On the nature of $X(2370)$

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We address the nature of the $X(2370)$ resonance observed in the J/ψ radiative decays, $J/\psi \to \gamma K^+ K^- \eta'$, $J/\psi \to \gamma K_S K_S \eta'$ and $J/\psi \to \gamma \pi^+ \pi^- \eta'$. By studying the invariant mass spectra we confirm that decays of the $X(2370)$ into three pseudo-scalars are well described by an effective chiral Lagrangian. We extract the branching ratio of $J/\psi \to X(2370)\gamma$ and show that it is an order of magnitude larger compared to the glueball production rate predicted by lattice QCD. This indicates that $X(2370)$ is not likely to be a glueball candidate.

Introduction.– Over the past decade important discoveries have been made about strong interactions especially in what regards the spectrum of hadrons. The observation of charged mesons and baryons with hidden charm, *e.g.,* the Z_c [1, 2], and P_c states, [3, 4] indicates the possibility of existence of compact multi-quark bound states that cannot be explained by the quark model. Furthermore, there is a growing evidence that gluons, besides confining quarks, can also act as hadron constituents resulting in quark-gluon *aka* hybrid states or pure glue made glueballs. For a recent review of the hybrid meson signatures and the phenomenological studies addressing the role of gluons as constituents of hadrons see, for example [5–12] and references therein.

Recent analysis of the BESIII data on J/ψ radiative decays to two pseudo-scalars have identified a multitude of iso-scalar states and argument have been put forward that there is a colorless, C-even pure glueball among them [13–15]. Furthermore, the recent observation of the odderon [16], in the high-energy pp and $p\bar{p}$ collisions may be related to existence of a C-odd glueball resonance in the direct channel. It is the gluon compound in J/ψ radiative decaying to three pseudo-scalars that we address in this paper. Lattice QCD (LQCD) predictions of [17–21] place a pseudo-scalar glueball mass above 2 GeV. For example the most recent computation from [21] gives $M_G \simeq 2395 \pm 14$ MeV. However, since these are quenched calculations it is difficult to access the systematic uncertainties.

Glueballs are expected to be produced in radiative decays of the J/ψ [22] because annihilation of the $c\bar{c}$ pair leaves behind a gluon rich component of the J/ψ wave function. Recently, the BESIII collaboration reported several, high statistics measurements of exclusive J/ψ radiative decays. A structure with mass around 2.37 GeV, referred to as the $X(2370)$ has been seen in,

$\pi^+ \pi^- \eta'$ [23] and $\bar{K} K \eta'$ [24] invariant mass distributions. In the former the mass and width of the $X(2370)$ are measured to be $M = 2376.3 \pm 8.7(stat)^{+3.2}_{-4.3}(syst)$ MeV and $\Gamma = 83 \pm 17(stat)^{+44}_{-6}(syst)$ MeV, with the statistic significance of 6.4σ, while in the latter decay, it was found that $M = 2341.6 \pm 6.5(stat) \pm 5.7(syst)$ MeV and $\Gamma = 117 \pm 10(stat) \pm 8(syst)$ MeV with the 8.3σ statistic significance.

There is no first principle method that would enable to distinguish a glueball from other inner components of a physical resonance. The two, however, are expected to have different phenomenological consequences and here we propose to compare the measured J/ψ radiative decay branching ratios to those predicted by LQCD to investigate if the recently observed $X(2370)$ resonance is a good candidate for the pseudo-scalar glueball.

The $X(2370)$ has been considered in previous studies [25, 26] where it was concluded that it may indeed correspond to the pseudo-scalar glueball that was found in the LQCD simulations. To verify this interpretation, it is necessary, however, to consider the production characteristics. Following [25] we postulate the effective interactions between the $X(2370)$ and the light scalar and pseudo-scalar meson resonances that appear to dominate its decay spectrum. The subsequent decays of these resonances are well studied, *e.g.* using the chiral theory [27]. Combining these processes we construct a reaction model to describe the J/ψ radiative decays for all the measured channels that contain the X, $J/\psi \to \gamma K^+ K^- \eta'$, $J/\psi \to \gamma K_S K_S \eta'$, $J/\psi \to \gamma \pi^+ \pi^- \eta'$ and $J/\psi \to \gamma \eta \eta \eta'$. With the branching ratios, $\mathrm{Br}[X \to PPP]$ of X decaying into three pseudo-scalars fixed by the reaction model and the branching ratio for the J/ψ radiative decay in the mass region of the X, $\mathrm{Br}[J/\psi \to \gamma X \to \gamma \bar{K} K \eta']$ determined by the experiment [24], we extract the branching ratio $\mathrm{Br}[J/\psi \to \gamma X]$

$$\mathrm{Br}[J/\psi \to \gamma X] = \frac{\mathrm{Br}[J/\psi \to \gamma X \to \gamma PPP]}{\mathrm{Br}[X \to PPP]} \quad (1)$$

