Electromagnetic flavor-changing lepton decays from Lorentz and CPT violation

V. Alan Kostelecký,[1] E. Passemar,[1] and N. Sherrill[2]
[1] *Physics Department, Indiana University, Bloomington, Indiana 47405, USA*
[2] *Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, UK*
(Dated: July 2022)

Lorentz- and CPT-violating effects initiating two-body electromagnetic flavor-changing decays of charged leptons are studied in the framework of Lorentz-violating effective field theory. An analysis of data from experiments at the Paul Scherrer Institute and at the Stanford Linear Accelerator measuring the branching ratios of these decays provides 576 constraints on independent flavor-changing effects in the charged-lepton sector, consistent with no Lorentz and CPT violation at the level of parts in 10^{-13} GeV^{-1} to 10^{-9} GeV^{-1}.

For many decades now, flavor-changing effects have played a central role in the discovery of new physics violating fundamental symmetries of nature, including the discrete symmetries charge conjugation C, parity inversion P, and time reversal T and the continuous internal symmetries of the minimal Standard Model (SM). In the SM, for example, charged weak interactions change fermion flavor, converting charged leptons to neutrinos or mixing quarks of different flavors. These effects underlie the observation of P violation in weak decays [1] and the detection of CP violation in kaon oscillations [2]. Also, flavor oscillations of neutrinos have provided evidence of physics beyond the SM [3, 4], involving breaking of the accidental SM global $\mathrm{U}_e(1)\times\mathrm{U}_\mu(1)\times\mathrm{U}_\tau(1)$ invariance by right-handed neutrino fields.

A key symmetry in nature is Lorentz invariance, which ensures that physical laws are unchanged under rotations or boosts and is accompanied by CPT invariance. While these invariances hold to an excellent approximation, they could be broken in an underlying theory that combines gravity and quantum physics such as strings [5], thereby leading to tiny observable effects of Lorentz violation (LV) at present energy scales. Although extensive experimental investigations of this idea have been performed [6], comparatively little is known about flavor-changing LV interactions. Instead, most studies of flavor-changing LV effects involve propagation. For example, neutral-meson oscillations are sensitive to Lorentz- and CPT-violating effects that are otherwise challenging to detect [7], but in these experiments the flavor changes are driven by known weak interactions while the Lorentz and CPT violation is diagonal in quark flavor.

The present work addresses this gap in the literature by investigating flavor-changing LV interactions that induce charged-lepton decays, in particular electromagnetic decays of the muon and tau. In conventional Lorentz-invariant models, flavor-changing decays of charged leptons occur only via suppressed one-loop processes with branching ratios $\lesssim 10^{-54}$ [8], so these processes offer exceptionally clean probes of new physics. Here, we perform a model-independent analysis of experimental data to search for dominant LV effects in these decays. Our results are consistent with no effects in 576 independent coefficients for LV, thereby excluding electromagnetic flavor-changing LV interactions of leptons at parts in 10^{-13} GeV^{-1} to 10^{-9} GeV^{-1}.

Given the absence of compelling evidence for LV to date, model-independent techniques are desirable and appropriate for analyses of prospective low-energy signals. A model-independent framework based on effective field theory, known as the Standard-Model Extension (SME) [9, 10], offers a powerful and widely adopted approach for experimental searches for LV [6, 11]. In Minkowski spacetime, the Lagrange density of the SME contains the SM extended by adding all observer-invariant terms formed by contracting LV operators with controlling coefficients. In effective field theory, CPT violation implies LV [9, 12], so the SME also describes all CPT-violating effects. Despite the substantial body of existing experimental measurements [6], many coefficients for LV remain unconstrained to date. Their magnitudes are generically undetermined by theory, with some "countershaded" ones challenging to detect despite being large [13], so model-independent experimental searches without prior assumptions about coefficient magnitudes acquire particular importance in this context.

