Polarized Antimatter in the Proton from Global QCD Analysis

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We present a global QCD analysis of spin-dependent parton distribution functions (PDFs) that includes the latest polarized W-lepton production data from the STAR collaboration at RHIC. These data allow the first data-driven extraction of a nonzero polarized light quark sea asymmetry $\Delta\bar{u}-\Delta\bar{d}$ within a global QCD framework with minimal theoretical assumptions. Within our simultaneous extraction of polarized PDFs, unpolarized PDFs, and pion and kaon fragmentation functions, we also extract a self-consistent set of antiquark polarization ratios $\Delta\bar{u}/\bar{u}$ and $\Delta\bar{d}/\bar{d}$.

Introduction.— Understanding the detailed decomposition of the proton spin into its constituent quark and gluon helicity and orbital angular momentum components promises to be one of the most significant accomplishments in nuclear and particle physics of this generation [1–3]. While the total light quark contributions to the helicity are well determined from polarized inclusive deep-inelastic scattering (DIS) data [4–17], and jet production in polarized pp collisions [18–25] provides constraints on the gluon helicity [26, 27], far less is known about the polarization of the antiquark sea. There have been some intriguing hints of a polarized antiquark asymmetry, $\Delta\bar{u}-\Delta\bar{d}$, from polarized semi-inclusive DIS (SIDIS) measurements [28–31], in analogy with the spin-averaged $\bar{u}-\bar{d}$ asymmetry inferred from unpolarized DIS and Drell-Yan measurements [32–36]. Various nonperturbative model calculations have also been performed [37–41], some of which predict [39–41] large positive $\Delta\bar{u}-\Delta\bar{d}$ asymmetries.

Recently more probes of antiquark polarization have been possible through W-lepton production in polarized pp collisions. In particular, the STAR [42–44] and PHENIX [45, 46] collaborations at RHIC have used polarized pp collisions at center of mass energy $\sqrt{s}$ = 510 GeV to measure the longitudinal single–spin asymmetry $A_L = (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$, where σ_+ (σ_-) is the cross section for positive (negative) proton helicity, for the leptonic decay channels $W^+ \to e^+\nu$ and $W^- \to e^-\bar{\nu}$. At leading order, these can be written as

$$A_L^{W^+} \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}, \tag{1a}$$

$$A_L^{W^-} \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}, \tag{1b}$$

where Δf (f) represents a polarized (unpolarized) PDF evaluated at momentum fraction x_1 (x_2) carried by the parton in the polarized (unpolarized) proton. Combined with the DIS observables, these asymmetries provide a vital new handle on the extraction of the polarized antiquark distributions $\Delta\bar{u}$ and $\Delta\bar{d}$.

Previous global analyses [26, 47–49] have sought to extract the asymmetry under various assumptions and with different methods for estimating uncertainties. De Florian *et al.* (DSSV) [47] extracted a positive $\Delta\bar{u}-\Delta\bar{d}$ asymmetry from spin-dependent data with fixed input for unpolarized PDFs and fragmentation functions (FFs), assuming PDF positivity and SU(3) symmetry for axial-vector charges within errors. The impact of the latter assumptions was examined in a simultaneous analysis of spin PDFs and FFs by the JAM collaboration [50], who found polarized light antiquark and strange PDFs consistent with zero when the constraints were relaxed. The Monte Carlo analysis by the NNPDF collaboration [49] generated prior samples from the DSSV fit [47], thus inheriting the corresponding biases. The NNPDF analysis also used a reweighting procedure involving χ^2-based weights, which, however, is inconsistent with the Gaussian likelihood used in the generation of the replicas [51].

Instead of relying on reweighting prescriptions and assumptions about PDF positivity or flavor symmetry, here we present a new simultaneous global QCD analysis of unpolarized and polarized PDFs and FFs, including for the first time STAR A_L^W data, along with data on inclusive and semi-inclusive polarized lepton-nucleon DIS and jet production in polarized pp collisions [27]. The Monte Carlo analysis allows us to more reliably quantify the uncertainties on all distributions, and examine the interplay between the sea asymmetry and parametrizations of FFs. The simultaneous determination of both types of PDFs also provides the first self-consistent extraction of the antiquark polarization ratios $\Delta\bar{u}/\bar{u}$ and $\Delta\bar{d}/\bar{d}$.