By comparing our results with those of the QCD predictions [21] and the models [25, 26], we can therefore

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distinguish whether the $X(2370)$ is more likely to be a glueball or a $q\bar{q}$ resonance.

Formalism.– The effective Lagrangian for $J/\psi \rightarrow \gamma X$, constrained by chiral and discreet symmetries, *e.g.,* charge conjugation and parity, can be written as

$$\mathcal{L} = g_\gamma \epsilon^{\mu\nu\alpha\beta} D_\mu \psi_\nu F_{\alpha\beta} X \,, \tag{2}$$

where $F_{\alpha\beta}$ is the usual electromagnetic field strength tensor. Following [25] we apply the chiral effective theory to describe the interactions between the X and nonets of scalar, S and pseudo-scalar, P fields which is given by,

$$\mathcal{L} = ig_X X(det\Phi - det\Phi^\dagger)\,. \tag{3}$$

with $\Phi = \Phi_0 + Z_S S(x) + iZ_P P(x)$, where Φ_0 is a constant matrix that contains various condensates and Z_S, Z_P are the wave function renormalization constants, For the pseudo-scalar nonet, P, we take the lightest π, K mesons, and include the η and η' in the standard way as a result of mixing between the η_0 and η_8 [28]. For the scalar nonet, S, there are two sets of resonances with mass below 2 GeV [29] that are relevant. The lighter set is associated with $\{\sigma, \kappa, a_0(980), f_0(980)\}$, and the heavier one with the $\{f_0(1370), K_0^*(1430), a_0(1450), f_0(1500)\}$ resonances. There is phenomenological evidence that the states in the heavier set are dominated by $\bar{q}q$ configurations [27], though a mixing with a glueball can not be neglected. The structure of the lighter scalars is still a mystery, but they have non-ignorable valance quark components [30–33]. Out of these we construct two scalar nonets $S = S_L, S_H$. Specifically, we use the heavier iso-scalar, scalar mesons $f_0(1370)$, $f_0(1500)$ and the $f_0(1710)$, mixed according to the model of [27] to extract the two iso-scalar elements of the heaver multiplet $S = S_H$ as well as the scalar glueball G. The other components of S_L and S_H originate from mixing between the physical states from the lighter set of resonances, $a_0(980)$ and κ for $I = 1$ and $I = 1/2$ respectively and the $a_0(1450)$ and the $K_0^*(1430)$ from the heavier one [27, 34]. The decays of the two nonets of the scalar resonances is described by the chiral effective Lagrangian from [27],

$$\begin{aligned}
\mathcal{L} =& c_d^H \langle S_H u_\mu u^\mu \rangle + c_m^H \langle S_H \chi_+ \rangle + \alpha_H \langle S_H u_\mu \rangle \langle u^\mu \rangle \\
&+ \beta_H \langle S_H \rangle \langle u_\mu u^\mu \rangle + \gamma_H \langle S_H \rangle \langle u_\mu \rangle \langle u^\mu \rangle \\
&+ c_d' G \langle u_\mu u^\mu \rangle + c_m' G \langle \chi_+ \rangle + \gamma' G \langle u_\mu \rangle \langle u^\mu \rangle\,, \\
&+ c_d^L \langle S_L u_\mu u^\mu \rangle + \alpha_L \langle S_L u_\mu \rangle \langle u^\mu \rangle + c_m^L \langle S_L \chi_+ \rangle\,. \tag{4}
\end{aligned}$$

The interactions among the pseudo-scalars are described by the chiral Lagrangian taken from [35, 36], $\mathcal{L} = F^2 \langle u_\mu u^\mu + \chi_+ \rangle/4$. Notice that the vector meson resonances, such as $\rho(770)$, $\omega(782)$, and $\phi(1020)$, do not appear as in the decay of the $X(2370)$. The reason is that the XVP vertex violates the C parity conservation. Similarly, the axial vectors such as $a_1(1260)$, $b_1(1235)$ do not appear as the intermediate states because APP is not allowed by the parity conservation.

