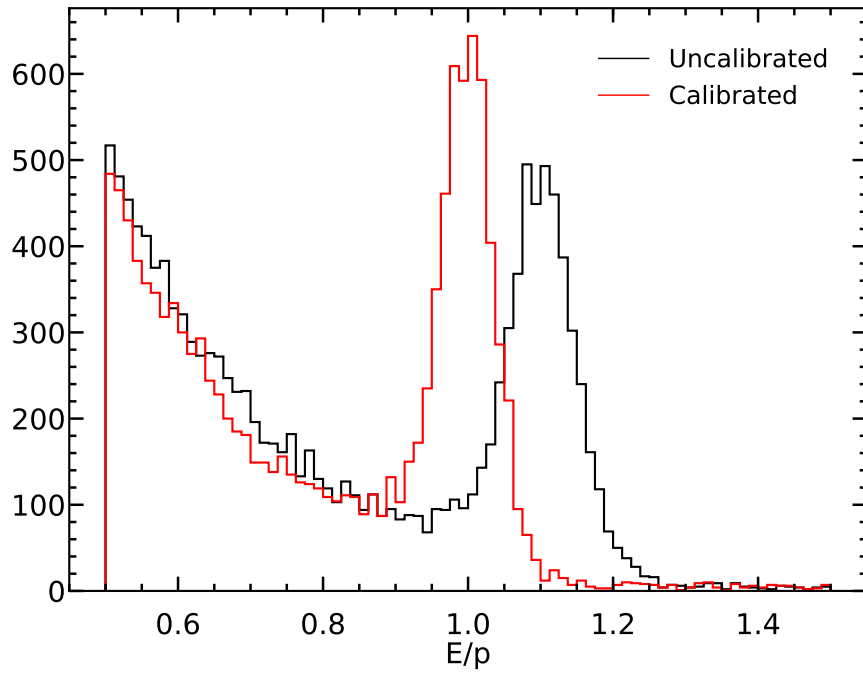


Figure 33: Comparison of  $E/p$  spectrum for RHRS before and after calibration.

## CHAPTER 5

### Data Analysis

#### 5.1 Cross Section Ratios

This chapter will explore the steps and procedures necessary for calculating experimental cross section ratios. Nuclear physics experiments such as MARATHON are fundamentally counting experiments, where the goal is to measure a certain reaction given the predetermined kinematics. For inclusive electron scattering, this is achieved by measuring the number of scattered electrons which satisfy a set of criteria. While in principle this is straightforward, care must be taken when selecting the electrons to account for background and contamination sources which would otherwise appear as good events. To extract absolute cross sections, one must either determine the acceptance function of each High Resolution Spectrometer or, simulate the yield using a Monte Carlo event generator and cross section model [47], both of which will introduce additional uncertainties to the final cross section ratio. These additional uncertainties, which can be large, may preclude any meaningful conclusions from being made from the final results. For this reason, the proposed plan of the MARATHON experiment was to instead measure cross section ratios. This is due to the fact that many of the systematic uncertainties cancel in the ratios. By measuring cross section ratios, it is possible to achieve the physics goals of the experiment while reducing

the overall systematic uncertainties. The cross section ratio effectively reduces to the ratio of the yields of the individual targets. The total yield,  $Y$ , is expressed by:

$$Y = \frac{N_e C}{LT Q \rho T_{cor}}, \quad (5.1)$$

where  $N_e$  is the number of scattered electrons,  $C$  represents corrections which are applied multiplicatively and includes charge symmetric and endcap background,  $LT$  is the electronic live time of the DAQ,  $Q$  is the total beam charge,  $\rho$  is the target density, and  $T_{cor}$  is a target density correction. In the following sections each of the components in Equation 5.1 will be discussed.

## 5.2 Event Selection

In order to accurately count electron events, sensible cuts must be applied on a series of quantities which act to ensure the quality of the data. These cuts should be restrictive enough to effectively limit the amount of background in the measured events while loose enough to record almost all electron events of interest. The quantities on which cuts were applied include the number of tracks in the VDCs, the Cherenkov ADC sum,  $E/p$ , the reaction vertex position ( $z_{tg}$ ), spectrometer acceptance, and  $W^2$ . Among the cuts listed, several were applied uniformly to both spectrometers and all kinematics. These include a T2 trigger signal (S0 & S2m & GC), a single track in the VDCs,  $W^2 > 3.24 \text{ (GeV}/c^2)^2$ , and  $E/p > 0.70$ . Table 6 lists those cuts which were either kinematic or spectrometer specific. Table 6 does not list cuts applied to either  $\phi_{tg}$  or  $\theta_{tg}$  due to the geometrical nature of the cuts, which will be discussed more in the following section. Also, definitions for  $z_{tg}$ ,  $\phi_{tg}$ , and  $\theta_{tg}$  are provided in Appendix A.

| Kin | Cherenkov Sum ADC | Reaction Vertex (m)      | $\delta_p$                  |
|-----|-------------------|--------------------------|-----------------------------|
| 0   | 1500              | $-0.08 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 1   | 1500              | $-0.08 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 2   | 1500              | $-0.08 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 3   | 1500              | $-0.08 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 4   | 1500              | $-0.08 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 5   | 1500              | $-0.09 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 7   | 1500              | $-0.09 < z_{tg} < 0.10$  | $-0.035 < \delta_p < 0.045$ |
| 9   | 1500              | $-0.095 < z_{tg} < 0.10$ | $-0.035 < \delta_p < 0.045$ |
| 11  | 1500              | $-0.095 < z_{tg} < 0.10$ | $-0.035 < \delta_p < 0.045$ |
| 13  | 1500              | $-0.10 < z_{tg} < 0.105$ | $-0.035 < \delta_p < 0.045$ |
| 15  | 1500              | $-0.10 < z_{tg} < 0.105$ | $-0.035 < \delta_p < 0.045$ |
| 16  | 2000              | $-0.15 < z_{tg} < 0.11$  | $-0.030 < \delta_p < 0.045$ |

Table 6: Event Cuts used for each kinematic. Note that kinematics 0-15 correspond to the LHRS while kinematic 16 corresponds to the RHRS.

### 5.2.1 Acceptance and Momentum Cuts

The reconstructed acceptance quantities of both spectrometers are determined through an optimization of the extraction of the optics matrix elements. The relevant quantities are  $\delta_p$ ,  $\theta_{tg}$ ,  $\phi_{tg}$ , and  $y_{tg}$  where  $\theta_{tg}$ ,  $\phi_{tg}$ , and  $y_{tg}$  are defined in terms of the target coordinate system as illustrated in Figure 61 in Appendix A. The quantities  $\theta_{tg}$  and  $\phi_{tg}$  are the in-plane and out-of plane angles from the central ray of the spectrometer,  $y_{tg}$  is the transverse position at the target, and  $\delta_p$  is the fractional momentum deviation from the central momentum, given by:

$$\delta_p = \frac{\Delta p}{p_0}, \quad (5.2)$$

where  $p$  is the momentum of the scattered particle and  $p_0$  is the central momentum setting of the spectrometer.

Cuts must be placed in these variables to exclude regions where the optics matrix elements are not well constrained. Explicit cuts were made on  $\delta$ ,  $\theta_{tg}$ ,  $\phi_{tg}$ , which

were similar to standard acceptance cuts traditionally used in Hall A. A number of different but reasonable combinations of the  $\delta$ ,  $\theta_{tg}$ ,  $\phi_{tg}$  cuts were applied to determine the overall reliability of the final cuts chosen. A maximum change of 0.2% was observed in the cross section ratio by altering the acceptance cuts. An uncertainty of 0.2% from the acceptance cuts is applied to each cross section ratio. Figures 34 and 35 illustrate the angular acceptance cuts used for the LHRs and RHRs systems, respectively.

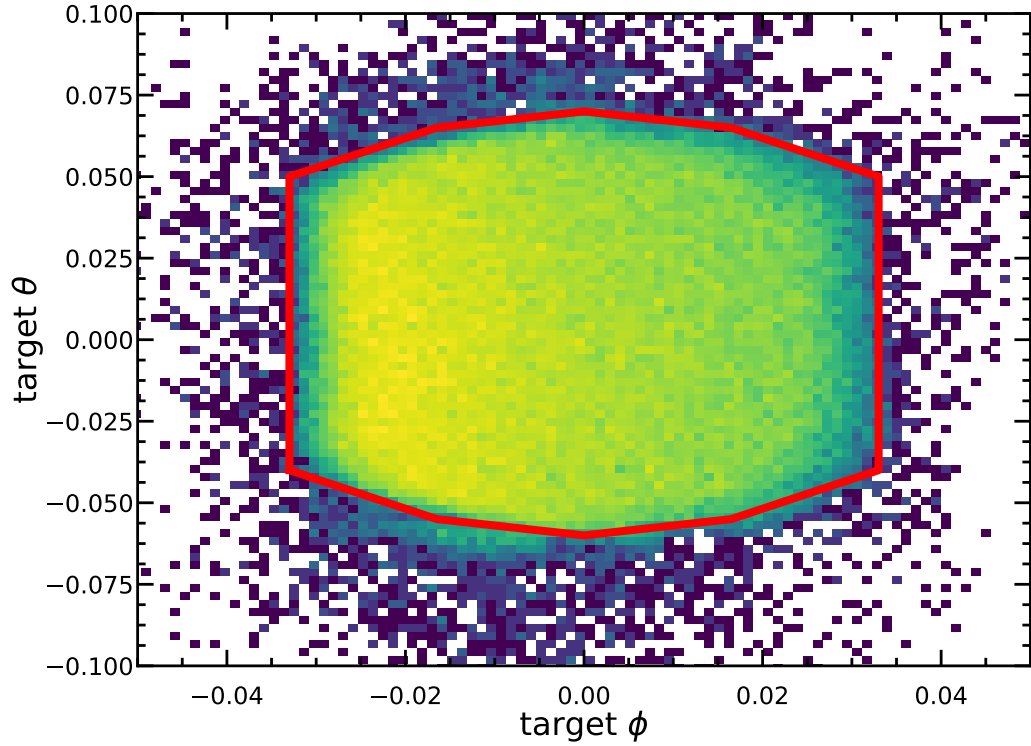


Figure 34: LHRs angular acceptance for the  $^3\text{H}$  target at kinematics 1. The red lines outline the angular acceptance cuts applied.

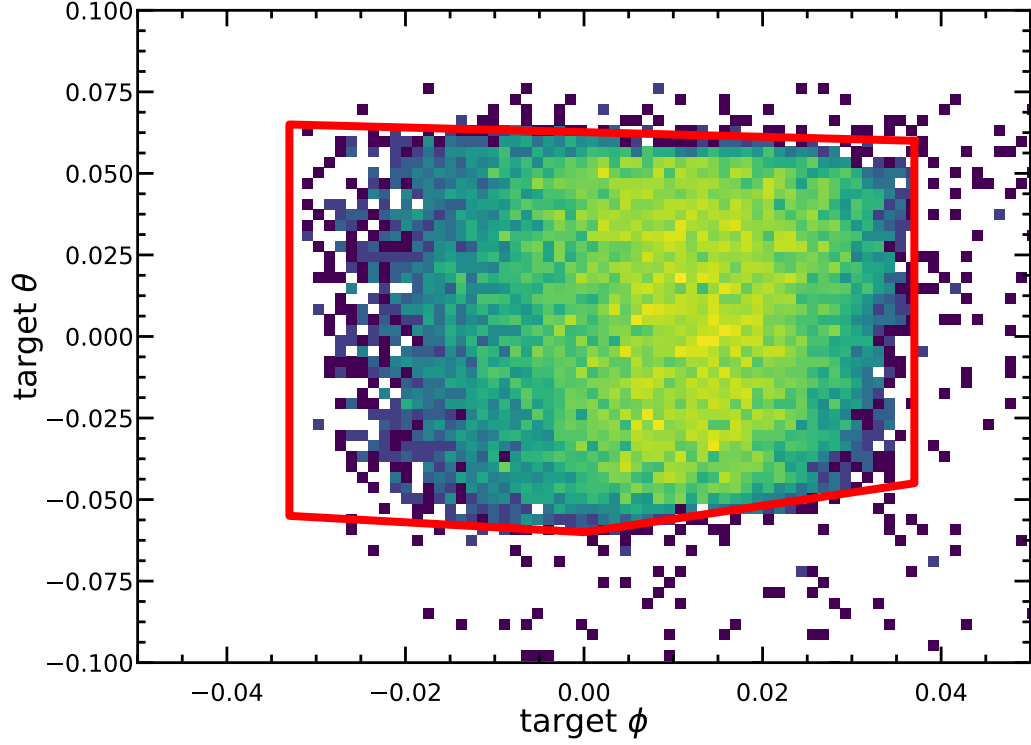


Figure 35: RHRs angular acceptance for the  $^3\text{H}$  target at kinematics 16. The red lines outline the angular acceptance cuts applied.

### 5.2.2 Particle Identification

The gas Cherenkov detector, along with the electromagnetic calorimeter, formed the particle identification (PID) set used to separate electrons from various background sources. For MARATHON, pions ( $\pi^-$ ) were the primary background. Both the gas Cherenkov and electromagnetic calorimeter detectors are designed to suppress the hadronic background, based on the  $E_{thresh}$  required to create Cherenkov radiation, and the total amount of Energy ( $E_{tot}$ ) deposited in the calorimeter. With these in mind, cuts were placed in each detector to minimize background sources. Cuts on the Cherenkov ADC sum and  $E/p$  spectrum constitute the PID cuts used in the analysis. Figure 36 shows the PID cuts placed on the Cherenkov ADC sum and  $E/p$

quantities. One can see that the combination of these cuts removes nearly all of the non-electron events. To check the validity of the cuts, sensible variations to the cuts were made to check the sensitivity of yields to the PID cuts applied. Variations of less than 0.1% were observed to the yield ratios, indicating little contamination as well as a reasonable choice in PID cuts.

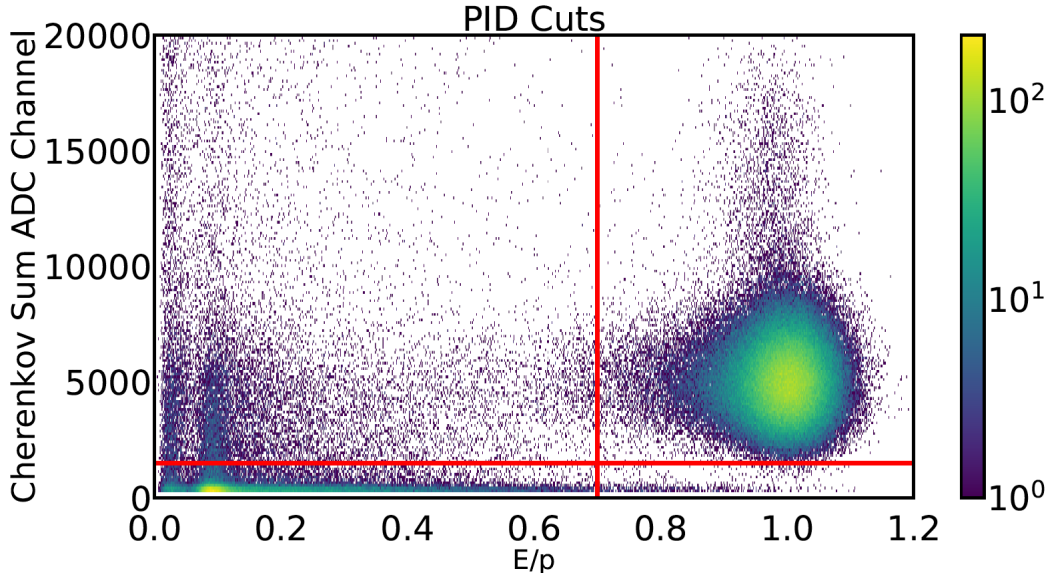


Figure 36: Cherekov ADC sum versus  $E/p$  for the  $^3\text{H}$  target at kinematics 1. The red lines indicate the cuts placed on the Cherenkov ADC sum and  $E/p$  quantities.

