

Detailed Study of Quark-Hadron Duality in Spin Structure Functions of the Proton and Neutron

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Background: The response of hadrons, the bound states of the strong force (QCD), to external probes can be described in two different, complementary frameworks: As direct interactions with their fundamental constituents, quarks and gluons, or alternatively as elastic or inelastic coherent scattering that leaves the hadrons in their ground state or in one of their excited (resonance) states. The former picture emerges most clearly in hard processes with high momentum transfer, where the hadron response can be described by the perturbative expansion of QCD, while at lower energy and momentum transfers, the resonant excitations of the hadrons dominate the cross section. The overlap region between these two pictures, where both yield similar predictions, is referred to as quark-hadron duality and has been extensively studied in reactions involving unpolarized hadrons. Some limited information on this phenomenon also exists for polarized protons, deuterons and ³He nuclei.

Purpose: In this paper, we present for the first time comprehensive and detailed results on the correspondence between the extrapolated deep inelastic structure function g_1 of both the proton and the neutron with the same quantity measured in the nucleon resonance region.

Method: We use a QCD parameterization of the world data on DIS spin structure functions, extrapolated into the nucleon resonance region and averaged over various intervals in the scaling variable x . We compare the results with the large data set collected in the quark-hadron transition region by the CLAS collaboration, averaged over the same intervals. We present this comparison as a function of the momentum transfer Q^2 .

Results: We find that, depending on the averaging interval and the minimum momentum transfer chosen, a clear transition to quark-hadron duality can be observed in both nucleon species. Furthermore, we show, for the first time, the scaling behaviour of g_1 measured in the resonance region at sufficiently high momentum transfer.

Conclusions: Our results can be used to quantify the deviations from the applicability of pQCD for data taken at moderate energies, and help with extraction of quark distribution functions from such data.

I. INTRODUCTION

Quantum Chromo-Dynamics (QCD) is the fundamental theory describing the interactions between quarks and gluons (partons), leading to their observed bound states (hadrons) and the strong nuclear force. At high spatial resolution (momentum scale), the QCD coupling constant becomes small (asymptotic freedom [1, 2]), and quark and gluon interactions can be calculated perturbatively (pQCD). This leads to the emergence of these partons as effective degrees of freedom in the description of hard processes like deep inelastic scattering where the observed cross section can be described approximately as an incoherent sum of scattering cross sections on individual point-like and structureless partons. On the other hand, at low momenta and long distance scales, the interaction becomes strong and a perturbative treatment is no longer possible. Instead, physical processes can be best described in terms of effective hadronic degrees of freedom, *e.g.*, the excitation of resonant hadronic states. By varying the resolution of a probe from short to long distances, physical cross sections displays a transition from the partonic to the hadronic domains. It remains an important question whether there is a region where both pictures apply simultaneously, *i.e.*, whether a parton-based description can reproduce the data in the kinematic region of hadronic resonances, at least on average. This phenomenon is known as Quark-Hadron Duality [3–6]. While strong evidence for duality has been found, it is

important to fully test the applicability of this concept in the case where spin degrees of freedom are present, and for different hadronic systems. If quark-hadron duality can be firmly established and its applicability quantitatively described, one can use measurements of hadronic observables to improve constraints on the parton structure of these hadrons. For instance, measurements of nucleon structure functions that are sensitive to high parton momentum fraction x are very difficult at high energies, which limits our knowledge of the very important behavior of the underlying Parton Distribution Functions (PDFs) as $x \rightarrow 1$. If the requirement of avoiding the region of nucleon resonances can be relaxed in a controlled manner, data taken at lower energies could contribute invaluable information on this asymptotic behavior.

In the present paper, we present new results on tests of duality in proton and neutron spin structure functions. Following this introduction, we introduce the relevant formalism and theoretical concepts, describe the data set we analyzed as well as the phenomenologically extracted spin structure functions from the JAM QCD global analysis to which we compare these data, and then present results and conclusions.

II. THEORETICAL BACKGROUND

In this paper, we focus on quark-hadron duality in polarized inclusive electron scattering off polarized nucleon

targets. In the single photon exchange approximation, an electron with four momentum l scatters with final momentum l' from a nucleon with momentum p by exchanging a space-like virtual photon with momentum $q = l - l'$ (see Fig. 1).

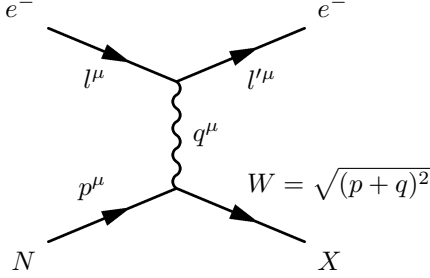


FIG. 1. Feynman Diagram for inclusive electron scattering off a nucleon target. W is the invariant mass of the unobserved final state X . All other symbols are explained in the text.

The invariant cross sections can then be written as [7]

$$E' \frac{d\sigma}{d^3l'} = \frac{2\alpha^2}{sQ^4} L_{\mu\nu} W^{\mu\nu} \quad (1)$$

where $Q^2 = -q^2$ is the virtuality of the exchanged photon, $L_{\mu\nu}$ is the leptonic tensor and $W_{\mu\nu}$ is the hadronic tensor. The latter can be written as a linear combination of unpolarized structure functions $F_{2,L}$ and the polarized structure functions $g_{1,2}$. The polarized structure functions can be experimentally accessed by measuring cross sections differences of the form

$$d\sigma^{\downarrow\uparrow} - d\sigma^{\uparrow\uparrow} \quad (2)$$

where $\downarrow\uparrow$ and $\uparrow\uparrow$ corresponds to anti-parallel and parallel beam and target spin configurations, respectively.