As mentioned in the introduction, we need to extract the radiative decay width of $J/\psi \rightarrow \gamma X$ to judge whether the $X(2370)$ is dominated by the glueball. The experiment [24] measured the product of the branching ratios $\mathrm{Br}[J/\psi \rightarrow \gamma X]\mathrm{Br}[X \rightarrow \gamma PPP]$. The total width of the $X(2370)$, irrespective of its nature, can be estimated by summing over all the decay channels with three light pseudo-scalar final states,

$$\Gamma_X(Q^2) = \sum_i \Gamma_{X\rightarrow(PPP)_i}, \tag{5}$$

with the sum ruining over $KK\pi$, $\pi\pi\eta$, $KK\eta$, $KK\eta'$, $\pi\pi\eta'$ $\eta\eta\eta$, $\eta\eta\eta'$, $\eta\eta'\eta'$ [25]. Since $\mathrm{Br}[X \rightarrow PPP]$ is given by a ratio of partial to total widths, the dependence on g_X cancels and it can be predicted directly from the amplitudes constructed from the effective Lagrangians in Eqs. (3,4). Thus the branching ratio $\mathrm{Br}[J/\psi \rightarrow \gamma X]$ for production of the $X(2370)$ can be extracted directly using Eq. (1) with the denominator fixed by the dynamics governing the $X(\rightarrow SP) \rightarrow PPP$ decays and the numerator given by the experiment.

Results and discussions.– The experiment gave the branching ratios of the $J/\psi \rightarrow \gamma X \rightarrow \gamma K^+ K^- \eta'$, $J/\psi \rightarrow \gamma X \rightarrow \gamma K_S K_S \eta'$ and upper limit of the branching ratio of $J/\psi \rightarrow \gamma X \rightarrow \gamma \eta \eta \eta'$. To estimate $\mathrm{Br}[X \rightarrow PPP]$, one needs to fix the couplings in $S \rightarrow PP$. The mixing angles of scalars and that of $\eta - \eta'$, as well as the coupling constants in Eqs. (4) were determined in [27] through analysis of PP production. In that analysis the scalar resonances do not appear as isolated Breit-Wigner amplitudes but decay into PP where the final state interactions (FSI) are taken into account. We perform two analyses in the present paper according to different ways to deal with $S \rightarrow PP$. In what we refer to as Sol. I, the parameters of $S \rightarrow PP$ are taken from [27], and then they are input into the analysis of J/ψ radiative decays. In contrast, in Sol. II, the $S \rightarrow PP$ decays are refitted without the FSI. The results of $\mathrm{Br}[X \rightarrow PPP]$ are shown in Table I. The results of Sol. I are only a slightly differ-

	Sol. I	Sol. II
$\mathrm{Br}[X \rightarrow K^+ K^- \eta']$	$0.50 \pm 0.03 \times 10^{-2}$	$0.50 \pm 0.02 \times 10^{-2}$
$\mathrm{Br}[X \rightarrow K_S K_S \eta']$	$0.24 \pm 0.02 \times 10^{-2}$	$0.24 \pm 0.01 \times 10^{-2}$
$\mathrm{Br}[X \rightarrow \eta\eta\eta']$	$3.27 \pm 0.26 \times 10^{-3}$	$4.62 \pm 0.23 \times 10^{-4}$

TABLE I. Predictions of branching ratios of the $X(2370)$ decays into $K^+ K^- \eta'$, $K_S K_S \eta'$ and $\eta\eta\eta'$.

ent from that of Sol. II, except for the $\mathrm{Br}[X \rightarrow \eta\eta\eta']$. The latter is caused by the difference in $f_0(1370) \rightarrow \eta\eta$ decay widths found between Sol. I and Sol. II with the former almost 4 times larger. From these branching ratios, it is easy to compute the branching ratios of the $J/\psi \rightarrow \gamma X$ directly from the experimental results for $\mathrm{Br}[J/\psi \rightarrow \gamma X \rightarrow PPP]$ through Eq. (1). In practice, to do so we perform a combined fit of $\mathrm{Br}[J/\psi \rightarrow \gamma X \rightarrow$