Theoretical framework.— Our theoretical framework is based on fixed order collinear factorization for high-energy scattering processes, including DIS, Drell-Yan lepton-pair production, and weak boson and jet production in hadronic collisions. The single-spin asymmetry A_L^W has unique sensitivity to both unpolarized and polarized PDFs, giving further motivation for performing a simultaneous analysis of both types of PDFs. The cross section for this process can be written as differential in the lepton pseudorapidity, η_ℓ, and its transverse momentum, p_T^ℓ. The renormalization and factorization scales are chosen to be the mass of the W boson, $\mu_R = \mu_F = M_W$, and the NLO expressions for the hard scattering kernels are found in Ref. [52].

The scale dependence of the PDFs is determined according to the DGLAP evolution equations [53–55], with the PDFs and α_s evolved at next-to-leading logarithmic accuracy with the boundary condition $\alpha_s(M_Z) = 0.118$. For light as well as heavy quarks the PDFs are evolved using the zero-mass variable flavor number scheme. The values of the heavy quark mass thresholds for the evolution are taken from the PDG as $m_c = 1.28$ GeV and $m_b = 4.18$ GeV in the $\overline{\rm MS}$ scheme [56].

Our PDF extraction procedure is based on Bayesian inference using the Monte Carlo techniques developed in previous JAM analyses [50, 57–60]. The parameterization of the unpolarized PDFs is discussed in Ref. [61], while for the polarized PDFs and FFs at the input scale $\mu_0 = m_c$ we use the form

$$f(x, \mu_0) = N x^\alpha (1-x)^\beta (1+\eta x), \qquad (2)$$

where N, α, β, and η are fit parameters. The polarized light quark PDFs Δu and Δd are parameterized as a sum of a valence and a sea component. For the sea quark $\Delta\bar{u}$, $\Delta\bar{d}$, Δs, and $\Delta\bar{s}$ PDFs we use two functions of the form (2), one of which is unique to each flavor while the other describes the low-x region and is shared between all four distributions.

The same template (2) is used for FFs, but with x replaced by the momentum fraction z of the parton carried by the hadron, and with $\eta = 0$. For the π^+ FFs, we assume charge symmetry, $D_u^{\pi^+} = D_{\bar{d}}^{\pi^+}$, $D_d^{\pi^+} = D_{\bar{u}}^{\pi^+}$, as well as $D_q^{\pi^+} = D_{\bar{q}}^{\pi^+}$ for heavier quarks $q = s, c, b$, while for the K^+ FFs we take $D_d^{K^+} = D_{\bar{u}}^{K^+} = D_{\bar{d}}^{K^+}$ and $D_q^{K^+} = D_{\bar{q}}^{K^+}$ for $q = c, b$, but allow the favored $D_u^{K^+}$ and $D_{\bar{s}}^{K^+}$ FFs to differ. The FFs for negatively charged mesons are related by $D_q^{\pi^-/K^-} = D_{\bar{q}}^{\pi^+/K^+}$ for all flavors. We use two shapes each for $D_u^{\pi^+}$, $D_d^{\pi^+}$, $D_u^{K^+}$, and $D_d^{K^+}$, while one shape for all other quark and gluon FFs.

The flexibility in the FF parametrization is necessary to avoid underestimating uncertainties on the polarized sea asymmetry. For example, with this flexibility we do not find any evidence for a nonzero asymmetry with DIS and SIDIS alone, while a positive asymmetry can be manufactured by removing the second shape for $D_u^{\pi^+}$ and assuming symmetric unfavored FFs for s, $\bar{s}$, $\bar{u}$ and d quarks. It is thus vital to assess the correlations between the FF parametrizations and the polarized PDF uncertainties. Adding further flexibility, however, such as $\eta \neq 0$, does not significantly impact the extracted PDFs. Overall, 30 leading twist PDFs and FFs are fitted with a total of 130 parameters. Including parameters for higher twist and off-shell corrections to structure functions, plus data normalizations, brings the number of parameters to 199.