In the kinematics of moderate $x = Q^2/2P \cdot q$ and Q^2 much larger than hadronic mass scales, the g_1 structure function can be approximated in collinear factorization schematically as

$$g_1(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi} \Delta f_{i/N}(\xi, Q^2) \Delta H_i\left(\frac{x}{\xi}, \alpha_S(Q^2)\right) + O\left(\frac{m}{Q}\right). \quad (3)$$

Here the sum runs over all parton flavors i . The term ΔH_i is the target-independent short-distance partonic coefficient function calculable in pQCD in powers of the strong coupling α_S and is convoluted with the spin-dependent Parton Distribution Function (PDF) Δf in the variable ξ . The factorization theorem is valid up to corrections of the order m/Q where m is a generic hadronic mass scales. The ξ variable is the light-cone momentum fraction of partons relative to the parent hadron, *i.e.*, $\xi = k^+/p^+$. At leading order in pQCD, the hard factor ΔH_i is proportional to $\delta(x - \xi)$; hence the structure

function g_1 has a leading order sensitivity to PDFs at $\xi = x$. Beyond the leading order however, the physical structure function receives PDF contributions in the range $x < \xi < 1$ due to the convolution in Eq. 3. The scale dependence on Q^2 in Δf is governed by the DGLAP evolution equations stemming from the renormalization of parton densities and are given as

$$\frac{d\Delta f_i}{d\ln\mu^2}(\xi, \mu^2) = \sum_j \int_\xi^1 \frac{dy}{y} \Delta P_{ij}\left(\frac{\xi}{y}, \alpha_S(\mu^2)\right) \Delta f_j(y, \mu^2) \quad (4)$$

where ΔP_{ij} are the Altarelli–Parisi space-like splitting functions. Finally we remark that the structure function g_2 has no leading power contribution.

Since the focus of our study is the behavior of g_1 in the large- x , moderate Q^2 regime, it is important to utilize a QCD global analysis frameworks that has a maximal kinematical overlap in x to allow us to study duality with minimal extrapolation. In [8], the Jefferson Lab Angular Momentum Collaboration (JAM) carried out a comprehensive analysis of the double spin asymmetries in DIS with an extend kinematic coverage in x and Q^2 . To cover lower Q^2 in this analysis, it was necessary to go beyond the leading power factorization and include additional power corrections (see last expression in Eq. 3) such as higher twist effects as well as target mass corrections using QCD operator product expansion of DIS in the moment space. In the following, we utilize the inferred g_1 from the JAM global analysis that has a kinematic convergence up to $x \sim 0.7$ and use DGLAP backward evolution to access the resonance region at high- x and lower Q^2 .

For moderate final hadronic state masses, $W < 2 - 3$ GeV, the cross section typically exhibits multiple resonance peaks that appear when the target is excited into other baryonic states before later decaying into final state products. This is illustrated in Fig. 2 for the F_2 structure function. This so-called resonance region can be best described in terms of hadronic degrees of freedom, where the cross section is expressed in terms of transition strengths to the various nucleon resonances, together with non-resonant hadron production contributions [9].

It is not a priori obvious how this resonant behavior is related to the underlying degrees of freedom of all hadrons, quarks and gluons, and their description in terms of PDFs, perhaps augmented by higher-twist terms in the OPE. This is addressed by the concept of Quark-Hadron Duality that was first introduced in a publication by Bloom and Gilman in 1970 [3, 4]. They found that the F_2 structure function measured in the nucleon resonance region approaches a smooth “scaling curve” as Q^2 increases, with the resonant troughs and peaks approximately averaging out to match an extrapolation of the deep inelastic structure function at high W into the resonance region (see Fig. 2).

In particular, Bloom and Gilman proposed that integrals over specific ranges in $\omega' = 1/x + M^2/Q^2$ (or just

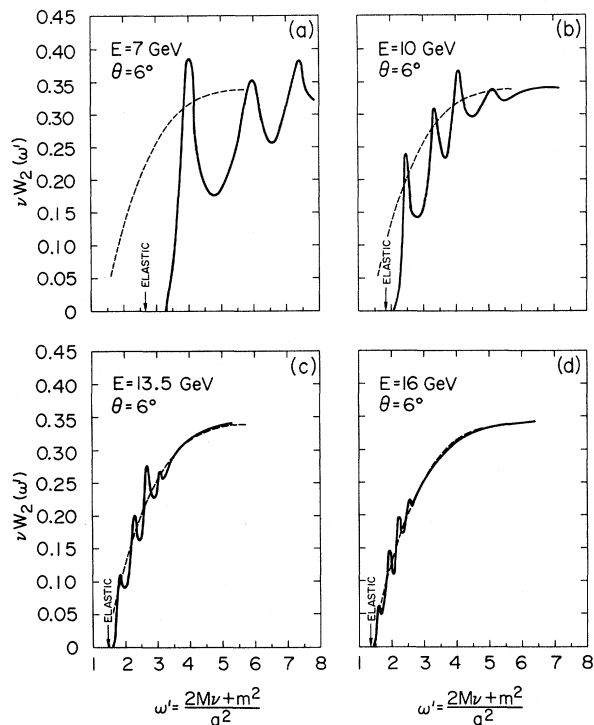


FIG. 2. Schematic dependence of the measured structure function F_2 in inelastic electron scattering off the nucleon on the variable $\omega' = W^2/Q^2 + 1$, which is close to $1/x$ at large Q^2 . Panels (a) through (d) are for increasing four-momentum transfer Q^2 . As can be observed, the resonance excitations of the nucleon are most prominent at low Q^2 , while at higher Q^2 the curve for F_2 approaches the scaling limit (dashed line), hence indicating a transition to quark-hadron duality in this observable. Reproduced from the paper by E. D. Bloom and F. J. Gilman [3], with the permission of AIP Publishing.

over x) of either the extrapolated DIS fits or the experimental data in the resonance region would give similar results. The case where the limits of integration cover only 100-200 MeV on either side of a single resonance peak is referred to as local duality, as opposed to global duality which covers the entire resonance region from threshold to $W = 2$ GeV, potentially also including the elastic peak. In either case, the relation can be summarized as

$$\int_{x_1(W_1, Q^2)}^{x_2(W_2, Q^2)} dx F_2^{res}(x, Q^2) = \int_{x_1}^{x_2} dx F_2^{DIS}(x, Q^2), \quad (5)$$

where F_2^{res} is the structure function **measured** in the resonance region, while F_2^{DIS} is extrapolated from a QCD global analysis. Here,

$$x(W, Q^2) = \frac{Q^2}{W^2 - M^2 + Q^2}. \quad (6)$$

Since the initial discovery by Bloom and Gilman in 1970, considerable progress has been made in the measurement of unpolarized structure functions at low to

moderate Q^2 and W and their interpretation in terms of quark-hadron duality, notably at the Thomas Jefferson National Accelerator Facility (also known as Jefferson Lab) [10–17].