### 5.2.3 $z$ Target Reconstruction

Each event with a track in the VDCs is reconstructed back to the target. The reconstructed location along the target cell is known as the target  $z$  reaction point. The spectrum of the events from the thick Aluminum endcaps of each target cell may have appreciable tails, as can be seen in Figure 37. If the target  $z$  cuts are too loose, these tails can extend into the gas electron sample, and conversely, if they are

too strict, they would remove otherwise good events and greatly reduce the overall statistics. The target  $z$  cuts applied are kinematic dependent and were checked by varying them and comparing the change in the final yield ratios. The target  $z$  cuts for each kinematic are listed in Table 6.

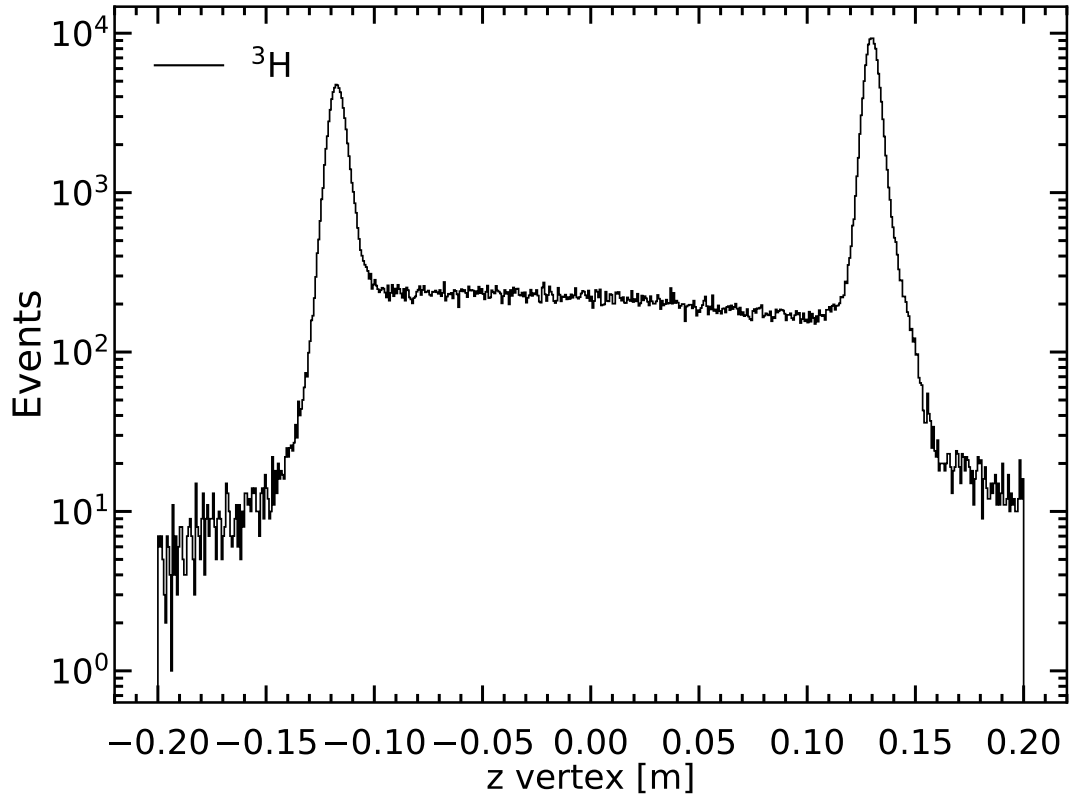


Figure 37: Reconstructed position of the origin of the events along the  $^3\text{H}$  target cell (for kinematics 1). The two large peaks are due to scattered electrons originating from the upstream and downstream target endcaps.

### 5.3 Charge Calculation

The Beam Charge Monitors (BCMs) measure the beam current delivered to the Hall as seen by the target. The beam current  $I$  is given by:

$$I = g * \text{BCM}_{freq} + b, \quad (5.3)$$

where  $g$  and  $b$  are the gain and offset of the BCM receiver, which are coefficients determined from the BCM calibration, and  $\text{BCM}_{freq}$  is the response of the BCM receiver. Equation 5.3 defines the current between 2 scaler events. To determine the total charge delivered,  $Q_{tot}$ , one must integrate the current over the total time,

$$Q_{tot} = \int I dt. \quad (5.4)$$

The overall stability of the BCMS during the experiment, and in particular, the dneu receiver, allowed for the determination of a single gain and an offset value, which could easily be applied over the entirety of the MARATHON experiment. These two coefficients were determined from a global fit to the calibrations performed during the experiment.

### 5.4 Target Density

When the electron beam interacts with the target, it can cause local heating and induce density reductions of the gas in the target cell. This effect, often referred to as “target boiling”, is both target configuration and beam current dependent. A thorough study of the effect for the gas targets was made during the run period previous to MARATHON, and an additional set of data was taken during MARATHON to confirm the results. To quantify the effect, data were taken over a series of currents ranging between 2.5 and 22.5  $\mu\text{A}$  for each gas target as well as the solid carbon target.

The solid target should exhibit no density fluctuation with current variations so it can be used as a benchmark to ensure the method applied is correct. For a given target, charge-normalized yields were calculated at each current setting. The data sets were taken at the same kinematic setting and over the same time period. Thus any changes in the normalized yields are due to density fluctuations of the target [57]. Figure 38 illustrates the effect of each of the four gas targets. The density reduction for each target was parameterized in terms of a second-order polynomial fit given by:

$$\rho_{cor} = p_0 + p_1 I + p_2 I^2, \quad (5.5)$$

where  $p_0$ ,  $p_1$  and  $p_2$  are the fit coefficients and  $I$  is the average current for a given run, calculated from Equation 5.3. This fit function was then applied to correct for the density fluctuation of each target on a run-by-run basis. The vast majority of data taken for MARATHON was with a current near  $22.5 \mu\text{A}$ , indicating that tritium and deuterium would experience approximately a 10% reduction in density.

## 5.5 Dead Time

Due to finite limitations of the Data Acquisition System (DAQ), not every event that produces a trigger will be recorded. Thus the total yield needs to be corrected to account for these missing events. When an event produces a trigger, the Trigger Supervisor will issue a Level 1 Accept (L1A) signal, which is distributed to each Readout Controller (ROC). While the trigger is being processed and the L1A signal is still latched, no new triggers can be processed by the Trigger Supervisor. Thus, a certain number of events that occur during this time frame will not be read out by the ADCs and TDCs. To determine the number of events that are lost, each trigger

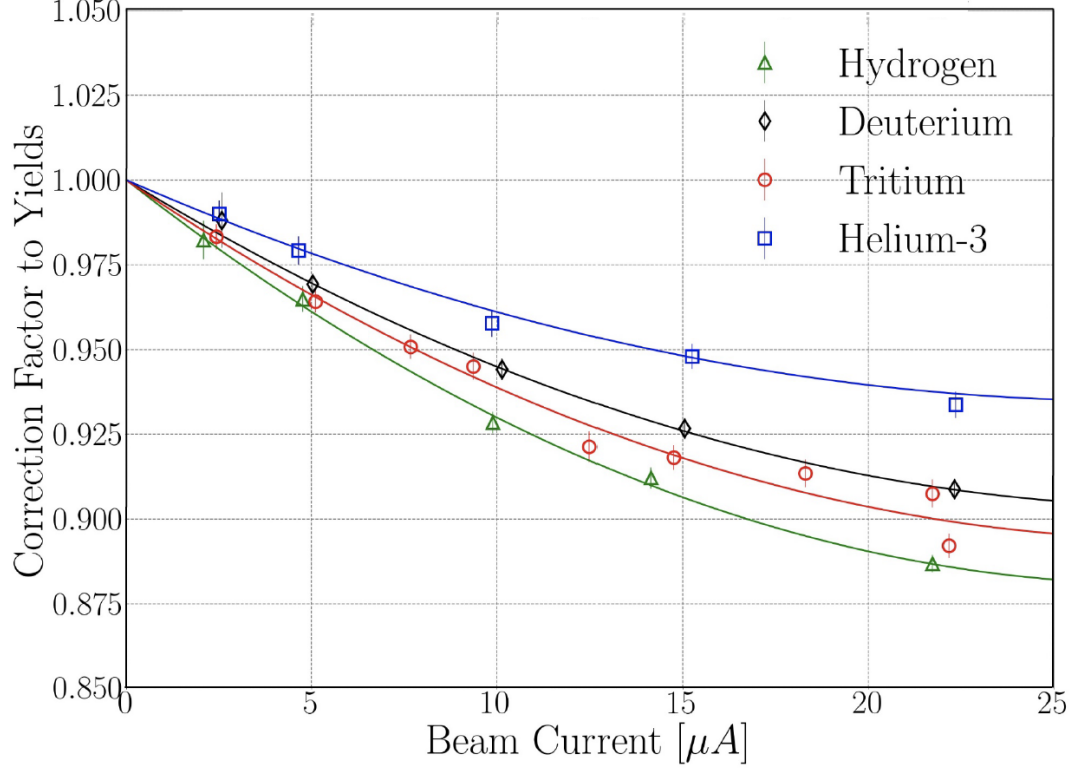


Figure 38: Beam-induced target density change along with the least-squares fit for each target. The majority of data was taken at  $22.5 \mu A$ , for which an approximately 10% density reduction can be seen for both  $^3H$  and  $^2H$  gases.

is also sent to and recorded by a scaler, which acts as a counter to record the total number of triggers during each data run. A direct comparison between the total number of the triggers counted by the scaler to the total number of triggers that were recorded by the DAQ will indicate the dead time of the DAQ. Therefore, the dead time for the  $T_2$  trigger is defined by the following formula:

$$DT_{T_2} = \frac{PS_{T_2} N_{T_2}^{DAQ}}{N_{T_2}^{scaler}}, \quad (5.6)$$

where  $PS_{T_2}$  is a prescale factor for the  $T_2$  trigger,  $N_{T_2}^{DAQ}$  is the number of  $T_2$  triggers recorded by the DAQ, and  $N_{T_2}^{scaler}$  is the total number of  $T_2$  triggers recorded by the

scaler. As mentioned in Section 3.6.2, T2 was the production trigger used during MARATHON. The prescale factor  $PS_{T2}$  was 1 for the entirety of the MARATHON experiment. The live time for each data run is then given by:

$$LT_{T2} = 1 - DT_{T2} . \quad (5.7)$$

The live time was kinematic dependent, as can be seen in Figure 39. The difference in live time among kinematics is due to overall event counting rate, which decreased at higher kinematics. The live time ranged between 94 and 100%.

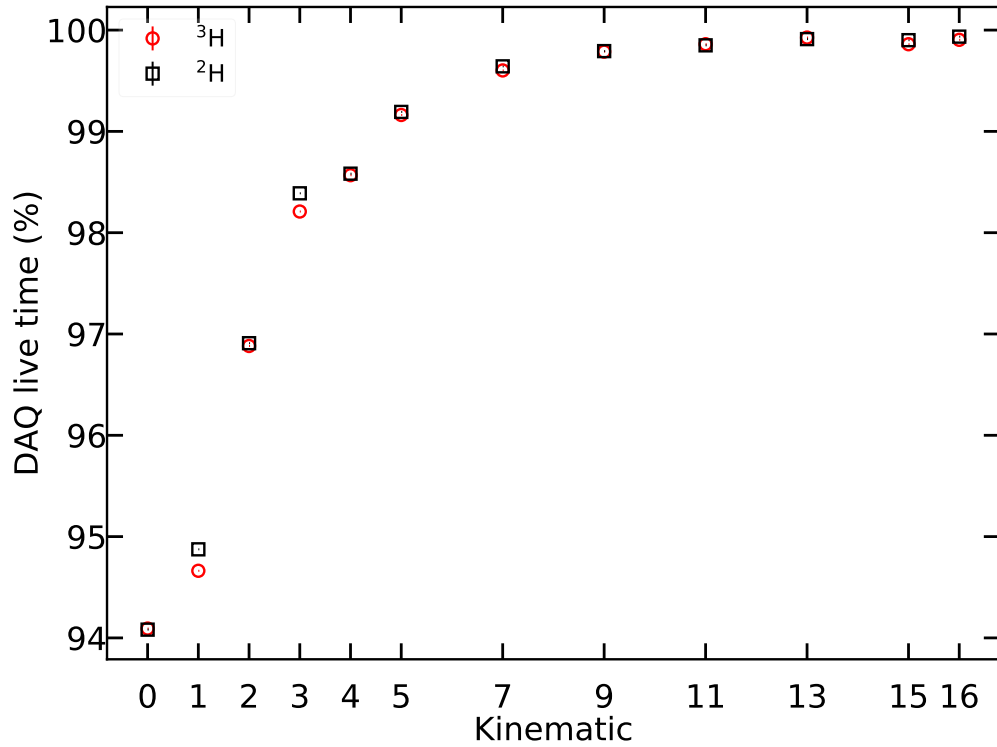


Figure 39: The DAQ live time for each kinematic. Shown are the live times for both the  $^3\text{H}$  and  $^2\text{H}$  targets.

## 5.6 Background Processes

Measuring electrons that have scattered off of the gas inside the target cell is the ultimate goal but one must ensure that the events recorded did not originate from a different physics process. Those events which were the result of an undesired reaction should be subtracted from the final sample. Since the detectors cannot discriminate the type of interaction that resulted in the electron event, we must use other techniques to estimate the amount they contribute to the final sample. In the following section, the general procedures for studying and quantifying the amount that these sources contaminate the final electron sample will be discussed.

### 5.6.1 Charge Symmetric Production

In the field of a nucleus, an electron can emit a photon which can subsequently decay into an electron-positron pair ( $\gamma \rightarrow e^+e^-$ ), which is a charge symmetric process. Similarly, an electron-positron pair can be produced by electroproduction of a neutral pion ( $\pi^0$ ), as the latter decays into photons. If this should occur inside the target cell and an electron reaches the detector hut, it would be wrongly classified as a good event, contaminating the sample of true scattering events. The contamination of such background events to the electron yield can be expressed as:

$$\frac{Y_{csb}^{e^-}}{Y_{Total}^{e^-}}, \quad (5.8)$$

where  $Y_{csb}^{e^-}$  is the electron yield from charge symmetric background and  $Y_{Total}^{e^-}$  is the total measured electron yield given by:

$$Y_{Total}^{e^-} = Y_{csb}^{e^-} + Y_{DIS}^{e^-}, \quad (5.9)$$

where  $Y_{DIS}^{e^-}$  is the total number of scattered electron events. While there is no way to directly discriminate between an electron scattering event and an electron produced

via photon decay, other ways can be used to estimate the amount of this background. Typically, this is done by measuring  $e^+$  events by reversing the polarity of all of the magnets. This was done during MARATHON by flipping the polarity of the LHRS spectrometer magnets to positive polarity, and measuring positron rates at kinematics in which charge symmetric background are known to be the largest. The positron and electron yields are calculated for each kinematic, following the same procedure, which includes applying identical acceptance and PID cuts as well as the same systematic corrections. The symmetric nature of the background implies that  $Y_{csb}^{e-} = Y_{csb}^{e+}$ , where  $Y_{csb}^{e+}$  is the positron yield. Thus, by measuring the positron yield we can determine the contamination  $Y_{csb}^{e-}$  to the measured electron yield  $Y_{meas}^{e-}$  from

$$Y_{DIS}^{e-} = Y_{meas}^{e-} \left( 1 - \frac{Y_{csb}^{e+}}{Y_{Total}^{e-}} \right). \quad (5.10)$$

A plot of  $Y_{csb}^{e-}/Y_{Total}^{e-}$  for both  $^2\text{H}$  and  $^3\text{H}$  can be seen in Figures 40 and 41 respectively. To extrapolate the correction to higher kinematics, a fit of the data was performed, which is also shown in Figures 40 and 41. The fit has the functional form:

$$\frac{Y_{csb}^{e+}}{Y_{Total}^{e-}} = e^{A+Bx}, \quad (5.11)$$

where  $A$  and  $B$  are constants determined from each fit.