The presence of LV allows the electromagnetic decays $\ell_A \to \ell_B + \gamma$ of a charged lepton ℓ_A into a charged lepton ℓ_B and a photon γ to proceed directly at tree level, in contrast to the suppressed loop-level decays in conventional Lorentz-invariant models. Since the decays are governed by energy scales on the order of m_A, SME operators of low mass dimension are expected to provide the dominant experimental signals in these and related processes [9, 14–22]. All terms in the SME Lagrange density with operators of mass dimension up to six are explicitly known [23]. In the lepton-photon sector, some of these operators affect propagation while others represent pure interactions. The former involve bilinears in the lepton fields and their spacetime derivatives. Setting the photon field to zero establishes the free-fermion Lagrange density and determines the propagating flavor eigenstates of the hamiltonian. By construction, these eigenstates represent the physical electron, muon, and tau fields relevant for laboratory experiments, and so during free propagation they preserve the corresponding

TABLE I. Dimension-five terms with $F_{\mu\nu}$ couplings. Note $(-)^\mu \equiv +$ for $\mu = 0$ and $(-)^\mu \equiv -$ for $\mu = 1, 2, 3$.

Term	Number	C	P	T	CP	CPT
$-\frac{1}{2}(m_F^{(5)})^{\alpha\beta}_{AB} F_{\alpha\beta}\overline{\psi}_A\psi_B$	54	$-$	$(-)^\alpha(-)^\beta$	$-(-)^\alpha(-)^\beta$	$-(-)^\alpha(-)^\beta$	$+$
$-\frac{1}{2}i(m_{5F}^{(5)})^{\alpha\beta}_{AB} F_{\alpha\beta}\overline{\psi}_A\gamma_5\psi_B$	54	$-$	$-(-)^\alpha(-)^\beta$	$(-)^\alpha(-)^\beta$	$(-)^\alpha(-)^\beta$	$+$
$-\frac{1}{2}(a_F^{(5)})^{\mu\alpha\beta}_{AB} F_{\alpha\beta}\overline{\psi}_A\gamma_\mu\psi_B$	216	$+$	$(-)^\mu(-)^\alpha(-)^\beta$	$-(-)^\mu(-)^\alpha(-)^\beta$	$(-)^\mu(-)^\alpha(-)^\beta$	$-$
$-\frac{1}{2}(b_F^{(5)})^{\mu\alpha\beta}_{AB} F_{\alpha\beta}\overline{\psi}_A\gamma_\mu\gamma_5\psi_B$	216	$-$	$-(-)^\mu(-)^\alpha(-)^\beta$	$-(-)^\mu(-)^\alpha(-)^\beta$	$(-)^\mu(-)^\alpha(-)^\beta$	$-$
$-\frac{1}{4}(H_F^{(5)})^{\mu\nu\alpha\beta}_{AB} F_{\alpha\beta}\overline{\psi}_A\sigma_{\mu\nu}\psi_B$	324	$+$	$(-)^\mu(-)^\nu(-)^\alpha(-)^\beta$	$(-)^\mu(-)^\nu(-)^\alpha(-)^\beta$	$(-)^\mu(-)^\nu(-)^\alpha(-)^\beta$	$+$

lepton numbers without flavor changes despite the presence of LV. With the photon reintroduced, the spacetime derivatives in all operators affecting propagation are covariant and symmetrized [23], so the photon fields appearing in these bilinears also preserve the eigenstates during propagation. It follows that no tree-level electromagnetic flavor-changing decays of the physical charged leptons can occur from these covariant-derivative couplings, contrary to the assumptions of pioneering works on these decays that adopted experimentally unphysical eigenstates for calculations. This flavor-conserving feature has the same origin as its analogue in the SM, and it can be understood as a consequence of the global symmetry $\mathrm{U}_e(1)\times\mathrm{U}_\mu(1)\times\mathrm{U}_\tau(1)$ of the free-fermion theory that is transmitted to any photon couplings associated with covariant derivatives.

Since we are interested in operators that change the flavor of a physical eigenstate while emitting a photon, the effects relevant here must instead involve direct couplings of the electromagnetic field strength to lepton bilinears. All such operators are independent of the propagation terms in the Lagrange density, so they can be off-diagonal in flavor space even in the basis of physical eigenstates relevant for experiments. They therefore can violate the $\mathrm{U}_e(1)\times\mathrm{U}_\mu(1)\times\mathrm{U}_\tau(1)$ symmetry, inducing observable charged-lepton decays in detectors. The dominant operators of this form have mass dimension five and are the focus of this work. Since the lepton decay rates contain the square of the decay amplitude, which itself is already at leading order in LV, any LV effects in propagation and the associated modifications of phase-space factors can be disregarded in what follows.