In addition to this, spin dependent structure functions in the same kinematic region have also been studied. Experiments at SLAC in the late 70's provided the first resonance region measurements for polarized proton-electron scattering [18, 19]. These experiments hinted at the applicability of Bloom-Gilman Duality to proton spin structure functions. They were followed in the early 90's by further experiments at SLAC which expanded the g_1 and g_2 measurements to the neutron as well [20] [21]. In the latter half of the 90's, both DESY (the HERMES Collaboration, [22]) and Jefferson Lab (Hall B, [23] and Hall A, [24]) contributed to the investigation of duality in spin structure functions with increased kinematics. The new century brought additional high-precision experiments at Jefferson Lab (Halls A [25–28], B [29–34] and C [35]). Studying quark-hadron duality in the spin sector is important, since polarization dependent observable can have both positive and negative sign, and hence offer a more stringent test of duality. In the present paper, we are presenting a new comparison of the most comprehensive data set on spin structure functions in the transition region between hadronic and partonic degrees of freedom, from the EG1b experiment [32, 33], to the recent JAM QCD global analysis [8] at high x .

III. INPUT DATA

For a detailed study of duality, one needs a dense set of data that cover the entire resonance region (conventionally from $W = 1.072$ GeV to 2 GeV) in fine W bins, for a large number of bins in Q^2 . The most comprehensive such data set was collected by the “EG1b” experiment carried out with CLAS [36] at Jefferson Lab during 2000-2001 [29–33]. The experiment used the polarized electron beam from the Continuous wave Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab, with beam energies of 1.6, 2.5, 4.2, and 5.7 GeV. Together with the large acceptance of CLAS, this set of beam energies yielded a large kinematic reach (with partially overlapping regions), covering nearly 2 orders of magnitude in Q^2 ($Q^2 = 0.06 \dots 5$) and W from threshold to about 3 GeV. A particular advantage of the wide acceptance of CLAS is that the data could be sorted into a pre-determined grid of Q^2 and W , with no need to interpolate between different data points.

The polarized nucleon targets were provided in the form of irradiated frozen ammonia and deuterated ammonia for measurements of proton and deuteron asymmetries, respectively. The target was polarized through Dynamic Nuclear Polarization and reached a polarization along the beam direction of approximately 75% for the protons and 30% for the deuterons [37].

The measured double-spin asymmetries were converted

into spin structure functions $g_1(W, Q^2)$ using a phenomenological fit to the world data on polarized and unpolarized structure functions. In the case of the neutron structure function g_1^n , a folding prescription [38] was used to relate the measured spin structure function of the deuteron to g_1^n for each kinematic point. This yielded the first data set of un-integrated neutron spin structure functions in the resonance region. Details about the experiment, the data analysis and the complete data sets can be found in [32, 33].

Extrapolated pQCD predictions for g_1^p and g_1^n , which are compared to the resonance region data in this paper, are taken from the JAM15 fits [8] of the world data on inclusive spin observables, including the EG1b data *outside* the resonance region (*i.e.*, for $W > 2$ GeV). The JAM fits used a novel iterative Monte Carlo fitting method that utilizes data resampling techniques and cross-validation for a robust determination of the uncertainty band of the fitted PDFs as well as any observables predicted from the fit. A total of 2515 data points from 35 experiments and 4 facilities (CERN, SLAC, DESY, and JLab) were included in the fit.

IV. ANALYSIS

#	Lower W limit	Upper W limit
1	0.939	2
2	1.08	2
3	0.939	1.38
4	1.08	1.38
5	1.38	1.58
6	1.58	1.82
7	1.82	2

TABLE I. W Ranges, in GeV

In this paper, we investigate two different but related tests of duality: a direct comparison between truncated integrals over measured spin structure functions and extrapolated pQCD fits, each covering a specific range in final-state mass W , and a study of the approach to scaling for g_1 averaged over a set of narrow ranges in x .

For the first test, we select seven different ranges of W as shown in table I. The first two of these cover the entire “canonical” resonance region, $W < 2$ GeV, either including (1) or excluding (2) the elastic contribution, to test “global” duality. The remaining 5 ranges cover specific prominent resonance peaks visible in inclusive unpolarized cross section data (see Fig. 2): the Delta resonance ($\Delta(1232)3/2^+$) (again either combined with the elastic peak (3) or without it (4)), the region of the $N(1440)1/2^+$, $N(1520)3/2^-$, and $N(1535)1/2^-$ resonances (5), the region of the $N(1680)5/2^+$ and nearby resonances (6), and the remaining region up to $W = 2$ GeV (7) which does not exhibit a strong peak in the inclu-

sive spin- averaged cross section but is known to contain several Delta-resonances that tend to have negative (virtual) photon asymmetries. We test whether local duality holds in each of these individual resonance regions.

Lower Q^2	Upper Q^2	Central Q^2
0.92	1.10	1.00
1.10	1.31	1.20
1.31	1.56	1.43
1.56	1.87	1.71
1.87	2.23	2.04
2.23	2.66	2.43
2.66	3.17	2.91
3.17	3.79	3.47
3.79	4.52	4.14
4.52	5.40	4.94
5.40	6.45	5.90

TABLE II. Experimental Q^2 Ranges, in GeV^2

For each of these W ranges, our analysis process is the same. Experimental data are first sorted into bins of Q^2 with limits shown in Table II. The extrapolated “pseudo-data” points provided by the JAM collaboration have been calculated from their PDF fits at the central Q^2 values of each bin.