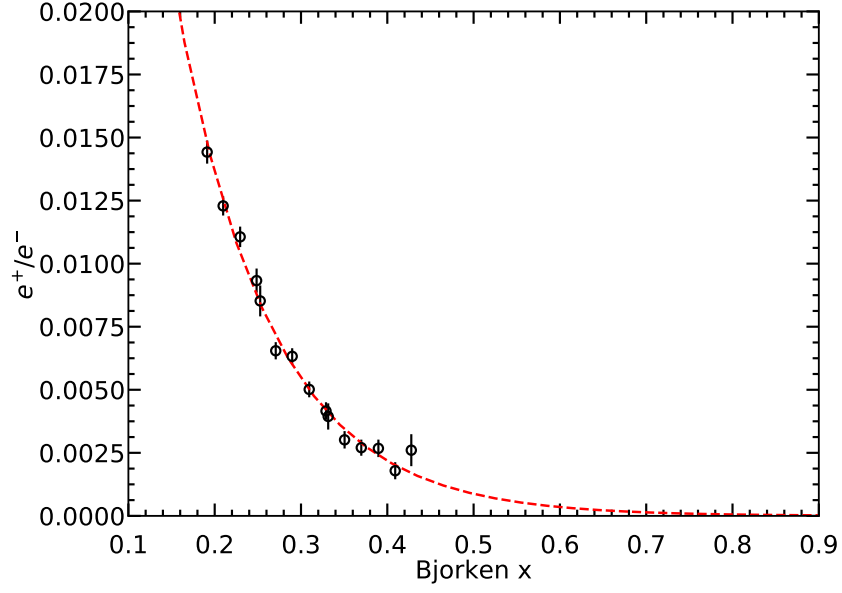


Figure 40: Ratio of  $e^+$  and  $e^-$  yields for the  $^2\text{H}$  kinematics. The red dashed line is the exponential fit used to subtract the charge symmetric background for the  $^2\text{H}$  targets.

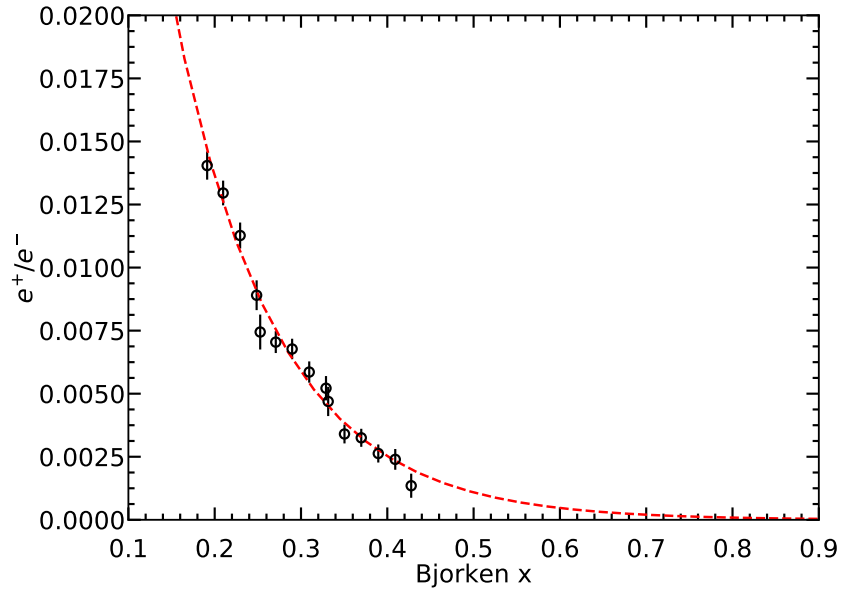


Figure 41: Ratio of  $e^+$  and  $e^-$  yields for the  $^3\text{H}$  kinematics. The red dashed line is the exponential fit used to subtract the charge symmetric background for the  $^3\text{H}$  targets.

### 5.6.2 Target Cell Subtraction

As can be seen in Figure 15, the target cells have thick aluminum endcaps through which the electron beam must pass in order to interact with the gas inside the target. Due to this fact, the vast majority of electrons that are detected in the spectrometer resulted from scattering off either the upstream or downstream endcap, which is illustrated in Figure 42. Even with tight target  $z$  cuts, tails from the endcaps can extend into and contaminate events originating from the gas. To quantify this effect, data was taken at each kinematic with an empty target cell, which is the same cell used for the gas targets, except evacuated of gas [46]. This allows for a comparison of the gas and empty target frame and thus the determination of the contamination due to the target endcaps. Figure 42 shows a comparison of gas cells with the empty target cell. To determine the amount of contamination to each gas target from the endcaps, the standard acceptance and PID cuts were applied to both empty target and gas target data. For empty target data, the  $z$  target cut applied associated events with either the upstream or downstream endcap. As an example, for kinematics 1, the  $z$  target cut applied to the gas target was  $-0.08 \text{ m} < z < 0.10 \text{ m}$  while for the empty target the same cut was divided such that:

- $-0.08 \text{ m} < z < 0.00 \text{ m}$  : Events are associated with the upstream endcap.
- $0.00 \text{ m} < z < 0.10 \text{ m}$  : Events are associated with the downstream endcap.

The charge normalized yield is a function of the target thickness for which the upstream and downstream endcaps of each target cell have different thicknesses, requiring the division above.

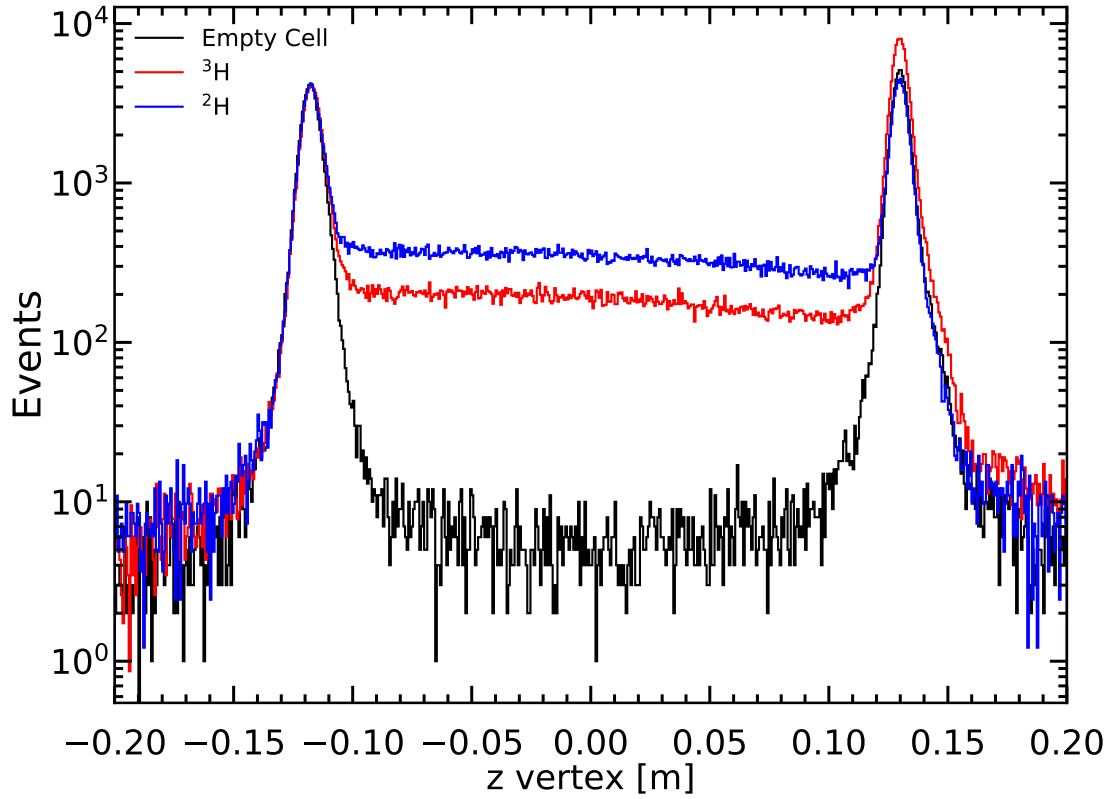


Figure 42: Comparison of events from cells filled with  $^3\text{H}$  and  $^2\text{H}$  gases and the empty cell (for kinematics 1). A small number of events from the endcaps can be seen to extend beyond the nominal  $z$  target cuts.

Table 7 lists the thickness for the upstream and downstream endcaps for each target as well as the reference empty target. The final empty cell yield is then the sum of the upstream and downstream yields:

$$Y_{empty} = Y_{up} + Y_{down} . \quad (5.12)$$

From the normalized yields for both the gas target and the empty cell, the endcap

| Position   | Empty             | $^3\text{H}$      | $^3\text{He}$     | $^2\text{H}$      | $^1\text{H}$      |
|------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Upstream   | $0.254 \pm 0.005$ | $0.253 \pm 0.004$ | $0.203 \pm 0.007$ | $0.215 \pm 0.004$ | $0.311 \pm 0.001$ |
| Downstream | $0.279 \pm 0.005$ | $0.343 \pm 0.047$ | $0.328 \pm 0.041$ | $0.294 \pm 0.056$ | $0.330 \pm 0.063$ |

Table 7: Endcap thickness for gas and empty cell targets. All units are in [mm].

contamination (ECC) for a single target is then given by:

$$ECC_{targ} = 1 - \frac{Y_{gas} - Y_{empty}}{Y_{gas}}. \quad (5.13)$$

The correction applied to the cross section ratios is the ratio of contamination from respective targets

$$\delta_{ECC} = \frac{ECC_{targ(1)}}{ECC_{targ(2)}}. \quad (5.14)$$

A fit of  $\delta_{ECC}$  as a function of  $x$  was performed for each cross section ratio of MARATHON, which is shown in Figure 43. Each fit gives the corresponding endcap contamination correction for each cross section ratio, and for each bin in  $x$ .

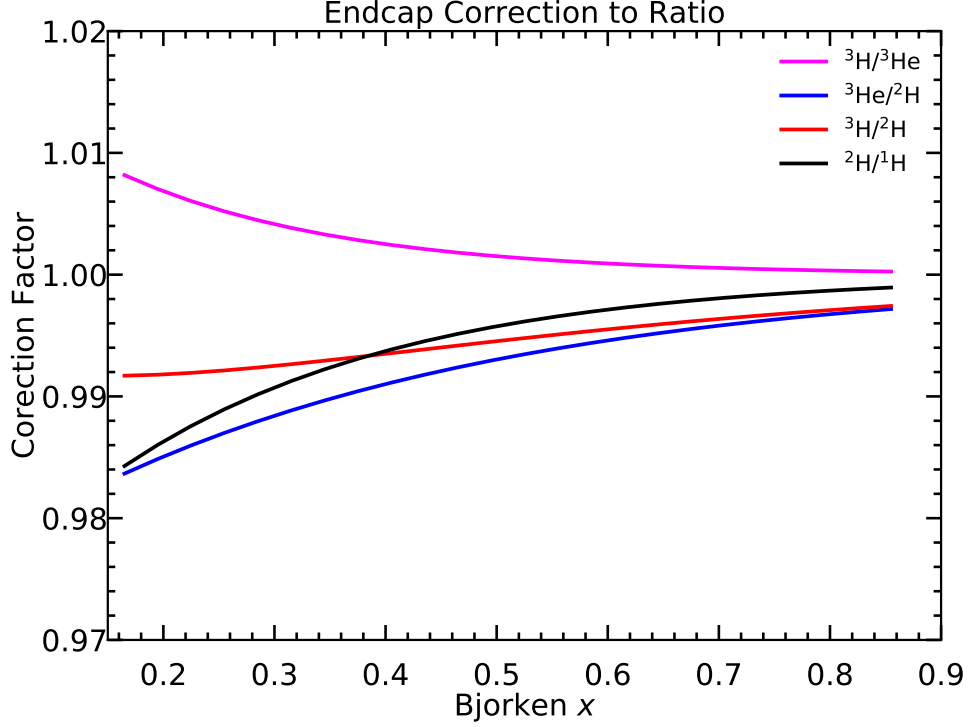


Figure 43: Fit of the Endcap correction factor which is applied to cross section ratios measured by MARATHON.

### 5.7 $^3\text{H}$ Decay

The  $^3\text{H}$  nucleus has a half-life,  $\tau$ , of  $4500 \pm 8$  days [58]. The decay of  $^3\text{H}$  proceeds via the  $\beta$ -decay reaction:

$$^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e. \quad (5.15)$$

Over the lifetime of the experiment, a portion of  $^3\text{H}$  in the target will be converted to  $^3\text{He}$ , introducing a source of contamination for the  $^3\text{H}$  electron sample. Figure 44 shows the percentage of tritium remaining in the target cell as a function of time.

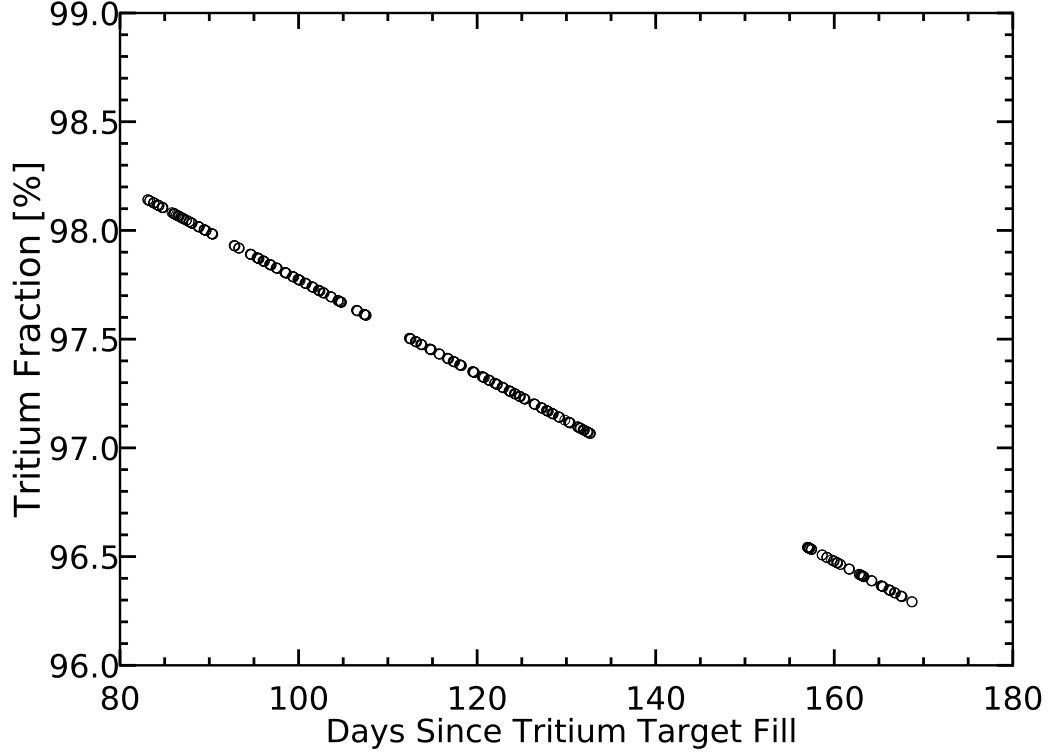


Figure 44: Percentage of tritium gas remaining in the  $^3\text{H}$  target cell. The open circles indicate recorded data runs. The plot begins at the start of the MARATHON experiment. The tritium target was filled  $\approx 80$  days before the start of MARATHON.