$\gamma K^+ K^- \eta'$], $\mathrm{Br}[J/\psi \to \gamma X \to \gamma K_S K_S \eta']$ and impose the upper limit of $\mathrm{Br}[J/\psi \to \gamma X \to \gamma \pi^+ \pi^- \eta']$ to extract the single parameter, $\mathrm{Br}[J/\psi \to \gamma X]$, The results are also named as Sols. I and II according to different $S \to PP$ inputs and are shown in Table.II. As can be found, when

	Sol. I	Sol. II	Results from exp. or LQCD
$\mathrm{Br}^{(1)}(10^{-5})$	1.45 ± 0.23	1.99 ± 0.38	$1.79 \pm 0.23 \pm 0.65$ [24]
$\mathrm{Br}^{(2)}(10^{-5})$	0.68 ± 0.11	0.94 ± 0.18	$1.18 \pm 0.32 \pm 0.39$ [24]
$\mathrm{Br}^{(3)}(10^{-6})$	9.20 ± 1.26	1.79 ± 0.25	< 9.2 [37]
$\mathrm{Br}^{\mathrm{tot}}(10^{-3})$	2.87 ± 0.68	3.95 ± 0.71	0.231 ± 0.090 [21]

TABLE II. Predictions of branching ratios of J/ψ radiative decays from our fit. Here the superscripts '1,2,3' represent for the processes of $J/\psi \to \gamma X \to \gamma K^+ K^- \eta'$, $J/\psi \to \gamma X \to \gamma K_S K_S \eta'$ and $J/\psi \to \gamma X \to \gamma \eta \eta \eta'$, respectively. The label 'tot' is for the process of $J/\psi \to \gamma X(2370)$. The $\chi^2_{d.o.f}$ is 0.15 and 0.60 for Sol. I and Sol. II, respectively.

the fitted value for $\mathrm{Br}[J/\psi \to \gamma X]$ is used to compute $\mathrm{Br}[J/\psi \to \gamma X \to PPP]$, the result agrees with the experiment within the experimental uncertainties. In Sol. I, and II, we find $\mathrm{Br}[J/\psi \to \gamma X] = 2.87 \pm 0.68 \times 10^{-3}$ and $\mathrm{Br}[J/\psi \to \gamma X] = 3.95 \pm 0.71 \times 10^{-3}$, respectively, while in quenched LQCD, the pure glueball production rate is found to be $\mathrm{Br}[J/\psi \to \gamma X] = 0.231 \pm 0.090 \times 10^{-3}$ [21], for $M_G = 2.395$ GeV. Our result is almost one order larger than that of LQCD and it implies that the glueball can not be the dominant component of the $X(2370)$.

To further study the nature of the $X(2370)$ and as a check of the amplitude model, we also perform a combined analysis of the branching ratios and the invariant mass spectra in $J/\psi \to \gamma K^+ K^- \eta'$, $J/\psi \to \gamma K_S K_S \eta'$ and $J/\psi \to \gamma \pi^+ \pi^- \eta'$. In particular in Fig.1 we show a sample fit result obtained for $J/\psi \to \gamma K^+ K^- \eta'$ and $J/\psi \to \gamma \pi^+ \pi^- \eta'$ mass spectra. In our analysis we
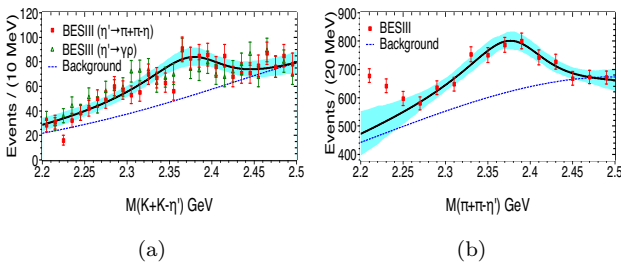

FIG. 1. Combined fit results of $K^+ K^- \eta'$ and $\pi^+ \pi^- \eta'$ invariant mass distributions. The experimental data displayed are from BESIII collaboration, [24] for the $J/\psi \to \gamma K^+ K^- \eta'$, and [23] for the $J/\psi \to \gamma \pi^+ \pi^- \eta'$, respectively. Two η' decay modes, i.e. $\eta' \to \pi^+ \pi^- \eta$ and $\eta' \to \gamma \rho^0$ are displayed for $J/\psi \to \gamma K^+ K^- \eta'$ process with red filled square and green open triangle. Our resluts are shown by the black solid lines with cyan bands. The backgrounds are shown with blue dotted lines.