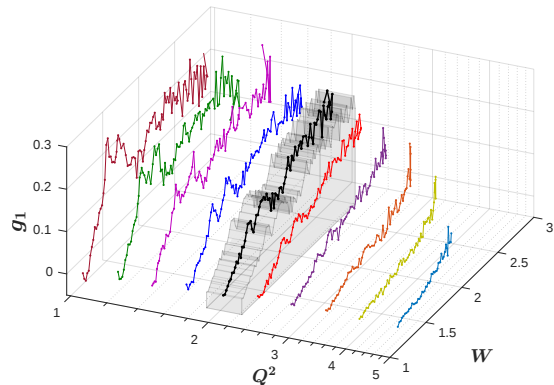


FIG. 3. Representation of the experimental data set used in this analysis. The measured data points are binned in bins in Q^2 , as indicated by the shaded area for the example of the bin $1.87 \text{ GeV}^2 < Q^2 < 2.23 \text{ GeV}^2$; see also Table II. The truncated integrals are then formed over specific regions in W as spelled out in Table I

As a second step, both the measured data and the JAM “pseudo-data” for each Q^2 bin are filtered to only include points within one of the given seven W regions. This is done by mapping the edges of each of our 7 regions to the corresponding values for x , following Eq. 6. Both can then be integrated over the corresponding x -ranges

to yield the truncated first moments of g_1 ,

$$\bar{\Gamma}_1(\Delta W, Q^2) = \int_{x_1(W_1, Q^2)}^{x_2(W_2, Q^2)} dx g_1(x, Q^2),$$

for each of our 7 regions, as a function of Q^2 bin.

For the W regions (1) and (3), the contribution from the elastic peak ($W = M$) was added “by hand” to the measured integrals to extend the truncated integrals up to $x_2 = 1$, since the EGIb data do not reach below a value of $W = 1.072$ GeV. This elastic contribution comes in the form

$$g_1^{el} = \frac{1}{2} \frac{G_E G_M + \tau G_M^2}{1 + \tau} \delta(x - 1),$$

where $G_M = F_1 + F_2$ and $G_E = F_1 - \frac{Q^2}{4M^2} F_2$ are the magnetic and electric Sachs form factors [39]. The corresponding integrals for the JAM “pseudo-data” are simply integrated up to $x = 1$ but don’t contain an elastic contribution (since they are based on DIS pQCD fits).

For the experimental data, the statistical and experimental errors are added in quadrature into the integration and displayed with corresponding error bars. For the JAM fits, we add or subtract one “theoretical standard deviation” to the central values, and the whole range between the corresponding integrals is displayed as band. Results for protons and neutrons are shown together on the same plot.

For our second investigation, we define a sequence of bins in x , each with a width of $\Delta x = 0.05$. The measured g_1 points within each of these x -bins are averaged over each of the Q^2 bins and the averages are plotted *vs.* the nominal Q^2 values. Again, the JAM “pseudo-data” are treated in the same way and shown as bands together with the data.

V. RESULTS

In this Section, we present the results of our two tests of duality. We begin by comparing the integral of g_1 over the entire resonance region (test of “global duality”), see Fig. 4. In the top panel of Fig. 4, we integrate the data over the entire region $0.938 \text{ GeV} < W < 2 \text{ GeV}$, including the elastic peak contribution. Correspondingly, the JAM PDF fit is integrated up to $x = 1$. For the proton (upper bands and data points), a clear (and non-trivial) agreement between data and PDF prediction is observed, starting around $Q^2 = 1.4 \text{ GeV}^2$. Initially, the data agree best with the JAM prediction that includes higher twist contributions and target mass corrections (solid band), but at higher Q^2 they tend to be a bit lower and closer to the striped band (which includes only leading twist). For the neutron (lower bands and data points), the predictions from the PDF fits as well as the data are mostly consistent with zero, and there is little difference between the extrapolation including higher twist and target mass

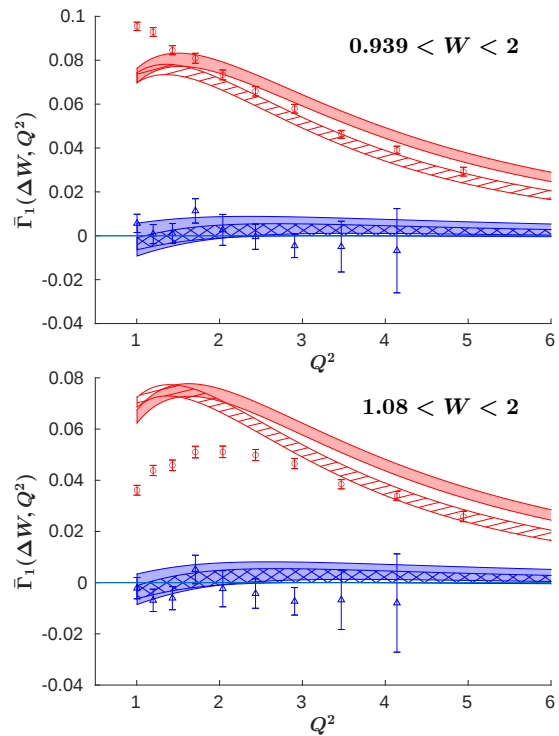


FIG. 4. [Color Online] Test of global duality. We show integrals $\bar{\Gamma}_1(\Delta W, Q^2)$ of the spin structure function g_1 over the entire resonance region, as a function of Q^2 . In the top panel, we include the elastic peak: $0.938 \text{ GeV} < W < 2 \text{ GeV}$, while the bottom panel shows only the inelastic part, $1.072 \text{ GeV} < W < 2 \text{ GeV}$. The top (red) bands and data points (circles) are for the proton, and the bottom (blue) bands and data points (triangles) are for the neutron. The data points are shown with statistical and systematic uncertainties added in quadrature (error bars). The solid bands show the full prediction from the extrapolated JAM fit, including target mass and higher twist contributions. The striped band (proton) and the cross-hatched band (neutron) show the results including only the leading twist contribution.

corrections (solid band) and the one including only leading twist (cross-hatched band). There may be a slight tendency for the data to fall below the PDF bands at high Q^2 , which would agree with the observation that the d -quark polarization appears to remain negative up to the highest x measured [25]; however, the experimental uncertainties are too large to make a definite statement.