Taking into account the initial amount of  $^3\text{H}$  and  $^3\text{He}$  present in the cell at the fill date [46], one can extrapolate to determine the amount of  $^3\text{H}$  and  $^3\text{He}$  in the  $^3\text{H}$  target cell at a given time, using the following formulas:

$$n_{^3\text{H}}(t) = n_{^3\text{H}}^0 e^{-t/\tau} \quad (5.16)$$

$$n_{^3\text{He}}(t) = n_{^3\text{He}}^0 + n_{^3\text{H}}^0 (1 - e^{-t/\tau}) , \quad (5.17)$$

where  $n_{^3\text{H}}^0$  and  $n_{^3\text{He}}^0$  are the initial number densities ( $t=0$ ) for  $^3\text{H}$  and  $^3\text{He}$  respectively

in the  $^3\text{H}$  target cell. The corrected  $^3\text{H}$  yield can then be found to be [59]:

$$Y_{^3\text{H}}^C = Y_{^3\text{H}}^{\text{raw}} \left( \frac{Q_{\text{tot}}}{Q_{\text{tot}} - \langle f_{^3\text{He}} \rangle} \right) - Y_{^3\text{He}} \left( \frac{\langle f_{^3\text{He}} \rangle}{Q_{\text{tot}} - \langle f_{^3\text{He}} \rangle} \right), \quad (5.18)$$

where  $Y_{^3\text{H}}^C$  and  $Y_{^3\text{H}}^{\text{raw}}$  are the corrected and raw  $^3\text{H}$  yields respectively,  $Y_{^3\text{He}}$  is the  $^3\text{He}$  yield,  $Q_{\text{tot}}$  is the total beam charge, and  $\langle f \rangle$  is the charge-weighted helium fraction. To account for the time dependence of the effect, this correction was applied on a run-by-run basis.

## 5.8 Radiative Corrections

The cross section depicted in the Feynman diagram in Figure 1 is the lowest order interaction in  $\alpha$ , referred to as the Born cross section. Experimentally, the measured raw cross section is a convolution of higher order processes, beyond the Born cross section, as well as additional processes due to electromagnetic interactions [60]. These are typically separated into external radiative effects that occur before and after the scattering of the electron, and internal radiative effects that occur at the scattering interaction vertex.

The above effects can introduce elastic, quasi-elastic, and inelastic tails to the measured cross section. These processes need to be estimated to accurately report the Born cross section. Figure 45 illustrates terms beyond the Born approximation which can contribute to the measured cross section. These terms include vacuum polarization, electron self-energy, and the electron vertex correction [60]. Along with these, bremsstrahlung radiation both before and after scattering are accounted for and are visualized in Figure 46.

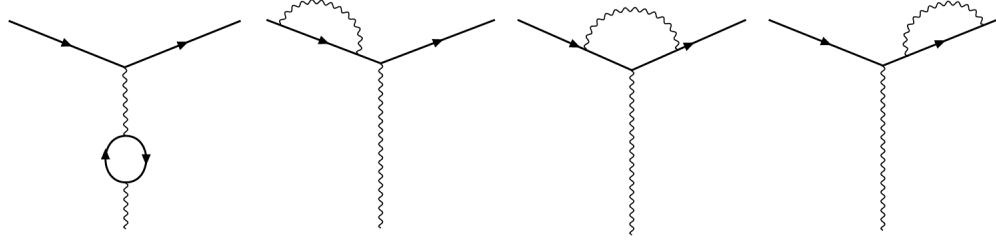


Figure 45: Higher order processes to the single photon exchange approximation for electron nucleus scattering.

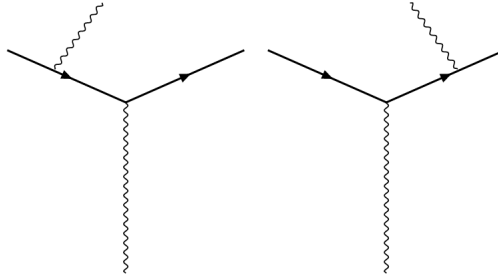


Figure 46: External bremsstrahlung radiation processes.

A package in the Hall A Single Arm Monte Carlo (SIMC), “T2\_Externals” [61], was used to calculate the fully radiated and Born cross section based on the seminal work of Mo and Tsai [60]. Reasonable model cross sections for each target are required to implement the correction.

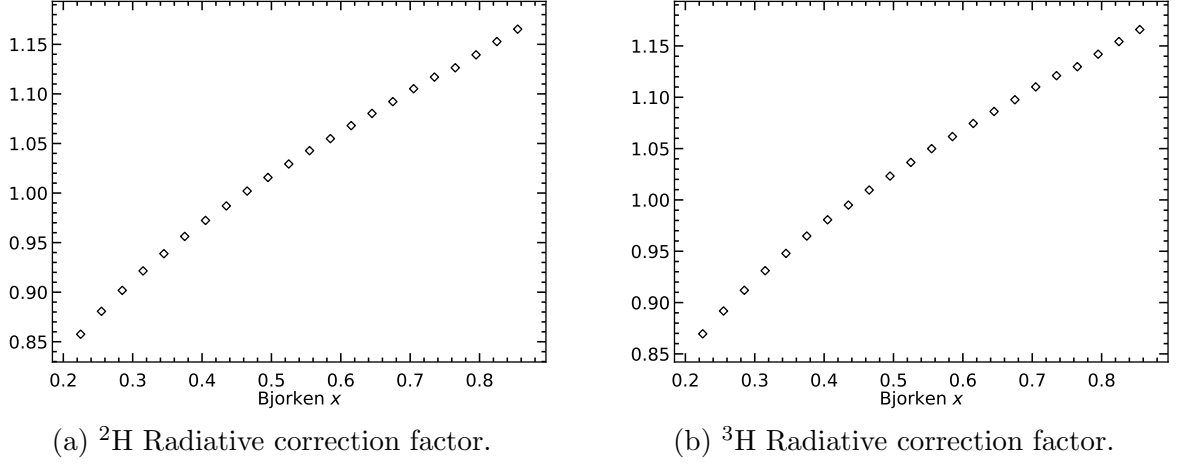


Figure 47: Radiative correction factor for the helium and deuteron DIS cross sections.

A comparison of the Born and fully radiated cross section allows for the determination of the radiative correction factor (RC) according to:

$$RC = \frac{\sigma_{Born}}{\sigma_{Radiated}}. \quad (5.19)$$

The Born and radiated cross sections were calculated for each kinematics. Figure 47 shows the radiative correction factors for the  $^3\text{H}$  and  $^2\text{H}$  DIS cross sections respectively, as a function of  $x$ . Finally, in order to determine the uncertainty associated with the radiative correction due to cross section input, several realistic cross section input models were used to calculate the correction. From a comparison of the corrections using different models, an uncertainty of  $\pm 0.5\%$  on the radiative correction is included in the final cross section ratios.

## 5.9 Coulomb Correction

The interaction of the incident electron with the Coulomb field of the target nucleus will shift the initial and scattered energies of the electron from the values calculated at the reaction vertex. To account for this shift, a “Coulomb correction”

needs to be applied to the cross section ratio. The Coulomb correction applied used is the “effective  $Q^2$  approximation”. In this approximation, it is noted that while the initial and scattered energies will change by some amount, they will change in the same proportion leaving the energy transfer  $\nu$  ( $E-E'$ ) unchanged. Recalling from the definition of  $Q^2$  and  $x$  given in Equations 1.4 and 1.28, the  $Q^2$  and  $x$  of the interaction at the vertex will change due to these shifts. An effective  $Q_{eff}^2$  at the vertex can be determined using [62]:

$$Q_{eff}^2 = \left(1 + \frac{3Z\alpha\hbar c}{2RE}\right) Q^2, \quad (5.20)$$

where

$$R = \sqrt{\frac{5\langle r^2 \rangle}{3}}, \quad (5.21)$$

is the hard-sphere-equivalent radius of the nucleus,  $Z$  is the number of protons,  $E$  is the beam energy, and the quantity  $\langle r^2 \rangle$  is the mean square radius of the nucleus and is given in Table 8 [63]:

| Nucleus      | $\langle r^2 \rangle$ [fm] |
|--------------|----------------------------|
| $^3\text{H}$ | 1.68                       |
| $^2\text{H}$ | 2.14                       |

Table 8: Mean square radius of the  $^3\text{H}$  and  $^2\text{H}$  nuclei [63].

With  $Q_{eff}^2$ , one can then determine the corresponding  $x_{eff}$ . Using a reliable model, the cross section can be determined at both the standard  $(x, Q^2)$  and shifted  $(x_{eff}, Q_{eff}^2)$  kinematics.

In the effective  $Q^2$  approximation, the Coulomb correction factor ( $C_{Coulomb}$ ) is found from a direct comparison of both cross sections, given by:

$$C_{Coulomb} = \frac{\sigma(x, Q^2)}{\sigma(x_{eff}, Q_{eff}^2)} . \quad (5.22)$$

Figure 48 shows the Coulomb correction factor for each of the 2 gas targets. As one can see, the Coulomb correction for both  $^3\text{H}$  and  $^2\text{H}$  increases with  $x$  to at most a 0.3% value per target.

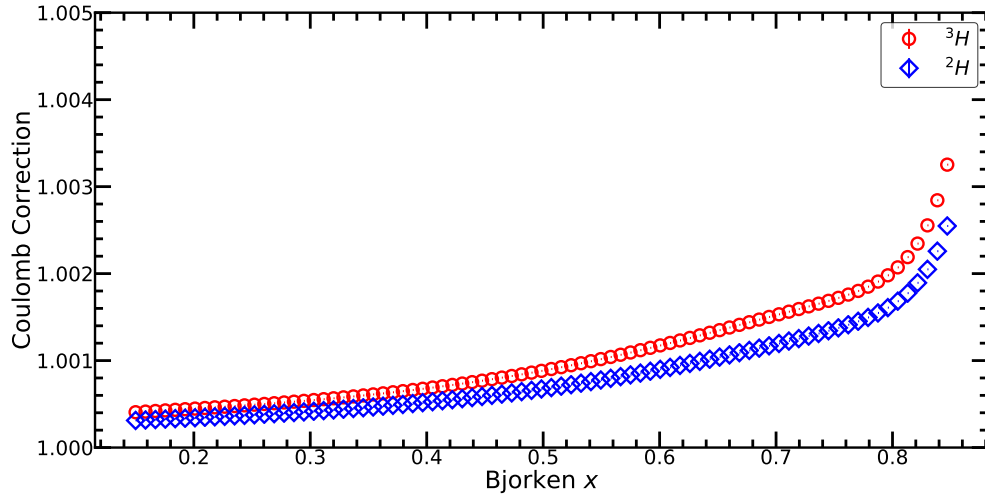


Figure 48: The Coulomb correction factor for  $^3\text{H}$  and  $^2\text{H}$  gas targets as a function of  $x$ . The largest correction is at high  $x$  where it reaches a maximum value of 0.3%.

### 5.10 Bin Centering Correction

Each spectrometer has a finite acceptance through which scattered electrons can make it to the detectors. The final yield is binned in  $x$  and is reported at each bin center. Depending on how strongly the cross section falls across the acceptance of the

spectrometer, the average value may differ significantly from that at the bin center. Unlike the elastic cross section which is known to fall drastically, the DIS cross section is not expected to vary so dramatically, but the variation must be accounted for. To investigate this effect on the final ratio, a model cross section was used to determine the magnitude of the bin centering correction needed to be applied. By integrating the cross section over the width of a given bin, one can determine the average cross section value in comparison with the cross section at the bin center. The difference in these cross sections, results in a bin center correction factor ( $BCC$ ) which should be applied to the final cross section ratio. The correction factor for a single target is given by:

$$BCC = \frac{\sigma_{Center}}{\sigma_{Average}} , \quad (5.23)$$

where  $\sigma_{Center}$  and  $\sigma_{average}$  are the model cross section at the bin center, and the average cross section over the bin of width  $\Delta x$ . Therefore, one can apply the correction to the measured yields as:

$$Y_{BC} = Y_{Average} * BCC , \quad (5.24)$$

where the  $Y_{BC}$  is the yield reported at the bin center and  $Y_{Average}$  is the measured yield.

## CHAPTER 6

### Results and Discussion

#### 6.1 $^3\text{H}/^2\text{H}$ Results

The final  $^3\text{H}/^2\text{H}$  cross section ratio values were analyzed from 12 different kinematic settings, with several of the spectrometer settings taken in different time periods. Table 9 lists the central kinematic values ( $E$ ,  $E'$ ,  $Q^2$ ,  $\theta$ , and  $x$ ) for each kinematic setting. After ensuring consistency, these coinciding kinematics were eventually combined. Likewise, due to the large acceptance of the HRS spectrometers, there was considerable overlap in  $x$  among different kinematics, especially at lower  $x$ . Figure 49 is a plot of  $Q^2$  versus  $x$  and illustrates the  $x$  overlap between adjacent kinematics. Each kinematic is analyzed separately but overlapping  $x$  bins need to be combined in order to report the final cross section ratio for each bin. These bins are combined by taking the weighted average of the corresponding data yields:

$$\bar{Y} = \frac{\sum_i Y_i}{\sum_i w_i}, \quad (6.1)$$

where the weight factor  $w_i$  is given by

$$w_i = \frac{1}{\sigma_i^2}, \quad (6.2)$$

and the  $\sigma_i$ 's are the statistical errors associated with each bin.

| Kin | $E$<br>(GeV) | $E'$<br>(GeV) | $Q^2$<br>[(GeV/c) <sup>2</sup> ] | $\theta$<br>(deg) | $x$  |
|-----|--------------|---------------|----------------------------------|-------------------|------|
| 0   | 10.59        | 3.1           | 2.79                             | 16.90             | 0.19 |
| 1   | 10.59        | 3.1           | 3.06                             | 17.58             | 0.22 |
| 2   | 10.59        | 3.1           | 3.61                             | 19.12             | 0.26 |
| 3   | 10.59        | 3.1           | 4.19                             | 20.58             | 0.29 |
| 4   | 10.59        | 3.1           | 4.76                             | 21.93             | 0.34 |
| 5   | 10.59        | 3.1           | 5.31                             | 23.21             | 0.38 |
| 7   | 10.59        | 3.1           | 6.47                             | 25.59             | 0.46 |
| 9   | 10.59        | 3.1           | 7.56                             | 27.77             | 0.54 |
| 11  | 10.59        | 3.1           | 8.71                             | 29.92             | 0.62 |
| 13  | 10.59        | 3.1           | 9.84                             | 31.73             | 0.70 |
| 15  | 10.59        | 3.1           | 10.96                            | 33.56             | 0.78 |
| 16  | 10.59        | 2.9           | 11.83                            | 36.12             | 0.82 |

Table 9: Kinematic settings of the MARATHON experiment.