The dimension-five terms of interest in the Lagrange density [23] are listed in the first column of Table I. The lepton fields are denoted ψ_A, $A = e, \mu, \tau$, and the electromagnetic field strength is $F_{\mu\nu}$. The behaviors of the terms under C, P, T, CP, and CPT are also displayed in the table. The results reveal the prospect of novel sources of discrete-symmetry breakdown in the presence of LV, including possible violations of the CPT theorem [24]. The coefficients for LV $(m_F^{(5)})^{\alpha\beta}_{AB}$, $(m_{5F}^{(5)})^{\alpha\beta}_{AB}$, $(a_F^{(5)})^{\mu\alpha\beta}_{AB}$, $(b_F^{(5)})^{\mu\alpha\beta}_{AB}$, $(H_F^{(5)})^{\mu\nu\alpha\beta}_{AB}$ have units of GeV^{-1} and by construction are antisymmetric on the index pairs (α, β) and (μ, ν). They can be viewed as complex matrices in flavor space, constrained by hermiticity of the Lagrange density. The number of independent real components of each coefficient is given in the second column of the table.

The SME coefficients transform as covariant tensors under observer Lorentz transformations and as scalars under particle transformations, so the terms in the first column generically violate Lorentz invariance [9]. However, some of the coefficients can contain components proportional to products of the Minkowski metric $\eta_{\mu\nu}$ and the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$, which are Lorentz-group invariants, and hence the corresponding terms are Lorentz invariant. For example, the coefficients $(H_F^{(5)})^{\mu\nu\alpha\beta}_{AB}$ contain two Lorentz-invariant pieces yielding the Lorentz-invariant terms $-(H_{F,1}^{(5)})_{AB}F^{\mu\nu}\overline{\psi}_A\sigma_{\mu\nu}\psi_B$ and $-(H_{F,2}^{(5)})_{AB}\widetilde{F}^{\mu\nu}\overline{\psi}_A\sigma_{\mu\nu}\psi_B$, where $\widetilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$ is the dual field strength. These terms describe anomalous magnetic and electric dipole moments. They correspond to the leading operators in the Low-energy Effective Field Theory (LEFT) [25, 26] for the decay $\ell_A \to \ell_B + \gamma$, which derive from dimension-six effects in the Standard-Model Effective Field Theory (SMEFT) upon matching operators at the electroweak scale [27]. For the magnetic dipole term in the two-flavor electron-muon limit, for instance, the connection appears explicitly by expanding the SME results into chiral field components and matching to the LEFT and SMEFT operators. More generally, the SMEFT can be viewed as a restriction of the SME to the Lorentz-invariant sector in Minkowski spacetime, SME $\supset$ SMEFT $\supset$ LEFT, with every SMEFT Wilson coefficient being a Lorentz-invariant combination of nonminimal SME coefficients in Minkowski spacetime and an appropriate power of the high-energy scale representing the onset of new physics.

The terms listed in Table I generate novel three-point vertex functions allowing the decays $\ell_A \to \ell_B + \gamma$. These terms leave unaffected the free propagation of the fermion and photon fields, so standard quantization techniques apply [28]. If ℓ_A has momentum p_μ and spin projection s, ℓ_B has momentum p'_μ and spin projection s', and the photon has momentum k_μ and helicity λ, then the contribution to the decay amplitude $\mathcal{M}_{AB}^{(s,s',\lambda)}(p, p', k)$ from a given term in the table takes the form

$$\mathcal{M}_{AB}^{(s,s',\lambda)} = \begin{cases} \overline{u}_B^{(s)}(p')V_{BA}^{\beta}(k)u_A^{(s')}(p)\epsilon_\beta^{(\lambda)*}(k), \\ \overline{v}_A^{(s)}(p)V_{AB}^{\beta}(k)v_B^{(s')}(p')\epsilon_\beta^{(\lambda)*}(k), \end{cases} \tag{1}$$