We repeat our test of “global duality” by also comparing the integrals excluding the elastic contribution (and, correspondingly, the region $x \rightarrow 1$), see lower panel of Fig. 4. While the prediction based on the extrapolated PDF fit changes only slightly (as expected, since $g_1 \rightarrow 0$ as $x \rightarrow 1$), we see that the data for the proton, excluding the elastic contribution, fall short of that expectation up to rather high $Q^2 > 3.5 \text{ GeV}^2$. (The agreement for the neutron is only slightly worse). This finding indicates that the elastic contribution must be included to get a fairly rapid convergence (low threshold) for global duality. This is due to the need to compensate for the rather

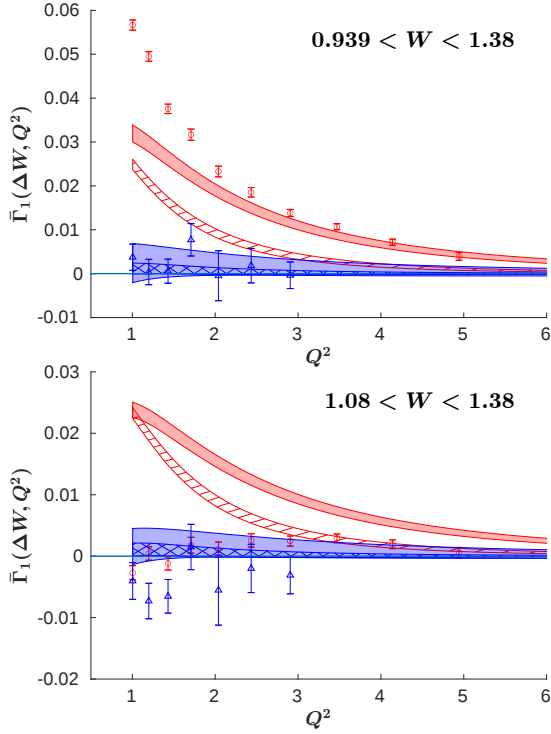


FIG. 5. [Color Online] Test of global duality in the region of the Δ resonance. Again, in the top panel we include the elastic peak: $0.938 \text{ GeV} < W < 1.38 \text{ GeV}$, while the bottom panel shows only the inelastic part $1.072 \text{ GeV} < W < 1.38 \text{ GeV}$. All symbols are the same as in Fig. 4

striking breakdown of “local duality” in the region of the Delta-resonance.

This breakdown can be seen in Fig. 5. Here, we integrate only over the peak of the lowest-lying Delta resonance, either with the addition (top panel of Fig. 5) of the elastic peak, or without it (bottom panel of Fig. 5). In the latter case, both the proton and the neutron data are either negative or close to zero, while the PDF extrapolation for both is positive (significantly so for the proton). Again, agreement between proton data and PDF extrapolation only begins around $Q^2 > 3 \text{ GeV}^2$. This is due to the well-known fact that the excitation of the Delta resonance is dominated by a $M1$ transition, for which the final state helicity $3/2$ has a stronger coupling than the final state helicity $1/2$, leading to a negative (virtual) photon asymmetry A_1 and, in consequence, a negative value for g_1 . Interestingly, this stark deviation from the PDF extrapolation is more than compensated by adding in the elastic peak (top panel of Fig. 5), which has $A_1 = 1$ by definition. In that case, the proton data overshoot the pQCD prediction, while the neutron data are largely in agreement.

The next two proton resonance regions (Figs. 6 and 7) show remarkably good agreement between the data and the extrapolated PDF bands (in particular the extrapolations including higher twist), indicating that “local duality” works well for these resonances. The remaining

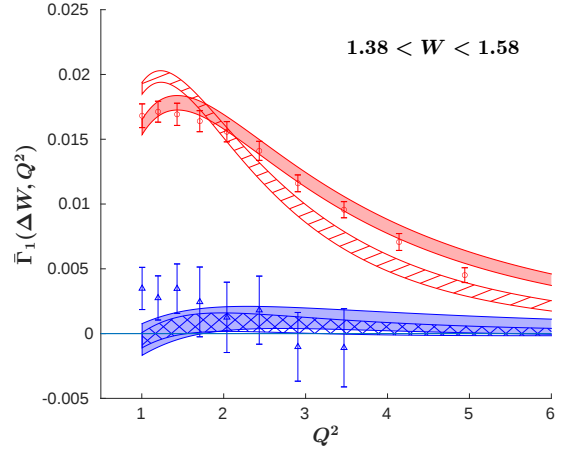


FIG. 6. [Color Online] Local duality test in the region of the $N(1440)1/2^+$, $N(1520)3/2^-$, and $N(1535)1/2^-$ resonances. All symbols as in Fig. 4.

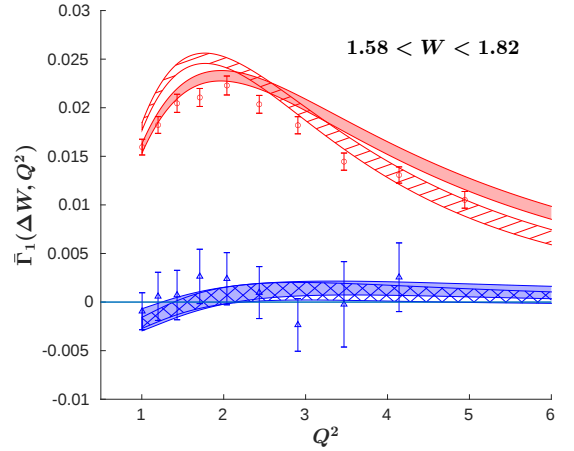


FIG. 7. [Color Online] Local duality test in the region of the $N(1680)5/2^+$ resonance. All symbols as in Fig. 4.