From these weighted average yields, the per nucleon cross section ratio can be formed

$$\frac{{}^3\text{H}}{{}^2\text{H}} \equiv \frac{Y({}^3\text{H})/3}{Y({}^2\text{H})/2}, \quad (6.3)$$

where  $Y({}^3\text{H})$  and  $Y({}^2\text{H})$  are the final yields per bin for tritium and deuterium respectively. Figure 50 shows the raw  ${}^3\text{H}/{}^2\text{H}$  per nucleon cross section ratio. Statistical and systematic errors have been added in quadrature. Table 10 in Appendix C lists the raw cross section ratio along with the statistical and systematic uncertainties for each bin. Finally, recalling Equation 2.4, the final cross section ratio reported can be interpreted as the structure function ratio of  $F_2^{{}^3\text{H}}/F_2^{{}^2\text{H}}$ .

## 6.2 Normalization

Along with the EMC effect of tritium, MARATHON also measured the EMC effect of helium-3, as well as the  ${}^3\text{H}/{}^3\text{He}$  cross section ratio to extract the  $F_2^n/F_2^p$  ratio. Each experimental cross section ratio depends on several quantities as discussed in chapter 5, including the individual target densities. A possible source of uncertainty revolved

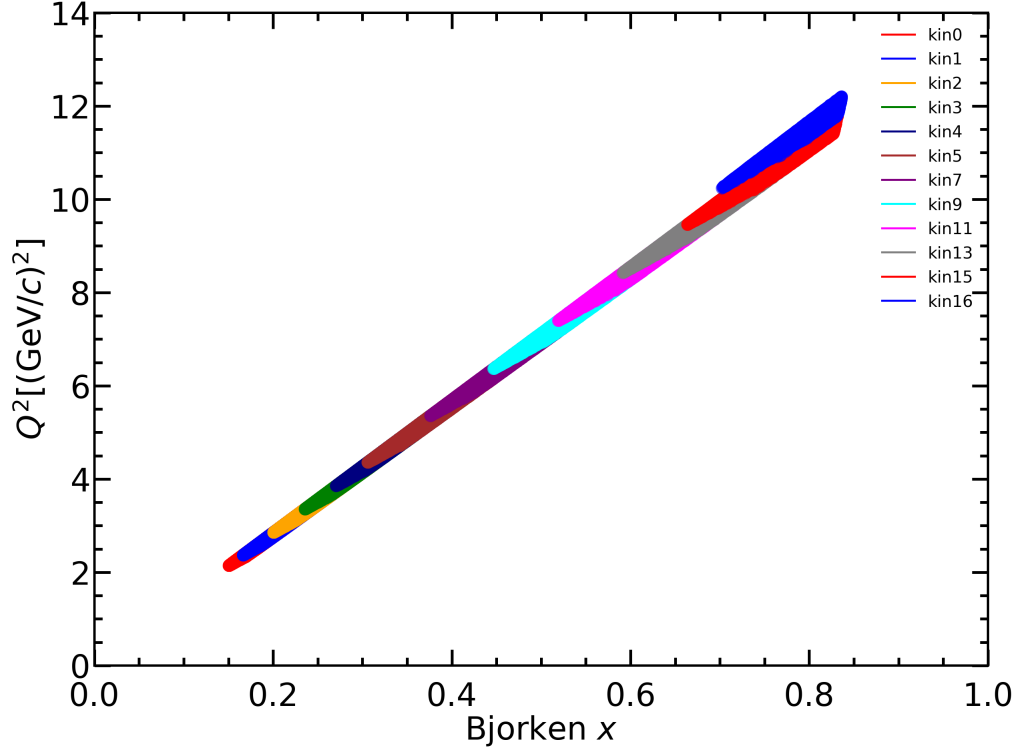


Figure 49: A plot of  $Q^2$  versus  $x$  to illustrate the  $x$  overlap between adjacent kinematics. Overlapping  $x$  bins were combined from the weighted average of corresponding  $x$  bins.

around the target densities of both tritium and helium-3. As can be seen in Table 4, the uncertainties for tritium and helium-3 are twice that of the deuterium and hydrogen. This was anticipated and the experimental proposal outlined a method of normalizing MARATHON's  $^3\text{H}/^3\text{He}$  cross section ratio by comparing the extracted  $F_2^n/F_2^p$  from the  $^3\text{H}/^3\text{He}$  cross section ratio to  $F_2^n/F_2^p$  extracted from the  $^2\text{H}/^1\text{H}$  cross section ratio at  $x \approx 0.3$  where nuclear effects are small and tend to cancel in the cross section ratio. For this reason, MARATHON took high statistics hydrogen data in this low  $x$  region. With the additional  $^1\text{H}$  data, we could first compare MARATHON's raw  $^2\text{H}/^1\text{H}$  cross section ratio with that of established results from the Stanford

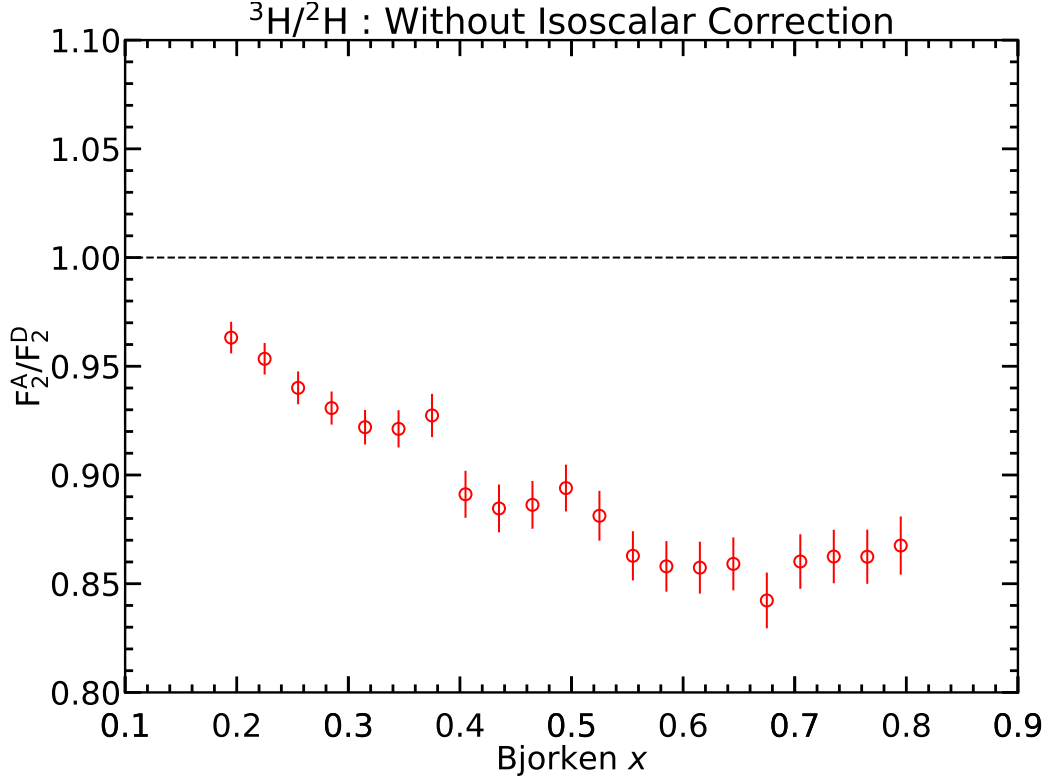


Figure 50: Raw  ${}^3\text{H}/{}^2\text{H}$  cross section ratio. No isoscalar correction has been applied to the ratio. Statistical and systematic uncertainties have been added in quadrature.

Linear Accelerator Center (SLAC). Such a comparison would provide confidence in the extracted  $F_2^n/F_2^p$ , given a reliable model, from the  ${}^2\text{H}/{}^1\text{H}$  cross section ratio. This could then be used to normalize the  ${}^3\text{H}/{}^3\text{He}$  cross section ratio. Along with  $F_2^n/F_2^p$  extractions from the  ${}^2\text{H}/{}^1\text{H}$  and  ${}^3\text{H}/{}^3\text{He}$  cross section ratios, extraction from the  ${}^3\text{H}/{}^2\text{H}$  and  ${}^3\text{He}/{}^2\text{H}$  cross section ratios is also possible, albeit with larger uncertainties. In the region of  $x \approx 0.3$ , within uncertainties, all four ratios should give similar results. The same procedure could then be used to normalize the  ${}^3\text{H}/{}^2\text{H}$  cross section ratio as well.

A theoretical input is required to extract the  $F_2^n/F_2^p$ . For this purpose, MARATHON

utilized a model provided by Sergey Kulagin and Roberto Petti (KP) [64]. This model provides reliable predictions suitable for the MARATHON experiment, and in particular for  $^1\text{H}$ ,  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  DIS cross sections as well as  $F_2^n$  and  $F_2^p$  structure functions. Also, the KP model has made predictions for a variety of other EMC ratios which have been shown to be in excellent agreement with experimental data. Using the KP model, the extraction of  $F_2^n/F_2^p$  from  $^2\text{H}/^1\text{H}$ ,  $^3\text{H}/^3\text{He}$ , and  $^3\text{H}/^2\text{H}$  can be shown to materialize through the equations:

$$\begin{aligned}\frac{F_2^n}{F_2^p} &= \frac{\mathcal{R}\left(\frac{^3\text{H}}{^2\text{H}}\right)}{R_2} - 1 \\ \frac{F_2^n}{F_2^p} &= \frac{2\frac{R_{32}}{R_{31}} - \mathcal{R}\left(\frac{^3\text{H}}{^2\text{H}}\right)}{2\mathcal{R}\left(\frac{^3\text{H}}{^2\text{H}}\right) - \frac{R_{32}}{R_{31}}} \\ \frac{F_2^n}{F_2^p} &= \frac{\frac{3R_2}{2R_{31}}\mathcal{R}\left(\frac{^3\text{H}}{^2\text{H}}\right) - 1}{2 - \frac{3R_2}{2R_{31}}\mathcal{R}\left(\frac{^3\text{H}}{^2\text{H}}\right)},\end{aligned}\tag{6.4}$$

where  $\mathcal{R}\left(\frac{^2\text{H}}{^1\text{H}}\right)$ ,  $\mathcal{R}\left(\frac{^3\text{H}}{^3\text{He}}\right)$ , and  $\mathcal{R}\left(\frac{^3\text{H}}{^2\text{H}}\right)$  are the experimentally measured cross section ratios and  $R_2$ ,  $R_{31}$  and  $R_{32}$  are theoretical inputs from the KP model, and are defined as:

$$\begin{aligned}R_2 &= \frac{F_2^{^2\text{H}}}{F_2^n + F_2^p} \\ R_{31} &= \frac{F_2^{^3\text{H}}}{2F_2^n + F_2^p} \\ R_{32} &= \frac{F_2^{^3\text{He}}}{F_2^n + 2F_2^p},\end{aligned}\tag{6.5}$$

where  $F_2^{^2\text{H}}$ ,  $F_2^{^3\text{He}}$ , and  $F_2^{^3\text{H}}$  are the model  $F_2$  structure functions for deuterium, helium-3, and tritium, respectively, while  $F_2^n$  and  $F_2^p$  are the model neutron and proton  $F_2$  structure functions. From the above formalism, one can determine both  $R_2$ ,  $R_{31}$ , and

$R_{32}$  for the experimentally binned  $x$  values and thus extract  $F_2^n/F_2^p$ . The  $^2\text{H}/^1\text{H}$  cross section ratio data were shown to be consistent with the SLAC  $^2\text{H}/^1\text{H}$  data [8] as well as the prediction from the KP model. Likewise,  $F_2^n/F_2^p$  extracted from the  $^2\text{H}/^1\text{H}$  cross section ratio was also consistent with the expected  $F_2^n/F_2^p$  at low  $x$ , providing a reasonable benchmark to which other observables can be compared to. When comparing  $F_2^n/F_2^p$  extracted from both the  $^3\text{H}/^3\text{He}$  and  $^2\text{H}/^1\text{H}$  cross section ratios, it became apparent that there was an inconsistency in the yield of either  $^3\text{H}$ ,  $^3\text{He}$ , or of both targets. A self-consistent check to determine the source(s) of the differences, involved the extraction  $F_2^n/F_2^p$  from  $^3\text{H}/^2\text{H}$  and  $^3\text{He}/^2\text{H}$  cross section ratios using the method detailed above. This procedure indicated that the helium-3 yield needed to be normalized up approximately +2.4 % and the tritium yield down approximately -0.4%, as can be seen in Figure 51. With these normalizations, the extracted  $F_2^n/F_2^p$  ratio from all four ratios are mutually consistent and agree with published SLAC data. All  $^3\text{H}/^2\text{H}$  cross section ratio results shown hereafter have been normalized down by 0.4%.

### 6.2.1 Isoscalar Correction

The EMC ratio is defined as the per nucleon structure  $F_2$  function ratio to deuterium. For nuclei with an equal number of protons and neutrons, the per nucleon ratio has a straightforward interpretation but for non-isoscalar nuclei where  $Z \neq A/2$ , a correction needs to be applied to account for differences in the proton and neutron  $F_2$  structure functions.

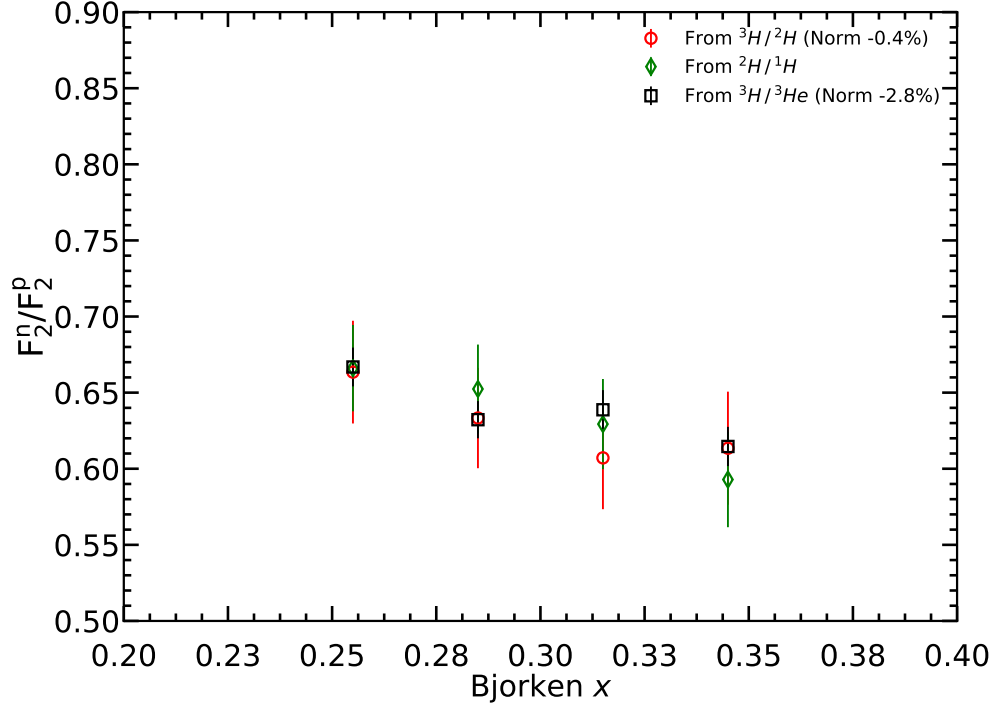


Figure 51:  $F_2^n/F_2^p$  extracted from  $^3\text{H}/^3\text{He}$  and  $^3\text{H}/^2\text{H}$  after normalizations of -2.8% and -0.4% were applied to  $^3\text{H}/^3\text{He}$  and  $^3\text{H}/^2\text{H}$  respectively.