Recently the question of PDF positivity beyond leading order in α_s in the $\overline{\rm MS}$ scheme has been debated [62, 63]. Such a constraint would require $|\Delta f(x, Q^2)| \leq f(x, Q^2)$ to hold for all flavors at all x and Q^2. To explore this question phenomenologically, we perform analyses with and without the positivity constraints. The baseline analysis, referred to in the following as "JAM", does not enforce positivity; however, when included, the positivity constraints are enforced approximately on each Monte Carlo replica by imposing a penalty on the χ^2 function when the bounds are violated [64].

Quality of fit.— Our analysis includes measurements of the DIS asymmetries $A_\parallel$ and A_1 for the proton, deuteron, and ^{3}He from EMC [4], SMC [5, 6], COMPASS [7–9], SLAC [10–15], and HERMES [16, 17]. To ensure the asymmetries are dominated by the leading twist g_1 structure function, with negligible contributions from g_2, we restrict the four-momentum transfer squared to $Q^2 > m_c^2$ and the hadronic final state masses to $W^2 > 10$ GeV^2. With the same cuts we include pion and kaon SIDIS measurements on polarized proton and deuteron targets from HERMES [29] and COMPASS [30, 31], with the fragmentation variable restricted to $0.2 < z_h < 0.8$ to ensure the applicability of the leading power formalism and avoid threshold corrections [60].

Beyond polarized lepton scattering, we describe jet production data in polarized pp collisions from STAR [18–24] and PHENIX [25], with a cut on the jet transverse momentum of 8 GeV [27]. We also include for the first time single-spin asymmetry A_L^W data from STAR [44] and $A_L^{W/Z}$ from PHENIX [45, 46], which provide the most direct constraints on the antiquark polarization. For unpolarized processes, we use data from inclusive DIS, Drell-Yan lepton-pair production, and in-

TABLE I. Summary of χ^2 values per number of points $N_{\rm dat}$ for the various datasets used in this analysis.

process	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
polarized		
inclusive DIS	365	0.93
inclusive jets	83	0.81
SIDIS (π^+, π^-)	64	0.93
SIDIS (K^+, K^-)	57	0.36
STAR $W^\pm$	12	0.53
PHENIX $W^\pm/Z$	6	0.63
total	**587**	**0.85**
unpolarized		
inclusive DIS	3908	1.11
inclusive jets	198	1.11
Drell-Yan	205	1.19
W/Z production	153	0.99
total	**4464**	**1.11**
SIA $(\pi^\pm)$	231	0.85
SIA $(K^\pm)$	213	0.49
total	**5495**	**1.05**

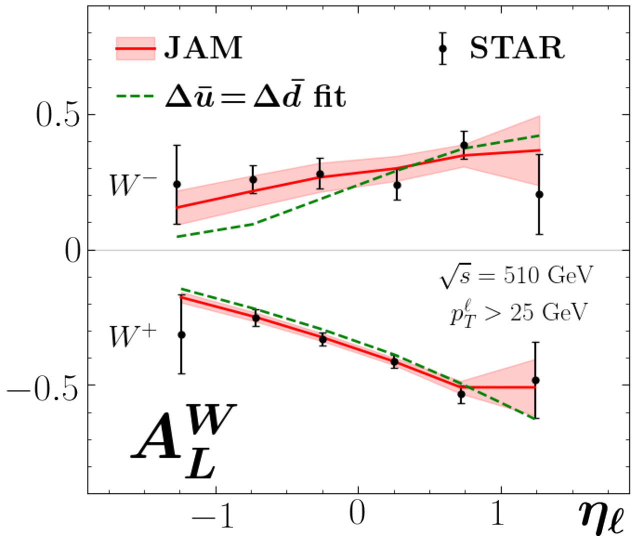

FIG. 1. Single-spin asymmetries A_L^W versus pseudorapidity η_ℓ from STAR [44] (black circles) at $\sqrt{s} = 510$ GeV and integrated over $p_T^\ell > 25$ GeV, compared with the full JAM fit (red solid lines and 1σ uncertainty bands) and with a fit where $\Delta\bar{u}$ is set equal to $\Delta\bar{d}$ (green dashed lines).