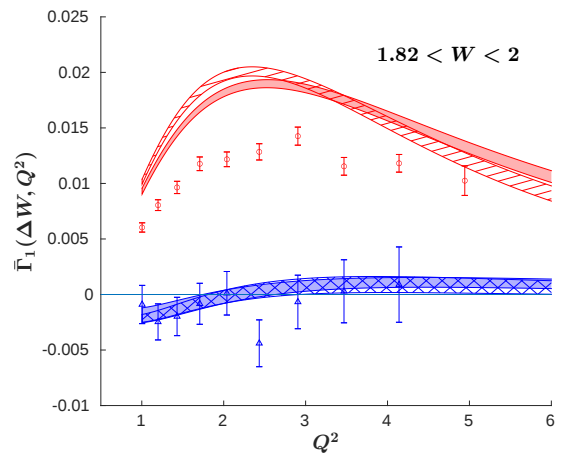


FIG. 8. [Color Online] Local duality test for the remainder of the resonance region, $1.82 \text{ GeV} < W < 2 \text{ GeV}$. All symbols as in Fig. 4.

region, up to $W = 2$ GeV, shows again a deviation of the data which tend to lie below the PDF fits. Once again, this is consistent with the assumption that this region has a strong contribution from various Delta-resonances, where the helicity-3/2 contribution dominates at small Q^2 . It is remarkable, coming back to Fig. 4, how the negative deviations in the lowest and highest W regions (both populated by Delta resonances) are compensated by the inclusion of the elastic peak to get a rather rapid approach to “global duality”.

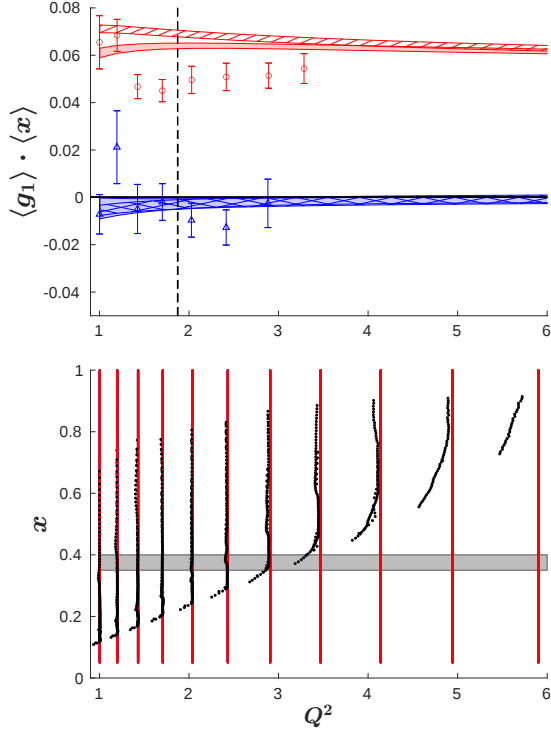


FIG. 9. [Color Online] Approach towards the scaling limit for the structure function $g_1(x, Q^2)$, averaged over the x -bin of $0.35 < x < 0.4$, as a function of Q^2 bin. Top: Data and JAM bands are shown multiplied with the average $x = 0.375$; symbols are as in Figs. 4-8. The vertical dashed line indicates the limit $W = 2$ GeV of the resonance region, which lies to the left. Bottom: kinematic location of all data points from EG1b in the x vs. Q^2 plane. The grey band indicates the interval in x over which the data are averaged, and the red vertical lines indicate the nominal central values of each Q^2 bin.

In our second analysis, we are integrating the EG1b data and JAM PDF fits over fixed intervals in x for each of our Q^2 bins, to study the approach towards scaling for the structure function $g_1(x, Q^2)$. The integrals are divided by the bin width $\Delta x = 0.05$ to obtain the average g_1 and then multiplied by the bin centroid in x for better visibility - see Figs. 9-10. In contrast to the previous analysis, we include in these figures *all* data from EG1b, from both the resonance and the DIS region, with the boundary between the two indicated by the vertical dashed line at $Q^2 = (W^2 - m^2)/(1/x - 1)$, with $W = 2$

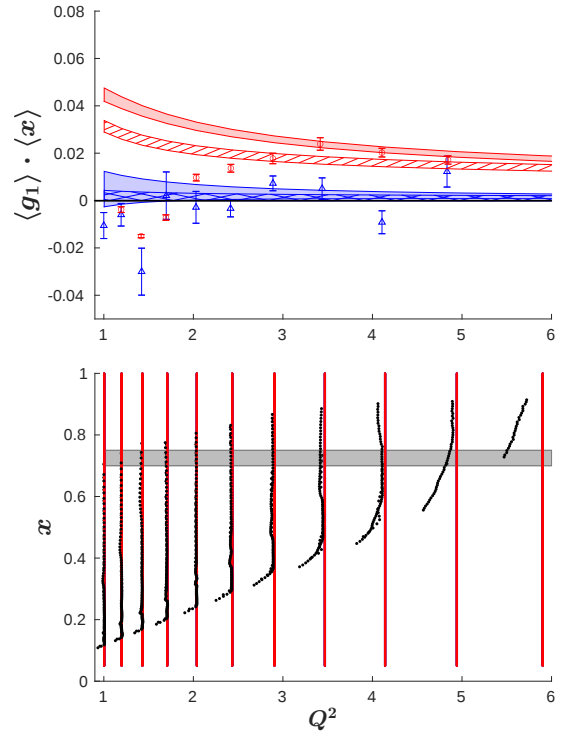


FIG. 10. [Color Online] Same as Fig. 9 except for the bin $0.7 < x < 0.75$.

GeV. The bottom half of each figure shows all of the EG1b data points, where the points included in each integral are those lying within the grey bands.

For the lower x bin (Fig. 9), both the proton data and the neutron data approach a smooth, largely flat behavior at increasing Q^2 , indicating the onset of scaling roughly around $Q^2 = 2$ GeV² which corresponds to $W > 2$ GeV. As in the earlier duality studies, the neutron data have larger uncertainties and are fully consistent with the expectations from the JAM PDF results, while the convergence of the proton data appears to be a bit slower, and they are closer to the bands including higher twist. In both cases there is no strong resonance structure visible beyond $W = 2$ GeV, while the proton data clearly exhibit some non-trivial structure below $W = 2$ GeV. Since in this case, we are not integrating over any particular structure in the resonance region but over a fixed x bin, different resonances contributed differently to each data point, spoiling the “local duality” agreement observed above for some of the prominent resonances.