where the first line holds for particle decay and the second for antiparticle decay. The quantity $V_{AB}^{\beta}(k) = (V_{BA}^{\beta}(k))^*$ is the member of the set $\{(m_F^{(5)})_{AB}^{\alpha\beta}k_\alpha,\ i(m_{5F}^{(5)})_{AB}^{\alpha\beta}\gamma_5 k_\alpha,\ (a_F^{(5)})_{AB}^{\mu\alpha\beta}\gamma_\mu k_\alpha,\ (b_F^{(5)})_{AB}^{\mu\alpha\beta}\gamma_\mu\gamma_5 k_\alpha,\ \frac{1}{2}(H_F^{(5)})_{AB}^{\mu\nu\alpha\beta}\sigma_{\mu\nu}k_\alpha\}$ corresponding to the chosen term in Table I. Note that the Ward identity ensuring gauge invariance of the amplitude (1) is enforced through the vanishing of $\mathcal{M}_{AB}^{(s,s',\lambda)}(p,p',k)$ under the replacement $\epsilon_\beta^{(\lambda)*}(k) \to k_\beta$.

Existing experimental limits on charged-lepton transitions are stringent and hence well suited for constraining small deviations from known physics. Tight bounds on flavor violations involving muons come from studies of two-body decays by the Mu to Electron Gamma (MEG) collaboration at the Paul Scherrer Institute [29], $\mathrm{BR}(\mu^+ \to e^+ + \gamma) \le 4.2 \times 10^{-13}$. The *BaBar* collaboration at the Stanford Linear Accelerator obtained constraints [30] both on decays of taus into muons, $\mathrm{BR}(\tau^\pm \to \mu^\pm + \gamma) \le 4.4 \times 10^{-8}$, and on decays into electrons, $\mathrm{BR}(\tau^\pm \to e^\pm + \gamma) \le 3.3 \times 10^{-8}$.

In the MEG experiment [31], polarized antimuons in a beam are stopped in a plastic target and subsequently decay at rest, producing back-to-back positrons and photons each carrying energy $m_\mu/2 \simeq 52.83$ MeV. The signal process therefore involves the calculation of the integrated decay rate of a polarized antimuon at rest to a positron and photon. The direction dependence arising from LV means that the calculation must take into account the restricted solid-angle acceptance window for the signal photon in the MEG detector and allow for all possible spin configurations in the final state. It is convenient to identify the beam direction as the detector z axis. Approximately $\simeq 11\%$ of the full phase space is accessible, and the limits on detector polar and azimuthal angles are $\theta \in (1.21, 1.93)$, $\phi \in (\frac{2\pi}{3}, \frac{4\pi}{3})$. For our purposes it suffices to approximate the antimuons as having initial polarization $P_\mu = -1$. In practice a small depolarization of the beam occurs during propagation, which could be taken into account in a future detailed data reconstruction. The polarized decay rate is therefore given by

$$\Gamma \approx \frac{1}{64\pi^2 m_\mu}\int_{\theta_{\min}}^{\theta_{\max}}\int_{\phi_{\min}}^{\phi_{\max}} \sin\theta d\theta d\phi\ |\mathcal{M}_{\mu e}(\theta,\phi)|^2, \tag{2}$$

where the antiparticle decay amplitude (1) is chosen. The explicit expression for $|\mathcal{M}_{\mu e}(\theta,\phi)|^2$ is obtained directly from Eq. (1) but is lengthy and omitted here. Multiplying the result by the muon lifetime $\tau_\mu \simeq 2.2 \times 10^{-6}$ s gives the theoretical branching ratio in terms of the coefficients for LV appearing in Table I, expressed in the frame of the MEG detector.

In the *BaBar* experiment [32, 33], the $\tau^\pm$ are produced via unpolarized and asymmetric $e^+e^- \to \tau^+\tau^-$ collisions near the $\Upsilon(4S)$ resonance. The emerging tau pairs retain nonzero longitudinal momentum, so their rest frame differs from the detector frame. However, the boost factor $\gamma \approx \sqrt{s}/(2m_\tau) \simeq \mathcal{O}(1)$ between the two frames is comparatively small and can be disregarded in studying LV effects, so the rest frame of the tau pairs can reasonably be taken as the detector frame. For present purposes it suffices to approximate the fiducial volume of the *BaBar* detector as spanning the full 4π steradians. In practice small cones involving $\simeq 10\%$ of the volume along the collider beamline directions are unavailable, and this could be incorporated into a future data analysis. The decay rates for the processes $\tau^\pm \to (\mu^\pm, e^\pm) + \gamma$ are therefore given by