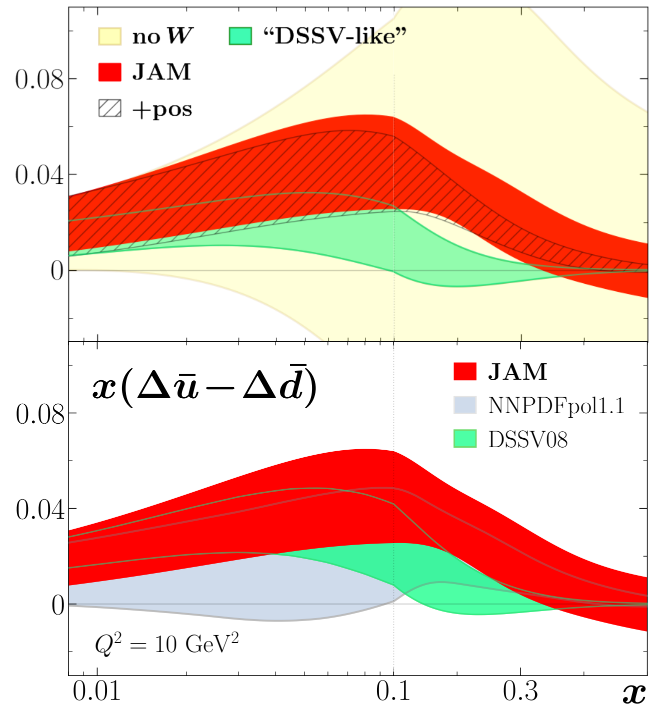

FIG. 2. Polarized sea quark asymmetry $x(\Delta\bar{u} - \Delta\bar{d})$ from JAM (red 1σ bands) at $Q^2 = 10$ GeV^2 compared with: [top panel] fit without RHIC W/Z data (yellow band), the result with positivity constraints (hatched band), and a "DSSV-like" analysis (see text) (green band), and [bottom panel] the NNPDFpol1.1 [49] and DSSV08 [47] analyses.

clusive $W^\pm$, Z and jet production in hadronic collisions, as in Ref. [65]. The FFs are constrained mainly by semi-inclusive pion and kaon production data in e^+e^- scattering, as discussed recently in Ref. [60].

The quality of our global analysis is summarized in Table I, which shows a global $\chi^2/N_{\rm dat} = 1.05$ for $N_{\rm dat} = 5495$ data points (587 for polarized, 4464 for unpolarized, and 444 for SIA). The $\chi^2/N_{\rm dat}$ for each experiment is generally stable whether PDF positivity constraints are imposed or not. When enforcing $\Delta\bar{u} = \Delta\bar{d}$, there are significant increases in $\chi^2/N_{\rm dat}$ for the STAR W data (from 0.53 to 2.51), PHENIX W/Z data at mid rapidity (0.18 to 1.49), and for the COMPASS $A_{1p}^{\pi^-}$ data (0.72 to 1.40, as observed in Ref. [50]). The STAR A_L^W measurement is compared with the JAM fit in Fig. 1 versus the pseudorapidity η_ℓ. When the asymmetry is forced to vanish, the quality of the fit suffers the most for $A_L^{W^-}$ at low η_ℓ. This can be understood from Eq. (1), which shows that the asymmetries are most sensitive to $\Delta\bar{u}$ and $\Delta\bar{d}$ at backward rapidity, where the first terms dominate due to x_2 being large and thus $q(x_2) \gg \bar{q}(x_2)$ for $q = u, d$.

QCD analysis.— The extracted unpolarized PDFs are nearly identical to those from the recent unpolarized JAM analyses [61, 65], while the pion and kaon FFs are consistent with those from Ref. [50]. In this work we focus on the polarized PDFs, extracted from ananalysis of over 900 Monte Carlo samples. The polarized antiquark asymmetry is shown in Fig. 2 and indicates a clear nonzero sea asymmetry for $0.01 < x < 0.3$. The inclusion of positivity constraints significantly reduces the uncertainties at $x \gtrsim 0.1$, since the polarized sea quarks are restricted by the size of the unpolarized sea quarks. Compared to the results without the RHIC W data, our analysis shows that $\Delta\bar{u} - \Delta\bar{d}$ constrained by polarized DIS and SIDIS data alone is consistent with zero, as previously found in Ref. [50]. This further emphasizes the importance of the STAR W data for the extraction of the polarized antiquark asymmetry.