also include backgrounds, which are caused by processes where thresholds for production of intermediate states

are far away from the $X(2370)$ region, for instance, $J/\psi \to \gamma \eta \eta(\eta') \to \gamma \bar{K} K \eta'$. These contribute with a smooth function of the invariant mass and when subtracted form the overall intensity the contribution of the $X(2370)$ is found to be very well described by a Breit-Wigner resonance with mass and width 2387.8 ± 1.3 MeV, and 119.6 ± 6.7 MeV, respectively. These values are close to those of the experimental analysis from [23, 24]. Finally, the combined analysis gives $\mathrm{Br}[J/\psi \to \gamma X] = 3.26 \pm 0.81 \times 10^{-3}$, very close to that of Sol. II.

A phenomenological analysis of the glueball production rate.– As is known the glueball is a pure state (G), but it mixes with quark components to form a physical states, X. Based on $U(1)_A$ anomaly, the dominant underlying mechanism of pseudo-scalar production in J/ψ radiative decay is via $c\bar{c}$ annihilation into two gluons and a photon [38–40], and the production rate fraction of the physical state X and pseudo-scalar meson η can be expressed as [26],

$$\frac{\mathrm{Br}[J/\psi \to \gamma X]}{\mathrm{Br}[J/\psi \to \gamma \eta]} = \left(\frac{\alpha_X}{\alpha_\eta}\right)^2 \left(\frac{M^2_{J/\psi} - M^2_X}{M^2_{J/\psi} - M^2_\eta}\right)^3. \quad (6)$$

Here α_i, $(i = X, \eta)$ stand for the matrix elements $\langle 0|\alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu}|i\rangle$, $G_{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ denote the gluon field strength tensor and its dual, respectively. We first assume that X is dominated the glueball, i.e. $|X\rangle = |G\rangle$. Then we can use the pure gauge lattice results from [21] for $\mathrm{Br}[J/\psi \to \gamma G] = 0.231 \pm 0.090 \times 10^{-3}$ and for $\alpha_X = \alpha_g = -0.054$ GeV^3[41]. Then using the experimental data for $\mathrm{Br}[J/\psi \to \gamma \eta] = 1.11 \pm 0.03 \times 10^{-4}$ [41], from Eq. (6) we obtain $\alpha_\eta = 0.031$ GeV^3. Now that we have determined α_η, we input this value into the model of [26], which enables to predict the physical matrix elements α_X for the physical state, that is a mixture of the glueball and the $q\bar{q}$ component. If this state was to be identified with the X, we can determine using Eq. (6) the production rate, obtaining $\mathrm{Br}[J/\psi \to \gamma X] = 0.487 \pm 0.143 \times 10^{-3}$. When compared with the result from our analysis of the experimental data, this further confirms our conclusion that the $X(2370)$ is not likely to be a glueball.

Summary.– In this work we constructed $J/\psi \to \gamma K^+ K^- \eta'$, $J/\psi \to \gamma K_S K_S \eta'$, $\gamma \pi^+ \pi^- \eta'$, and $\gamma \eta \eta \eta'$ decay amplitudes using chiral effective Lagrangian. It is found that the $\mathrm{Br}[J/\psi \to \gamma X]$ can be directly extracted from the experiment measurement of $\mathrm{Br}[J/\psi \to \gamma X \to \gamma PPP]$, where the $\mathrm{Br}[X \to PPP]$ is fixed by these amplitudes. The branching ratio of $\mathrm{Br}[J/\psi \to X\gamma]$ is found to be $2.87 \pm 0.68 \times 10^{-3}$ or $3.95 \pm 0.71 \times 10^{-3}$ depending on the choice of parameters and are one order of magnitude larger than that of LQCD, $0.231 \pm 0.090 \times 10^{-3}$, or $0.487 \pm 0.143 \times 10^{-3}$ from a LQCD motivated phenomenology analysis. This is confirmed by a refined analysis of the amplitude model that also considers the invari-

ant mass spectra. Our result is a strong evidence that the $X(2370)$ is not dominated by a glueball component. Future experiments by BESIII at BEPCII and BelleII at superKEKB, which focus on the branching ratios of X decays, would be rather helpful to study the nature of the $X(2370)$.

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