The form of the isoscalar correction can be determined by considering first the case of isoscalar nuclei ( $Z = A/2$ ). For any isoscalar nuclei, it can be shown that

$$\frac{1}{2} (F_2^n + F_2^p) = \frac{1}{A} [Z F_2^p + (A - Z) F_2^n] . \quad (6.6)$$

By substituting  $Z = A/2$  on the right side of Equation 6.6, one finds an isoscalar correction of  $f_{iso} = 1$ .

Thus, the isoscalar correction can be written as

$$\frac{1}{2}(F_2^n + F_2^p) = f_{iso} \frac{1}{A} (ZF_2^p + (A - Z)F_2^n), \quad (6.7)$$

and by rearranging terms and dividing through by  $F_2^p$ :

$$f_{iso} = \frac{\frac{1}{2} \left(1 + \frac{F_2^n}{F_2^p}\right)}{\frac{1}{A} \left(Z + (A - Z) \frac{F_2^n}{F_2^p}\right)}. \quad (6.8)$$

The only input needed for the isoscalar correction is the  $F_2^n/F_2^p$  ratio. As mentioned in Section 6.2,  $F_2^n/F_2^p$  was also measured in the MARATHON experiment. Figure 52 shows the fit of the normalized  $F_2^n/F_2^p$  extracted from the MARATHON  $^3\text{H}/^3\text{He}$  cross section ratio [65] and used in the isoscalar correction. Figure 53 shows the magnitude of the isoscalar correction applied to the  $^3\text{H}/^2\text{H}$  cross section ratio.

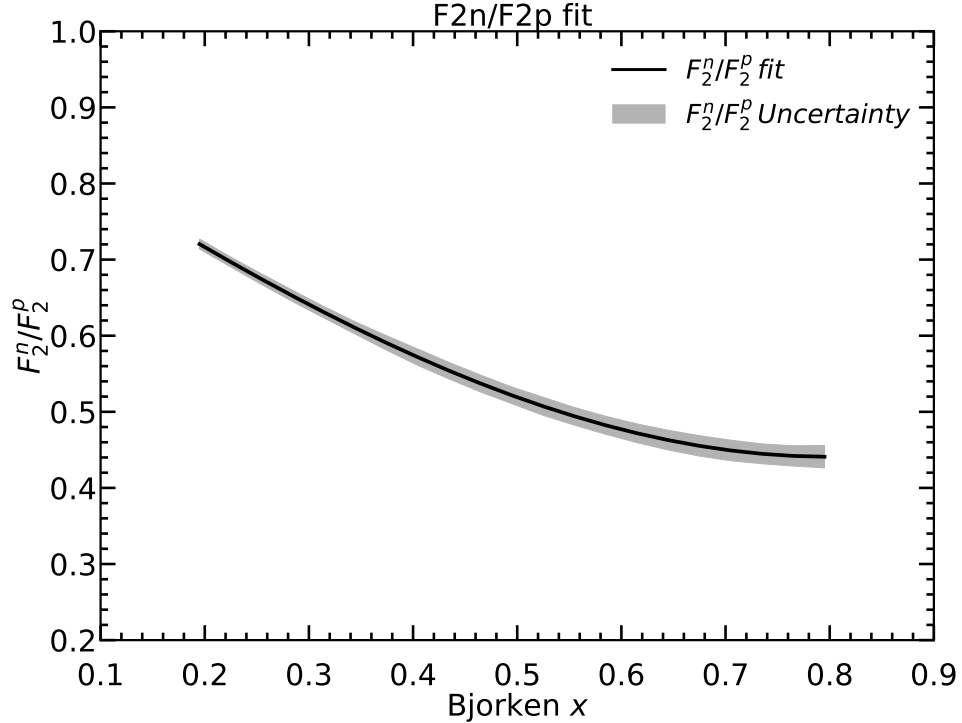


Figure 52:  $F_2^n/F_2^p$  extracted from MARATHON and used in the Isoscalar correction.

The grey band is the fit uncertainty.

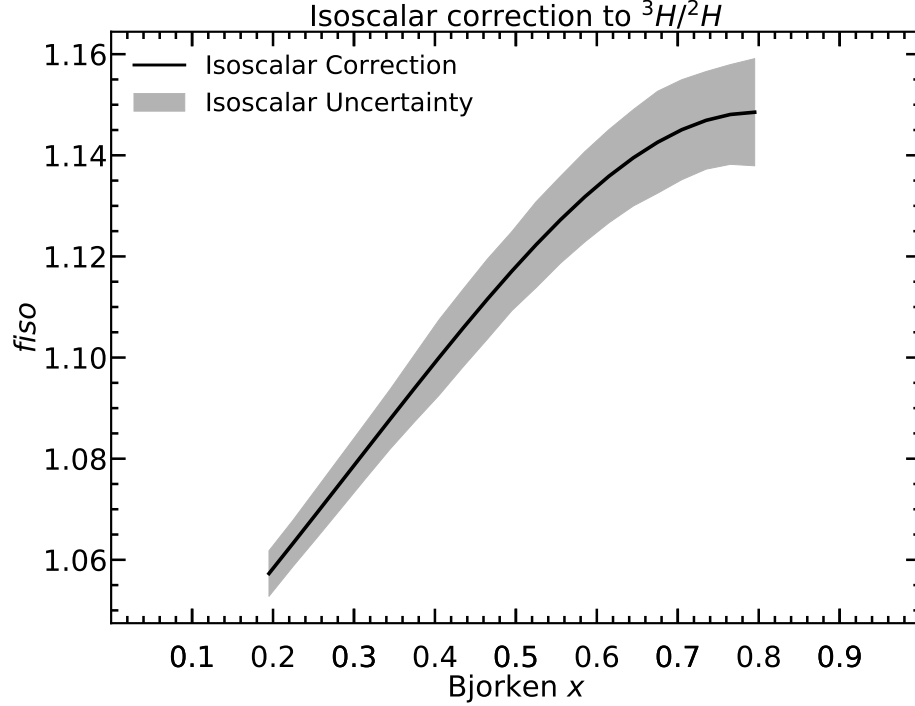


Figure 53: Isoscalar correction applied to  $^3\text{H}/^2\text{H}$  data. The  $F_2^n/F_2^p$  ratio came from MARATHON's extraction from  $^3\text{H}/^3\text{He}$ . The grey band is the isoscalar uncertainty.

### 6.2.2 Tritium EMC Effect

After including the normalization factor determined from the comparison of extracted  $F_2^n/F_2^p$  values as well as the isoscalar correction, all corrections have been applied to the  $^3\text{H}/^2\text{H}$  cross section ratio. Figure 54 shows the first measured EMC effect of  $^3\text{H}$  along with the predictions from the KP model. Table 11 in Appendix C lists the isoscalar-corrected data along with uncertainties. One key feature to note is, as with all measured EMC effects, there is a unity crossing at  $x \approx 0.3$  for the  $^3\text{H}$  EMC effect, as can be seen in Figure 54. Also, there is a very good agreement between the data and the KP model. The new EMC measurement of  $^3\text{H}$  will allow for further investigations, into the source of effect, and will be discussed later in this chapter.

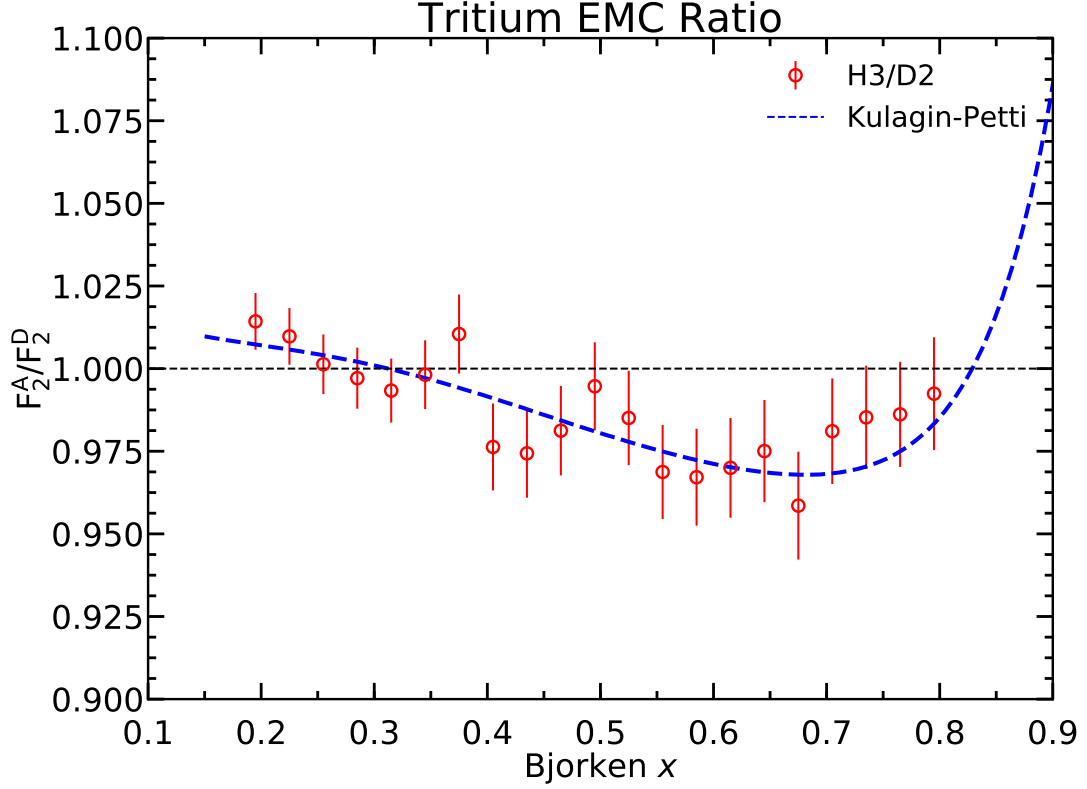


Figure 54: Isoscalar Corrected  $^3\text{H}/^2\text{H}$  cross section ratio from the MARATHON experiment. Error bars include statistical, systematic, and isoscalar uncertainties added in quadrature.

### 6.3 Cross Section Ratio Uncertainties

According to Equation 2.3, the ratio of cross sections reduces to the ratio of  $F_2$  structure functions, resulting in

$$\frac{\sigma_a}{\sigma_b} = \frac{F_2^a}{F_2^b}. \quad (6.9)$$

The advantage of looking directly at the cross section ratio is that many of the systematic errors associated with cross section results will cancel in the direct ratio. Those systematic errors which cancel include the detector, trigger, and tracking efficiencies. Still, not all of the errors will cancel in the ratio, such as those associated

with the systematic corrections discussed earlier, which must be included. These include the target density, charge symmetric background, endcap contamination,  $^3\text{H}$  decay, Coulomb correction, bin centering, and radiative corrections. Uncertainties from corrections such as bin centering were negligible (less than 0.1 %) and were not included. The following section will give a brief overview of the error propagation for the dominant sources of uncertainty.

### 6.3.1 Target Density

The target density correction applied to the data comes from a fit to the normalized yields measured at several different beam current values. The uncertainty for these fits must be included in the overall systematic uncertainty. The form of the fit for both  $^2\text{H}$  and  $^3\text{H}$  is given by:

$$T_{cor} = p_0 + p_1 I + p_2 I^2. \quad (6.10)$$

where  $I$  is the beam current and  $p_0$ ,  $p_1$ , and  $p_2$  are to-be-determined coefficients. The uncertainty in the correction factor is given by:

$$\Delta_{T_{cor}} = \sqrt{\left(\frac{\partial T_{cor}}{\partial p_0}\right)^2 \delta_{00}^2 + \left(\frac{\partial T_{cor}}{\partial p_1}\right)^2 \delta_{11}^2 + \left(\frac{\partial T_{cor}}{\partial p_2}\right)^2 \delta_{22}^2 + \Delta_{cov}}, \quad (6.11)$$

where  $\delta_{00}$ ,  $\delta_{11}$ , and  $\delta_{22}$  correspond to the diagonal elements of the covariance matrix. Likewise,  $\Delta_{cov}$  represents the terms due to the off-diagonal elements, namely  $\delta_{01}$ ,  $\delta_{02}$ , and  $\delta_{12}$ . The uncertainty must be propagated through to the yields via Equation 5.1,

$$\Delta_{Y_{T_{cor}}} = \sqrt{\left(\frac{\partial Y}{\partial T_{cor}}\right)^2} \Delta_{T_{cor}}^2. \quad (6.12)$$

### 6.3.2 Charge Symmetric Background

The charge symmetric background correction is a fit to  $e^+/e^-$  normalized yields at low  $x$  and extrapolated to higher  $x$  values. The fit is of the form:

$$CS = \frac{e^+}{e^-} = e^{Ax+B}. \quad (6.13)$$

where  $A$  and  $B$  are fit coefficients. The uncertainty can be shown to be written as:

$$\Delta_{CS} = \sqrt{\left(\frac{\partial CS}{\partial A}\right)^2 \delta_A^2 + \left(\frac{\partial CS}{\partial B}\right)^2 \delta_B^2 + \left(\frac{\partial CS}{\partial A}\right) \left(\frac{\partial CS}{\partial B}\right) \delta_{AB}}, \quad (6.14)$$

where  $\delta_A$  and  $\delta_B$  are the diagonal terms of the covariance matrix and  $\delta_{AB}$  is the off-diagonal term. Similarly, as in the density correction, the uncertainty is propagated to the yield by

$$\Delta_{Y_{CS}} = \sqrt{\left(\frac{\partial Y}{\partial CS}\right)^2} \Delta_{CS}^2. \quad (6.15)$$

### 6.3.3 Isoscalar Correction

In order to correct for the non-isoscalarity of  $^3\text{H}$ , an isoscalar correction is applied, using the  $F_2^n/F_2^p$  extracted from the MARATHON  $^3\text{H}/^3\text{He}$  cross section ratio. The  $F_2^n/F_2^p$  ratio has an associated error that needs to be included in the  $^3\text{H}/^2\text{H}$  isoscalar corrected cross section ratio. The uncertainty  $\Delta_{f_{iso}}$  is propagated via Equation 6.8 as follows:

$$\Delta_{f_{iso}} = \sqrt{\left(\frac{\partial f_{iso}}{\partial \left(\frac{F_2^n}{F_2^p}\right)}\right)^2 \delta_{F_2^n/F_2^p}^2}. \quad (6.16)$$

## 6.4 Discussion

As mentioned in Chapter 2, it has become commonplace to discuss the EMC effect in terms of the slope of the effect between  $0.3 < x < 0.7$ . The slope provides a unique

comparison among different experiments, each of which has normalization uncertainties which cause an overall shift of the data but will not change the magnitude of the slope. Correlations between the strength of the EMC effect with a series of calculated or experimentally measured quantities have been investigated to illuminate possible connections. These quantities include the atomic mass number ( $A$ ), average nuclear density, scaled average nuclear density, average nucleon separation energy  $\langle \varepsilon \rangle$ , and short-range correlation scaling factor  $a_2$ .