In Fig. 2 we also compare our results to the asymmetries from the DSSV [47] and NNPDF [49] groups. Interestingly, the DSSV analysis [47], which included SIDIS data, found a positive asymmetry for $x \lesssim 0.1$. We find that such an extraction depends strongly on positivity constraints, as well as on the propagation of FF uncertainties, and polarized PDF parametrization choice. To demonstrate this, we have carried out a "DSSV-like" analysis that excludes RHIC W/Z data, imposes positivity constraints and SU(3) symmetry, fixes the FFs to the DSS fit [66], and uses the polarized PDF parametrization from Ref. [47]. These choices, none of which involve the addition of data, greatly reduce the uncertainties on the asymmetry and generate a positive $\Delta\bar{u} - \Delta\bar{d}$ for x below ≈ 0.1, suggesting that the asymmetry from the DSSV analysis [47] may be driven by parametrization bias and theory assumptions rather than by data.

The NNPDF result [49], on the other hand, shows only

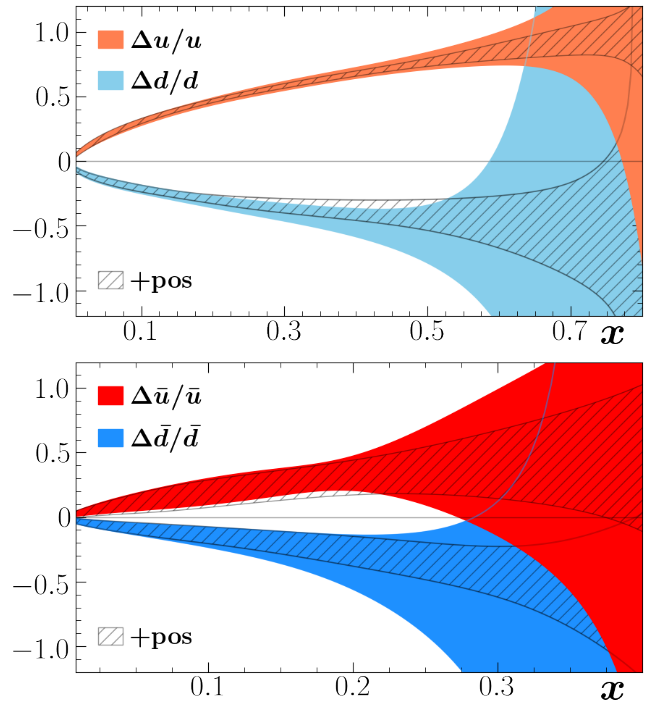

FIG. 3. Light sea quark polarization ratios $\Delta q/q$ at $Q^2 = 10$ GeV^2: [top panel] u and d (coral and skyblue 1σ bands), [bottom panel] $\bar{u}$ and $\bar{d}$ (red and blue 1σ bands), compared with results with positivity constraints (hatched bands).

a slight deviation from zero at high values of x. This is consistent with this fit taking the DSSV result [47] as the prior for $\Delta\bar{u}$ and $\Delta\bar{d}$, but with 4σ uncertainty, and including the older STAR W data [43] in their reweighting analysis. Our analysis is thus the first data-driven extraction of a nonzero polarized antiquark asymmetry.

The results for the light quark polarization ratios $\Delta q/q$ are shown in Fig. 3. As is well known, the polarization is positive for u quarks and negative for d quarks. Without positivity constraints, a nonzero ratio can be extracted for u up to $x \approx 0.8$ and for d up to $x \approx 0.6$. With positivity constraints this is extended further up to $x \approx 0.85$ and $x \approx 0.7$ for u and d, respectively. Given the phenomenological interest in the behavior of $\Delta q/q$ as $x \to 1$ [67–69], our simultaneous extraction of unpolarized and helicity PDFs including the W-lepton data provides the most reliable determination of the ratios to date.