In contrast to the above, the data for the x -range $0.7 < x < 0.75$ are all inside the resonance region, since the kinematic reach of EG1b (with a maximum beam energy of 5.7 GeV) was not sufficient to reach the limit $W = 2$ GeV for this rather high x . Both the proton and the neutron data show prominent resonance structure at low Q^2 , while approaching a more smooth behavior (and the JAM PDF predictions) at moderately higher Q^2 , above $Q^2 \approx 3$ GeV². This corresponds to an average W of only 1.4 GeV, still well into the resonance region. Thus

it appears as if the approach to scaling may set on early at larger x values, which would be very beneficial for the goal of extracting the behavior of spin structure functions at large x , a topic of continuing high interest [40]. For tables for all x ranges and for all data plotted in Figs. 4-8 see the Supplemental Material at [URL will be inserted by publisher].

VI. CONCLUSION

In this paper, we present the most detailed study of quark-hadron duality in the spin structure function g_1 to date, for both the proton and the neutron. We study several different formulations of duality, and find that duality seems to hold much better (at smaller momentum transfer) in some cases than in others. In particular, we conclude the following:

- When forming integrals over kinematic regions corresponding to specific resonance peaks, we observe good agreement between the measured data and the extrapolation from pQCD fits whenever several resonances with different spins contribute, *i.e.* the for the regions $W = 1.38 \text{ GeV} \dots 1.58 \text{ GeV}$ (including the $N(1440)1/2^+$, $N(1520)3/2^-$, and $N(1535)1/2^-$ resonances) and $W = 1.58 \text{ GeV} \dots 1.82 \text{ GeV}$ (several higher-lying resonances). In contrast, in the region dominated by the ground-state Delta-resonance ($W = 1.08 \text{ GeV} \dots 1.38 \text{ GeV}$) and the region $W = 1.82 \text{ GeV} \dots 2 \text{ GeV}$ with several Delta resonances, we observe a much slower approach of the measured integrals towards the extrapolated PDF fits with Q^2 . This is likely due to the fact that, at least at moderately low Q^2 , for the excitation of Delta resonances the transition to final-state helicity $3/2$ dominates.
- If we integrate over the entire resonance region up to $W = 2$, including the elastic peak at $W = 0.938 \text{ GeV}$, a rather rapid convergence towards the ex-

trapolated PDF fits is observed: Global duality seems to work in spin structure functions.

- If instead we integrate over a fixed bin in x , with different resonances contributing at different Q^2 , we find that for lower values of x , the transition with Q^2 towards a smooth scaling curve occurs only if the value of Q^2 is high enough so that $W > 2 \text{ GeV}$. Conversely, for the highest x values, we observe that the approach towards a smooth scaling curve (and the extrapolated PDF fits) occurs even below $W = 2 \text{ GeV}$, albeit at a higher Q^2 . This may be due to the fact that at higher Q^2 , resonant and non-resonant contributions with different asymmetries average out, leading to a “precocious” approach to scaling (or a different form of local duality). This observation supports the idea that, for high enough x and Q^2 , even data in the resonance region may be used to constrain (polarized) parton distribution functions. Being able to include data in the resonance region and *a fortiori* at moderate $W^2 < 10 \text{ GeV}^2$ - a limit often imposed on PDF fits - will help with the goal to pin down more precisely the quark polarization of both types of valence quarks in the limit $x \rightarrow 1$, which is still an open question at this time.

ACKNOWLEDGMENTS

We would like to thank our collaborators on CLAS experiment EG1b and the members of the JAM collaboration for their help with the data presented. We are particularly indebted to W. Melnitchouk for his valuable comments and to J. Ethier, who provided us with the extrapolated JAM results. This work was supported by the Department of Energy under Contract DE-FG02-96ER40960.

VII. BIBLIOGRAPHY

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- [1] D. J. Gross and F. Wilczek, Ultraviolet Behavior of Nonabelian Gauge Theories, *Phys. Rev. Lett.* **30**, 1343 (1973).
 - [2] H. Politzer, Reliable Perturbative Results for Strong Interactions?, *Phys. Rev. Lett.* **30**, 1346 (1973).
 - [3] E. D. Bloom and F. J. Gilman, Scaling, Duality, and the Behavior of Resonances in Inelastic electron-Proton Scattering, *Phys. Rev. Lett.* **25**, 1140 (1970).
 - [4] E. D. Bloom and F. J. Gilman, Scaling and the Behavior of Nucleon Resonances in Inelastic electron-Nucleon Scattering, *Phys. Rev. D* **4**, 2901 (1971).
 - [5] A. De Rujula, H. Georgi, and H. Politzer, Demythification of Electroproduction, Local Duality and Precocious Scaling, *Annals Phys.* **103**, 315 (1977).
 - [6] W. Melnitchouk, R. Ent, and C. Keppel, Quark-hadron duality in electron scattering, *Phys. Rept.* **406**, 127 (2005), arXiv:hep-ph/0501217.
 - [7] J. Collins, *Foundations of perturbative QCD*, Vol. 32 (Cambridge University Press, 2013).
 - [8] N. Sato, W. Melnitchouk, S. Kuhn, J. Ethier, and A. Accardi (Jefferson Lab Angular Momentum), Iterative Monte Carlo analysis of spin-dependent parton distributions, *Phys. Rev. D* **93**, 074005 (2016), arXiv:1601.07782 [hep-ph].
 - [9] A. N. Hiller Blin *et al.*, Nucleon resonance contributions to unpolarised inclusive electron scattering, *Phys. Rev.*