The next step is to extract the slope of the EMC effect of  ${}^3\text{H}$ , and to include it in the correlation plots mentioned above, where available results or calculations for  ${}^3\text{H}$  exists. Figure 55 shows the slope extracted from the Tritium EMC effect. The slope was determined from a linear fit to the data between  $0.35 \leq x \leq 0.7$  and is of the form:

$$y = mx + b, \tag{6.17}$$

where  $m$  is the slope and  $b$  is the intercept of the fit. The extracted slope of the tritium EMC effect is  $-0.105 \pm 0.042$ . The uncertainty of the slope is calculated by adding the fit and target density uncertainties in quadrature. With the extracted slope, one can now compare all the EMC slopes from published data.

For a comparison of experimental results, data from SLAC experiment E139 [27] and JLab experiment E03-103 [66] were used. The results from these experiments are of high quality, with relatively small total uncertainties. One possible source of disagreement when comparing results is that each experiment reports their final EMC cross section ratios with a slightly different isoscalar correction. This difference is mainly due to the choice of  $F_2^n/F_2^p$  parameterization used. In order to ensure consistency, each isoscalar correction was first removed and then applied with a common

isoscalar correction utilizing the MARATHON  $F_2^n/F_2^p$  parameterizations. The slopes were then recalculated between  $0.35 \leq x \leq 0.7$ . As a final note, since there is an overlap of measurements from different experiments, the slopes are shown for each experiment as opposed to combining a weighted average result. This is to visualize the agreement between data sets.

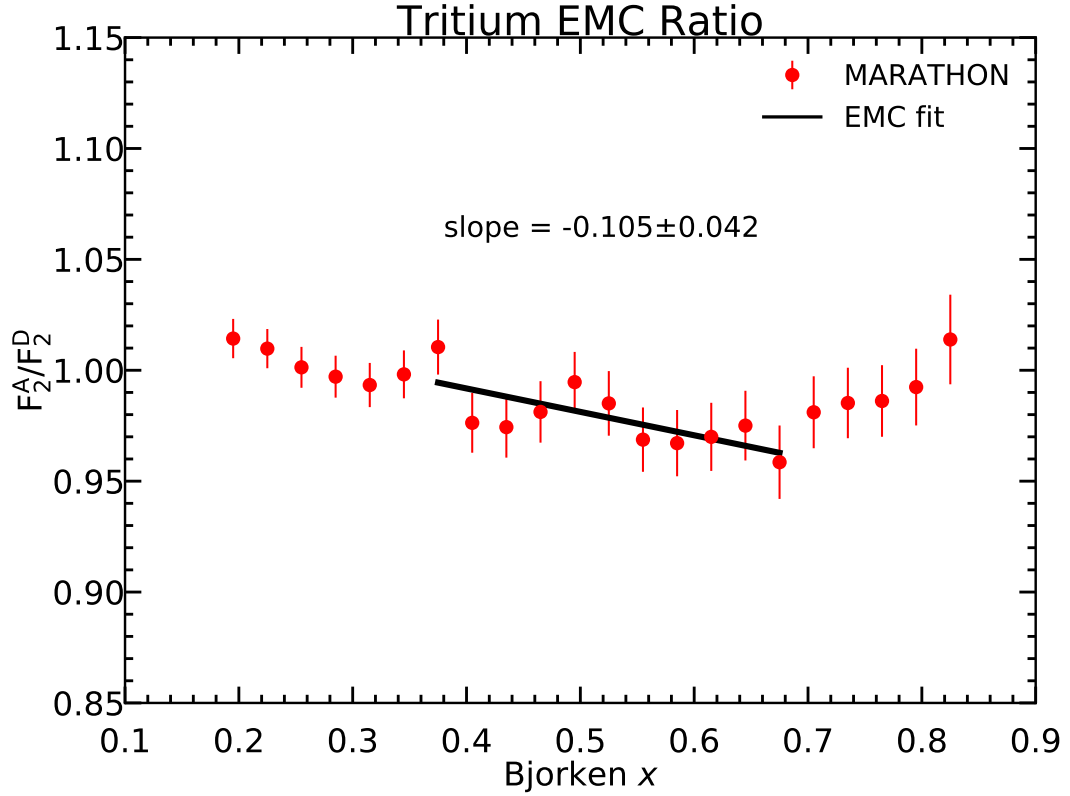


Figure 55: Slope of  $^3\text{H}$  EMC Effect. The grey band signifies the uncertainty of the slope. The uncertainty is calculated from the fit and target density uncertainty added in quadrature.

#### 6.4.1 Atomic Mass Number

Results from experiment E139 at SLAC [27], shown in Figure 8, indicate that the magnitude of the depletion in the EMC region increases with the total number of

protons and neutrons in nuclei, otherwise known as the atomic mass number ( $A$ ). Figure 56 is a plot of the EMC slope versus the atomic mass number. One can see what appears to be a logarithmic dependence of the effect with increasing mass number.

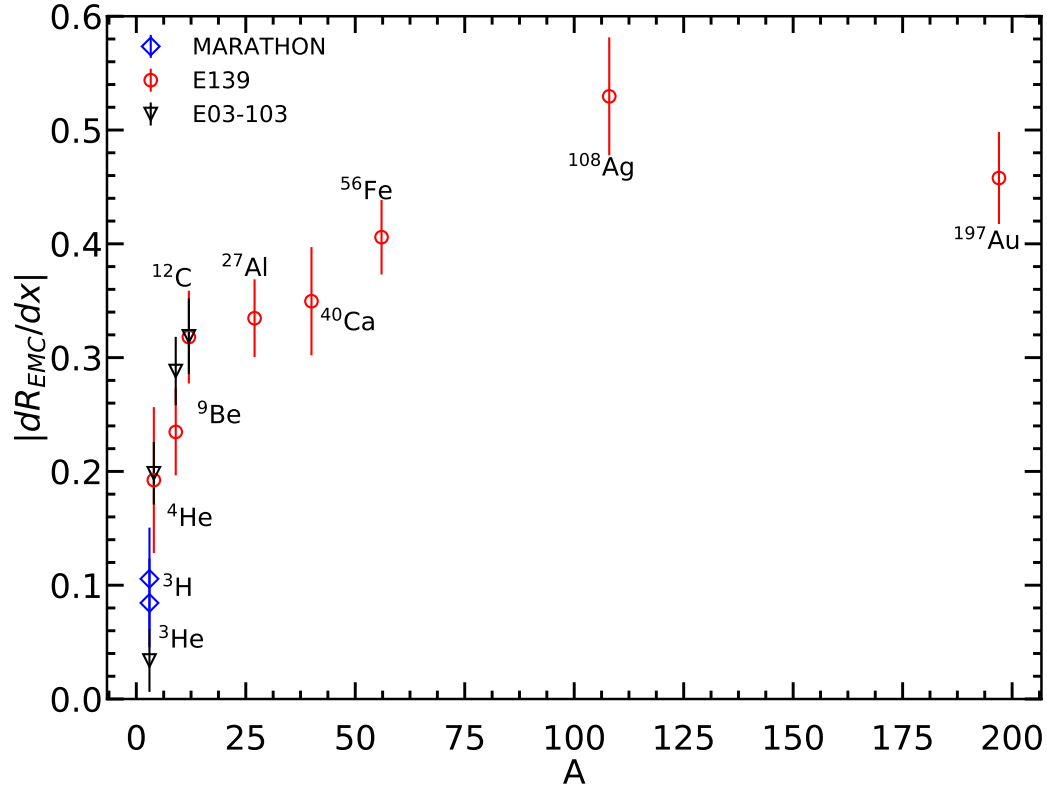


Figure 56: EMC slope versus atomic mass number. Error bars include both statistical and systematic uncertainties. The MARATHON  ${}^3\text{He}$  datum is from Reference [54].

#### 6.4.2 Average Nuclear Density

Beyond the atomic mass number, one can instead consider the EMC effect in terms of the average density of each nucleus. Experiment E139 noted a possible relationship between the EMC effect and the average nuclear density but focused on the entire  $x$

range measured [27]. The average nuclear density,  $\rho$ , is given by

$$\rho = \frac{3A}{4\pi R^3}, \quad (6.18)$$

where  $R$  is the hard-sphere equivalent radius of the nucleus given by Equation 5.21.

Figure 57 shows the EMC slope  $|dR_{EMC}/dx|$  versus the average nuclear density. There does not seem to be good agreement between the slope and the average density. If instead, we look at the “scaled” nuclear density, where the density is scaled by a factor of  $A/(A-1)$ , the correlation appears to improve as shown in Figure 58.

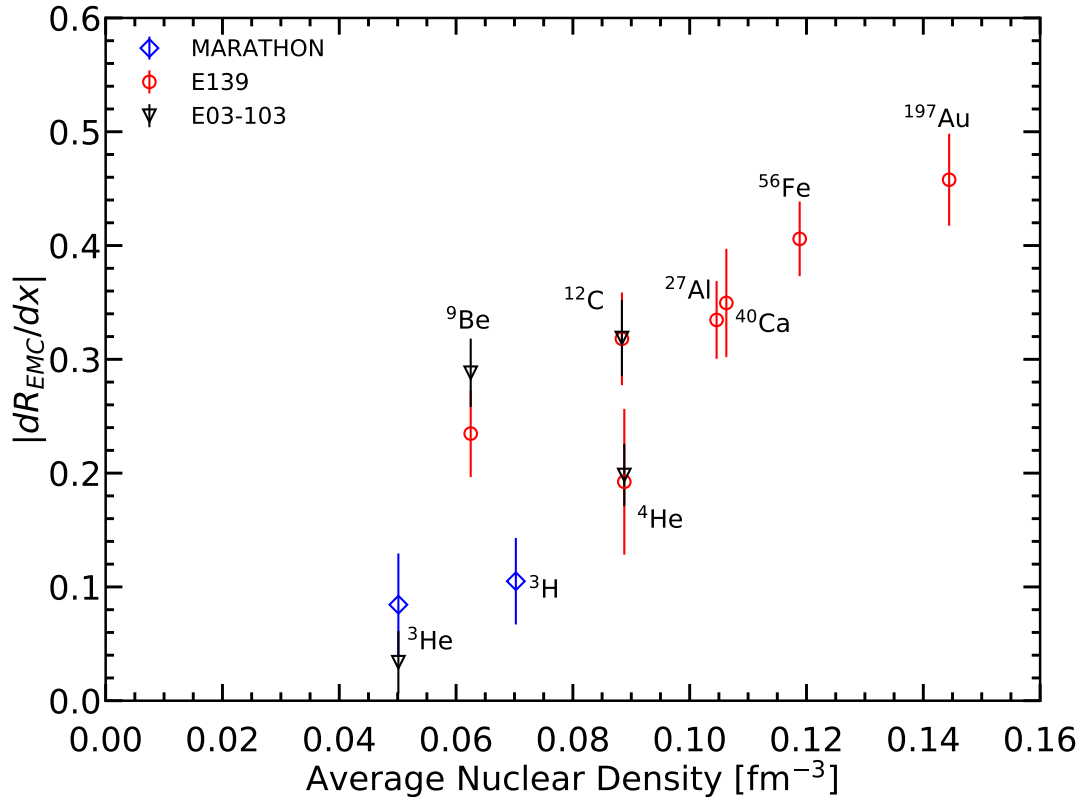


Figure 57: EMC slope versus average nuclear density. Error bars include both statistical and systematic uncertainties. The MARATHON <sup>3</sup>He datum is from Reference [54].

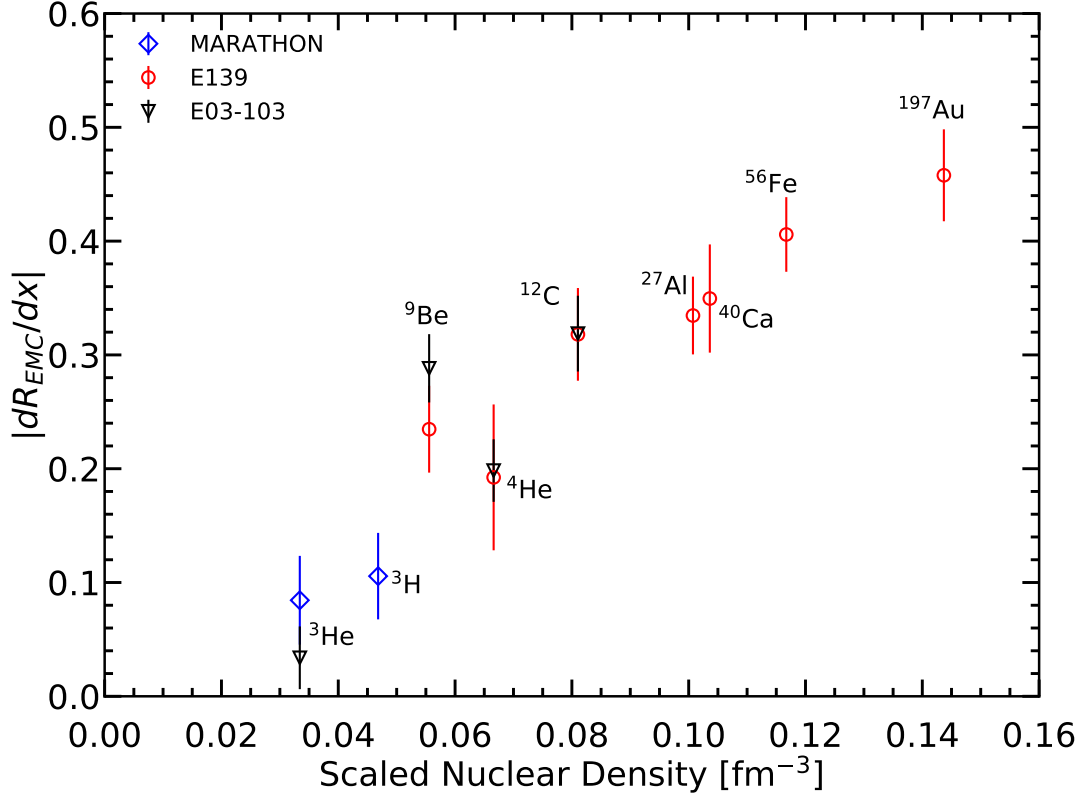


Figure 58: EMC slope versus average scaled nuclear density. Error bars include both statistical and systematic uncertainties. The MARATHON <sup>3</sup>He datum is from Reference [54].

#### 6.4.3 Average Separation Energy

The average nucleon separation or removal energy,  $\langle \varepsilon \rangle$ , is the average amount of energy needed to remove a nucleon from a given nucleus. Calculations of the average separation energy for different nuclei have been performed [64] and Figure 59 shows the relationship between the slope of the EMC effect versus  $\langle \varepsilon \rangle$ . There is currently no available calculation for the average separation energy of <sup>3</sup>H. In Figure 59 the assumption made is that  $\langle \varepsilon \rangle$  for <sup>3</sup>H is the same <sup>3</sup>He. A positive correlation between

the quantities appears to exist. A future calculation of  $\langle \varepsilon \rangle$  for  $^3\text{H}$  could help to provide further evidence of the possible relationship.