The inclusion of the latest W data also provides unambiguous signs for $\Delta\bar{u}$ and $\Delta\bar{d}$, leading to a positive $\Delta\bar{u}/\bar{u}$ and a negative $\Delta\bar{d}/\bar{d}$, matching their quark counterparts. Without (with) positivity constraints, $\Delta\bar{u}/\bar{u}$ can be distinguished from zero up to values of $x \approx 0.25$ ($x \approx 0.35$), while for $\Delta\bar{d}/\bar{d}$ it can be distinguished from zero up to $x \approx 0.35$ ($x \approx 0.4$). As with the asymmetry, the inclusion of positivity constraints makes little difference below $x = 0.1$ for both the quarks and antiquarks but reduces the uncertainties at larger x.

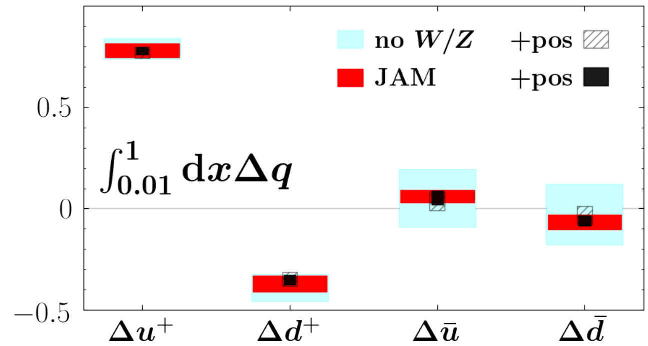

FIG. 4. Truncated integrals $\int_{0.01}^{1} \mathrm{d}x\, \Delta q(x)$ at $Q^2 = 4$ GeV^2 for Δu^+, Δd^+, $\Delta\bar{u}$ and $\Delta\bar{d}$ from this analysis (red rectangles) compared with the fit without the RHIC W/Z data (cyan) and with positivity constraints (small hatched squares without RHIC, and black squares with RHIC). The vertical height of the bands represents 1σ uncertainty.

Finally, in Fig. 4 we show the truncated integral $\int_{0.01}^{1} \mathrm{d}x\, \Delta q(x)$ at $Q^2 = 4$ GeV^2 for the light quarks and antiquarks before and after including the RHIC W data. The lower limit of integration is chosen to roughly match the lower x limit of the data. We see an improvement for Δu^+ and Δd^+ of roughly 25 to 50%, while for the light antiquarks the improvement is as much as 80%. While prior to the inclusion of RHIC W data the sign of the antiquark contributions to the proton spin was unknown, after including these data we find that $\Delta\bar{u}$ ($\Delta\bar{d}$) provides a small but unambiguously positive (negative) contribution to the proton spin. Prior to the inclusion of the RHIC data, the results for $\Delta\bar{u}$ and $\Delta\bar{d}$ depend heavily on the inclusion of positivity constraints. When the RHIC data are included, however, this dependence is significantly reduced, allowing for an extraction that is far less dependent on theoretical assumptions.

Our truncated moments for Δu^+ and Δd^+, with values 0.779(34) and $-0.370(40)$, respectively, are only slightly smaller in magnitude than the corresponding full moments from lattice QCD calculations, which find 0.864(16) for Δu^+ and $-0.426(16)$ for Δd^+ [70]. This comparison suggests that the contributions to the light quark moments below $x = 0.01$ must be small. Interestingly, we note that the contributions from $\Delta\bar{u}$ and $\Delta\bar{d}$ ($+0.061(30)$ and $-0.065(35)$, respectively) approximately cancel in the sum.

Outlook.— Our analysis provides the first data-driven extraction of a nonzero polarized sea asymmetry, using the latest W-lepton data from RHIC, within a simultaneous global QCD analysis of polarized PDFs, unpolarized PDFs, and pion and kaon FFs. This also provides the first self-consistent extraction of the light quark polarizations and shows a nonzero contribution to the proton's spin from the light antiquarks.

With the Jefferson Lab 12 GeV upgrade and the Electron-Ion Collider (EIC), future experiments will access new information on the spin structure of the proton [71, 72]. In particular, the high-luminosity CLAS12 SIDIS experiment using K production [73] will provide precise SIDIS data to complement the W-lepton production data from RHIC. The EIC should bring forth new information on all polarized PDFs, in particular the strange and gluon PDFs [74], while also extending the kinematic coverage of polarized DIS experiments to lower x and higher Q^2.

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