- C **100**, 035201 (2019), arXiv:1904.08016 [hep-ph].
- [10] I. Niculescu *et al.*, Experimental verification of quark hadron duality, Phys. Rev. Lett. **85**, 1186 (2000).
 - [11] I. Niculescu *et al.*, Evidence for valencelike quark hadron duality, Phys. Rev. Lett. **85**, 1182 (2000).
 - [12] M. Osipenko *et al.* (CLAS), A Kinematically complete measurement of the proton structure function F_2 in the resonance region and evaluation of its moments, Phys. Rev. D **67**, 092001 (2003), arXiv:hep-ph/0301204.
 - [13] I. Niculescu *et al.*, Direct observation of quark-hadron duality in the free neutron F_2 structure function, Phys. Rev. C **91**, 055206 (2015), arXiv:1501.02203 [hep-ex].
 - [14] M. E. Christy and P. E. Bosted, Empirical fit to precision inclusive electron-proton cross- sections in the resonance region, Phys. Rev. C **81**, 055213 (2010), arXiv:0712.3731 [hep-ph].
 - [15] V. Tvaskis *et al.*, Measurements of the separated longitudinal structure function F_L from hydrogen and deuterium targets at low Q^2 , Phys. Rev. C **97**, 045204 (2018), arXiv:1606.02614 [nucl-ex].
 - [16] S. P. Malace *et al.* (Jefferson Lab E00-115), Applications of quark-hadron duality in F_2 structure function, Phys. Rev. C **80**, 035207 (2009), arXiv:0905.2374 [nucl-ex].
 - [17] S. P. Malace, Y. Kahn, W. Melnitchouk, and C. E. Keppel, Confirmation of quark-hadron duality in the neutron F_2 structure function, Phys. Rev. Lett. **104**, 102001 (2010), arXiv:0910.4920 [hep-ph].
 - [18] G. Baum *et al.*, Measurement of Asymmetry in Spin Dependent e p Resonance Region Scattering, Phys. Rev. Lett. **45**, 2000 (1980).
 - [19] G. Baum *et al.*, A New Measurement of Deep Inelastic e p Asymmetries, Phys. Rev. Lett. **51**, 1135 (1983).
 - [20] P. L. Anthony *et al.* (E142), Determination of the neutron spin structure function., Phys. Rev. Lett. **71**, 959 (1993).
 - [21] K. Abe *et al.* (E143), Measurements of the proton and deuteron spin structure functions g_1 and g_2 , Phys. Rev. D **58**, 112003 (1998), arXiv:hep-ph/9802357.
 - [22] A. Airapetian *et al.* (HERMES), Evidence for quark hadron duality in the proton spin asymmetry A_1 , Phys. Rev. Lett. **90**, 092002 (2003), arXiv:hep-ex/0209018.
 - [23] J. Yun *et al.* (CLAS), Measurement of inclusive spin structure functions of the deuteron, Phys. Rev. C **67**, 055204 (2003), arXiv:hep-ex/0212044.
 - [24] Z. E. Meziani *et al.*, Higher twists and color polarizabilities in the neutron, Phys. Lett. B **613**, 148 (2005), arXiv:hep-ph/0404066.
 - [25] X. Zheng *et al.* (Jefferson Lab Hall A), Precision measurement of the neutron spin asymmetries and spin-dependent structure functions in the valence quark region, Phys. Rev. C **70**, 065207 (2004), arXiv:nucl-ex/0405006.
 - [26] X. Zheng *et al.* (Jefferson Lab Hall A), Precision measurement of the neutron spin asymmetry A_1^n and spin flavor decomposition in the valence quark region, Phys. Rev. Lett. **92**, 012004 (2004), arXiv:nucl-ex/0308011.
 - [27] D. S. Parno *et al.* (Jefferson Lab Hall A), Precision Measurements of A_1^n in the Deep Inelastic Regime, Phys. Lett. B **744**, 309 (2015), arXiv:1406.1207 [nucl-ex].
 - [28] D. Flay *et al.* (Jefferson Lab Hall A), Measurements of d_2^n and A_1^n : Probing the neutron spin structure, Phys. Rev. D **94**, 052003 (2016), arXiv:1603.03612 [nucl-ex].
 - [29] K. V. Dharmawardane *et al.* (CLAS), Measurement of the x- and Q^2 -dependence of the asymmetry A_1 on the nucleon, Phys. Lett. B **641**, 11 (2006), arXiv:nucl-ex/0605028.
 - [30] Y. Prok *et al.* (CLAS), Moments of the Spin Structure Functions g_1^p and g_1^d for $0.05 < Q^2 < 3.0 \text{ GeV}^2$, Phys. Lett. B **672**, 12 (2009), arXiv:0802.2232 [nucl-ex].
 - [31] P. E. Bosted *et al.* (CLAS), Quark-hadron duality in spin structure functions g_1^p and g_1^d , Phys. Rev. C **75**, 035203 (2007), arXiv:hep-ph/0607283.
 - [32] N. Guler *et al.* (CLAS), Precise determination of the deuteron spin structure at low to moderate Q^2 with CLAS and extraction of the neutron contribution, Phys. Rev. C **92**, 055201 (2015), arXiv:1505.07877 [nucl-ex].
 - [33] R. Fersch *et al.* (CLAS), Determination of the Proton Spin Structure Functions for $0.05 < Q^2 < 5 \text{ GeV}^2$ using CLAS, Phys. Rev. C **96**, 065208 (2017), arXiv:1706.10289 [nucl-ex].
 - [34] R. G. Fersch (CLAS), Quark-Hadron Duality of Spin Structure Functions in CLAS EG1b Data, Few Body Syst. **59**, 108 (2018).
 - [35] F. R. Wesselmann *et al.* (RSS), Proton spin structure in the resonance region, Phys. Rev. Lett. **98**, 132003 (2007), arXiv:nucl-ex/0608003.
 - [36] B. A. Mecking *et al.* (CLAS), The CEBAF Large Acceptance Spectrometer (CLAS), Nucl. Instrum. Meth. A **503**, 513 (2003).
 - [37] C. D. Keith *et al.*, A polarized target for the CLAS detector, Nucl. Instrum. Meth. A **501**, 327 (2003).
 - [38] Y. Kahn, W. Melnitchouk, and S. A. Kulagin, New method for extracting neutron structure functions from nuclear data, Phys. Rev. C **79**, 035205 (2009), arXiv:0809.4308 [nucl-th].
 - [39] R. C. Walker *et al.*, Measurements of the proton elastic form-factors for $1 \leq Q^2 \leq 3 \text{ GeV}^2$ at SLAC, Phys. Rev. D **49**, 5671 (1994).
 - [40] T. Liu, R. S. Sufian, G. F. de Téramond, H. G. Dosch, S. J. Brodsky, and A. Deur, Unified Description of Polarized and Unpolarized Quark Distributions in the Proton, Phys. Rev. Lett. **124**, 082003 (2020), arXiv:1909.13818 [hep-ph].