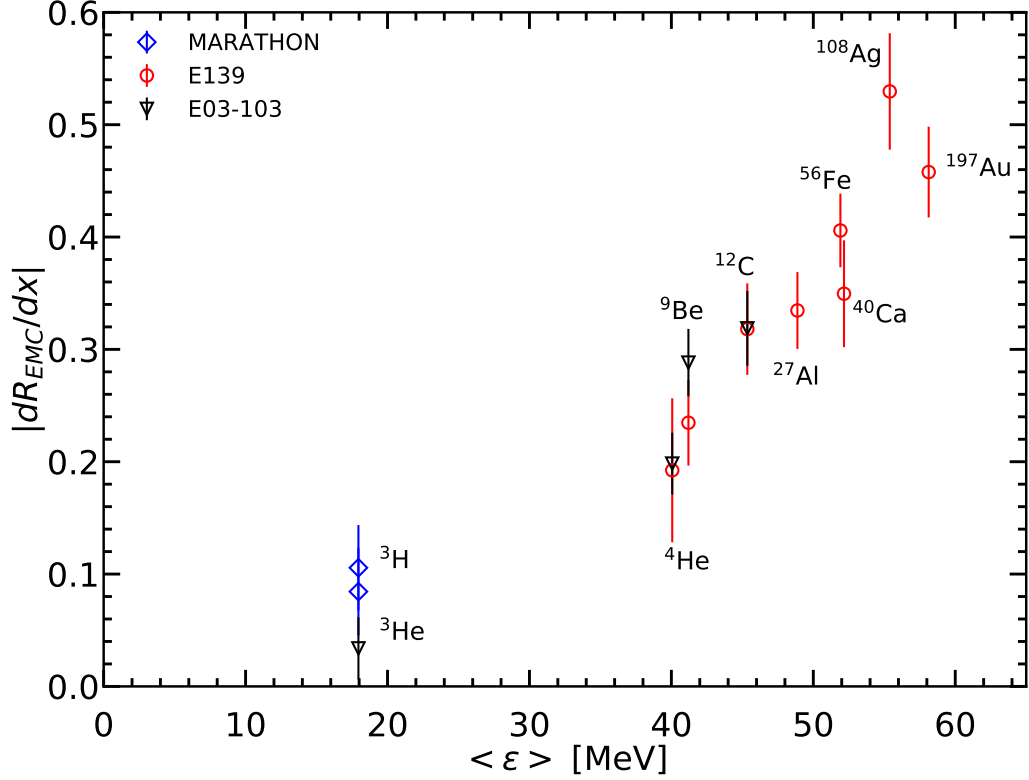


Figure 59: EMC slope versus average nucleon separation energy. Error bars include both statistical and systematic uncertainties. The MARATHON  $^3\text{He}$  datum is from Reference [54].

#### 6.4.4 Short-Range Correlations

The most recent observation has been of the correlation between the slope of the EMC effect and Short-Range Correlations (SRCs). SRCs were discussed in Section 2.2.2. SRC Experiments performed at SLAC [67] and JLab [68] have measured the scaling behavior of nuclei with respect to deuterium, using inclusive quasielastic scattering. The cross section is given by Equation 2.9 in Section 2.2.2, where the ratio of

cross sections corresponds to the probability to find high momentum nucleons in the nucleus [34]. The quantity  $a_2(A)$  is referred to as the “scaling factor”. In the region  $x > 1$ , the cross section ratio in Equation 2.9 plateaus and is independent of  $x$ . Figure 60 shows the compilation of data for the slope of the EMC effect versus the scaling value  $a_2(A)$  [69]. For the purpose of the plot, the  $a_2(A)$  quantity for  $^3\text{H}$  is assumed to be the same as that of  $^3\text{He}$ . A recent experiment at JLab has measured  $a_2(A)$  for  $^3\text{H}$ . With the addition of the EMC slope of  $^3\text{H}$  from MARATHON, the positive correlation between the slope of the EMC effect and  $a_2(A)$  is still apparent.

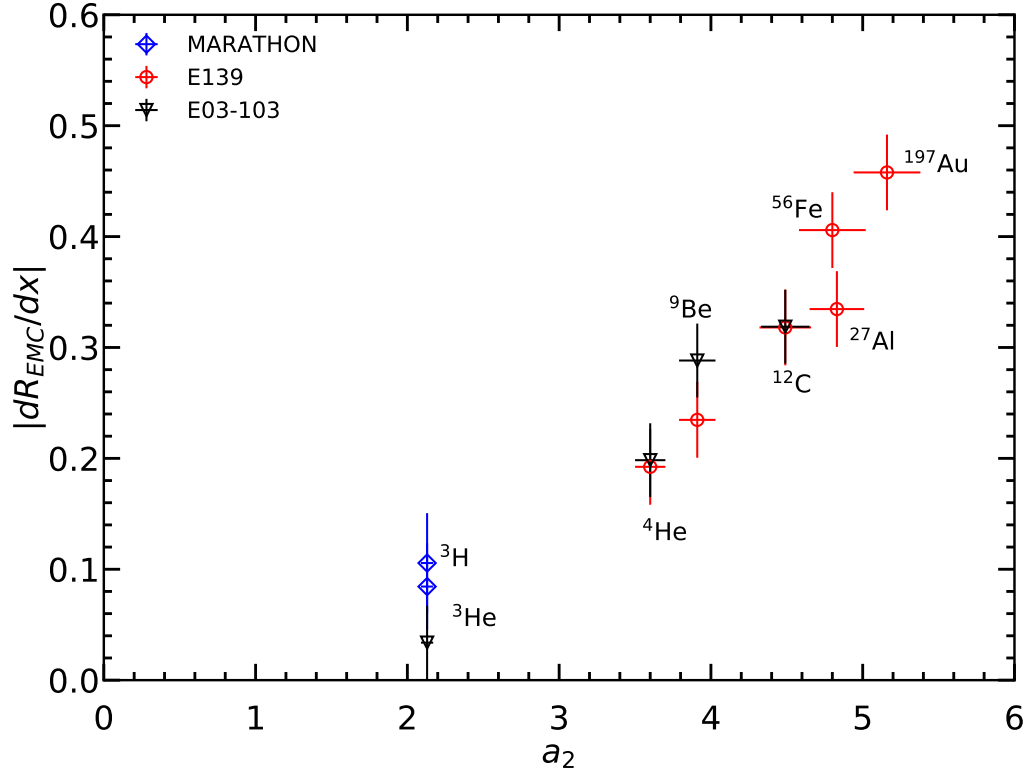


Figure 60: EMC slope versus SRC scaling factor. Error bars include both statistical and systematic uncertainties. The MARATHON  $^3\text{He}$  datum is from Reference [54].

## 6.5 Conclusions

The  $^3\text{H}/^2\text{H}$  deep inelastic cross section ratio was measured in the MARATHON experiment in the Hall A facility of Jefferson Lab, which provided the first measurement of the EMC effect of tritium. The cross section ratio measured over 12 different kinematic settings. The range of  $Q^2$  and  $W$  was  $3 < Q^2 < 12 \text{ (GeV}/c^2\text{)}$  and  $1.8 < W < 3.25 \text{ GeV}/c^2$ , respectively. The cross section results are of high precision with total uncertainties below 2.0%. The addition of the new tritium EMC effect result, along with the complementary helium-3 MARATHON result [54], provides an opportunity to further test the current proposed explanations for the effect. A detailed study was performed by compiling existing results to extract EMC slopes using the MARATHON isoscalar correction. This reinforced the consistency among the existing data sets and provided the opportunity to utilize the newest extraction of  $F_2^n/F_2^p$ , a complementary result from the MARATHON experiment. The tritium result, overall, appears to agree with previous results, but more data are needed to further discern between possible sources of the effect. Finally, future experiments aimed at measuring the spin-dependent EMC effect will be vital in further discerning between the possible sources of the EMC effect, due the differences in the predictions of the effect between the competing models.

## HRS Coordinate System

Each High Resolution Spectrometer in Hall A has a defined coordinate system, referred to as the target coordinate system (TCS) [70]. The description of the TCS for both the LHRS and RHRS is similar and is illustrated in Figure 61 [71].



The TCS is a right handed coordinate system, with the  $\hat{z}_{tg}$  axis pointing in the direction of the central ray of the spectrometer and intersecting with the central sieve hole and the  $\hat{x}_{tg}$  axis pointing vertically down. In this formulation, the in-plane and out-of-plane angles,  $\phi_{tg}$  and  $\theta_{tg}$ , respectively can be expressed as:

$$\begin{aligned}\phi_{tg} &= \frac{dy}{dz} \\ \theta_{tg} &= \frac{dy}{dz} .\end{aligned}\tag{A.1}$$

## APPENDIX B

### KP Model

#### B.1 Kulagin and Petti Model

The model used for comparisons of data ratios as well as used in the determination of systematic corrections applied to data (bin centering and Coulomb correction) was provided by Kulagin and Petti [64]. This model has been extensively compared to the world DIS data and is used to make reliable a prediction for the unknown EMC effect of the  $^3\text{H}$  nucleus. Also, experiments have compared their results to the predictions made by the model [72]. The following will be a brief description of the Kulagin and Pettit (KP) model.

#### B.2 Overview

The model provides a comprehensive treatment of nuclear effects, incorporating contributions from the exchange of pions (strong force), off-shell effects of bound nucleons, and nuclear shadowing due to the possible hadronic intermediate states of the virtual photon ( $\gamma^*$ ) [64]. Deuteron and more complicated nuclei are described in terms of their spectral functions. The dependence on the deuteron wavefunction  $\Phi_D$  was checked by using different wavefunctions. The spectral function for complicated nuclei incorporates the mean-field spectral function ( $P_{MF}$ ) as a model spectral function to account for nucleon-nucleon correlations ( $P_{NN}$ ). The model requires the determination of 3 parameters, related to the nuclear effect corrections mentioned above, which are deduced from fits to combined data sets for the same target ratios

$(F_A/F_D)$ . These parameters should be common to each ratio and a weighted average of the coefficients was found to describe all nuclei. The data sets used for the parameter extraction include data from a number of fixed target experiments, covering a large range in  $x$  and  $Q^2$ . Electron and muon beam experiments were included. The Data sets used came from the NMC, SLAC E139, BCDMS, and FNAL E665 experiments.

## APPENDIX C

### Data

#### C.1 Data Tables

| $x$   | $W^2$<br>(GeV/c <sup>2</sup> ) <sup>2</sup> | $Q^2$<br>[(GeV/c) <sup>2</sup> ] | <sup>3</sup> H/ <sup>2</sup> H | $\Delta_{stat}$ | $\Delta_{syst}$ |
|-------|---------------------------------------------|----------------------------------|--------------------------------|-----------------|-----------------|
| 0.195 | 12.2                                        | 2.75                             | 0.963                          | ±0.003          | ±0.007          |
| 0.225 | 11.6                                        | 3.16                             | 0.953                          | ±0.003          | ±0.006          |
| 0.255 | 11.3                                        | 3.58                             | 0.939                          | ±0.004          | ±0.006          |
| 0.285 | 10.9                                        | 3.98                             | 0.931                          | ±0.004          | ±0.006          |
| 0.315 | 10.5                                        | 4.36                             | 0.922                          | ±0.004          | ±0.006          |
| 0.345 | 10.1                                        | 4.82                             | 0.921                          | ±0.005          | ±0.007          |
| 0.375 | 9.66                                        | 5.23                             | 0.927                          | ±0.007          | ±0.007          |
| 0.405 | 9.25                                        | 5.64                             | 0.891                          | ±0.008          | ±0.007          |
| 0.435 | 8.82                                        | 6.05                             | 0.884                          | ±0.008          | ±0.007          |
| 0.465 | 8.40                                        | 6.59                             | 0.886                          | ±0.009          | ±0.007          |
| 0.495 | 7.98                                        | 6.95                             | 0.894                          | ±0.008          | ±0.007          |
| 0.525 | 7.56                                        | 7.30                             | 0.881                          | ±0.009          | ±0.007          |
| 0.555 | 7.14                                        | 7.83                             | 0.863                          | ±0.009          | ±0.007          |
| 0.585 | 6.71                                        | 8.20                             | 0.858                          | ±0.009          | ±0.007          |
| 0.615 | 6.30                                        | 8.62                             | 0.857                          | ±0.010          | ±0.007          |
| 0.645 | 5.88                                        | 9.07                             | 0.859                          | ±0.010          | ±0.006          |
| 0.675 | 5.45                                        | 9.44                             | 0.842                          | ±0.011          | ±0.006          |
| 0.705 | 5.04                                        | 9.86                             | 0.860                          | ±0.011          | ±0.006          |
| 0.735 | 4.63                                        | 10.5                             | 0.862                          | ±0.010          | ±0.006          |
| 0.765 | 4.21                                        | 10.8                             | 0.862                          | ±0.011          | ±0.006          |
| 0.795 | 3.80                                        | 11.2                             | 0.867                          | ±0.012          | ±0.006          |

Table 10: Raw <sup>3</sup>H/<sup>2</sup>H cross section ratio results and associated uncertainties.

| $x$   | $W^2$<br>(GeV/ $c^2$ ) <sup>2</sup> | $Q^2$<br>[(GeV/ $c$ ) <sup>2</sup> ] | $(^3\text{H}/^2\text{H})_{iso}$ | $\Delta_{Total}$ |
|-------|-------------------------------------|--------------------------------------|---------------------------------|------------------|
| 0.195 | 12.2                                | 2.75                                 | 1.010                           | $\pm 0.009$      |
| 0.225 | 11.6                                | 3.16                                 | 1.010                           | $\pm 0.009$      |
| 0.255 | 11.3                                | 3.58                                 | 1.001                           | $\pm 0.009$      |
| 0.285 | 10.9                                | 3.98                                 | 0.997                           | $\pm 0.010$      |
| 0.315 | 10.5                                | 4.36                                 | 0.993                           | $\pm 0.010$      |
| 0.345 | 10.1                                | 4.82                                 | 0.998                           | $\pm 0.011$      |
| 0.375 | 9.66                                | 5.23                                 | 1.010                           | $\pm 0.012$      |
| 0.405 | 9.25                                | 5.64                                 | 0.976                           | $\pm 0.014$      |
| 0.435 | 8.82                                | 6.05                                 | 0.974                           | $\pm 0.014$      |
| 0.465 | 8.40                                | 6.59                                 | 0.981                           | $\pm 0.014$      |
| 0.495 | 7.98                                | 6.95                                 | 0.995                           | $\pm 0.014$      |
| 0.525 | 7.56                                | 7.30                                 | 0.985                           | $\pm 0.015$      |
| 0.555 | 7.14                                | 7.83                                 | 0.969                           | $\pm 0.015$      |
| 0.585 | 6.71                                | 8.20                                 | 0.967                           | $\pm 0.015$      |
| 0.615 | 6.30                                | 8.62                                 | 0.970                           | $\pm 0.015$      |
| 0.645 | 5.88                                | 9.07                                 | 0.975                           | $\pm 0.016$      |
| 0.675 | 5.45                                | 9.44                                 | 0.959                           | $\pm 0.017$      |
| 0.705 | 5.04                                | 9.86                                 | 0.981                           | $\pm 0.016$      |
| 0.735 | 4.63                                | 10.5                                 | 0.985                           | $\pm 0.016$      |
| 0.765 | 4.21                                | 10.8                                 | 0.986                           | $\pm 0.016$      |
| 0.795 | 3.80                                | 11.2                                 | 0.992                           | $\pm 0.017$      |

Table 11: Isoscalar Corrected  $^3\text{H}/^2\text{H}$  cross section ratio results and associated uncertainty.

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