$$\Gamma_{AB}^{\pm} \approx \frac{1}{64\pi^2 m_\tau}\int_{4\pi} d\Omega \sum_{s,s',\lambda} \tfrac{1}{2}|\mathcal{M}_{AB}^{\pm(s,s',\lambda)}(\theta,\phi)|^2, \tag{3}$$

where θ, ϕ are the polar and azimuthal angles of the photon with respect to the $+z$ axis of the detector frame, $A = \tau$ and $B = \mu$ or e, and the signs $\pm$ correspond to the lepton charge in the process. The full integrand $\overline{|\mathcal{M}(\theta,\phi)|^2}$ is lengthy, but if attention is restricted to any given operator in Table I then it takes the form

$$\overline{|\mathcal{M}(\theta,\phi)|^2} = -\tfrac{1}{2}\eta_{\mu\nu}\mathrm{Tr}\big[(\not{p}\mp m_A)V_{AB}^{\mu}(\not{p}'\mp m_B)V_{AB}^{\dagger\nu}\big]. \tag{4}$$

Explicit evaluation gives

$$\begin{aligned}
\sum\overline{|\mathcal{M}_{m_F^{(5)}}|^2} &= 2(m_A m_B + p\cdot p')(m_F^{(5)})_{AB}^{k\mu}(m_F^{(5)})^*_{AB\mu}{}^{k}, \\
\sum\overline{|\mathcal{M}_{m_{5F}^{(5)}}|^2} &= 2(m_A m_B - p\cdot p')(m_{5F}^{(5)})_{AB}^{k\mu}(m_{5F}^{(5)})^*_{AB\mu}{}^{k}, \\
\sum\overline{|\mathcal{M}_{a_F^{(5)}}|^2} &= 2(m_A m_B - p\cdot p')(a_F^{(5)})_{AB}^{\mu\nu k}(a_F^{(5)})^*_{AB\mu\nu}{}^{k} \\
&\quad -2\big((a_F^{(5)})_{AB}^{pk\nu}(a_F^{(5)})_{AB}^{*p'k}{}_{\nu} + \mathrm{h.c.}\big), \\
\sum\overline{|\mathcal{M}_{b_F^{(5)}}|^2} &= 2(m_A m_B + p\cdot p')(b_F^{(5)})_{AB}^{\mu\nu k}(b_F^{(5)})^*_{AB\mu\nu}{}^{k} \\
&\quad -2\big((b_F^{(5)})_{AB}^{pk\nu}(b_F^{(5)})_{AB}^{*p'k}{}_{\nu} + \mathrm{h.c.}\big), \\
\sum\overline{|\mathcal{M}_{H_F^{(5)}}|^2} &= \tfrac{1}{2}(m_A m_B + p\cdot p') \\
&\quad \times(H_F^{(5)})_{AB}^{\mu\nu\alpha k}(H_F^{(5)})^*_{AB\nu\mu\alpha}{}^{k} \\
&\quad +2\big((H_F^{(5)})_{AB}^{\mu pk\nu}(H_F^{(5)})^*_{AB\mu}{}^{p'k}{}_{\nu} + \mathrm{h.c.}\big),
\end{aligned} \tag{5}$$

where the sums are over spins and indices p, p', k represent contraction with the corresponding momenta. Substituting these expressions into the decay rate (3) and multiplying by the tau lifetime $\tau_\tau \simeq 2.9 \times 10^{-13}$ s yields the theoretical branching ratio in terms of coefficients for LV, expressed in the detector frame.

The presence of LV means that the explicit values of the coefficients listed in Table I are frame dependent, so experimental results must be reported in a specified frame. The coefficients can be taken as spacetime constants in cartesian inertial frames near the Earth [10].

TABLE II. Constraints deduced from Ref. [29].

Coefficients	Constraint (GeV^{-1})
$(m_F^{(5)})^{TJ}_{\mu e}, (m_F^{(5)})^{JZ}_{\mu e}, (m_{5F}^{(5)})^{TJ}_{\mu e}, (m_{5F}^{(5)})^{JZ}_{\mu e}, (a_F^{(5)})^{TTJ}_{\mu e}, (a_F^{(5)})^{TJZ}_{\mu e}, (a_F^{(5)})^{JTJ}_{\mu e}, (a_F^{(5)})^{JTK}_{\mu e}, (a_F^{(5)})^{JJZ}_{\mu e}, (a_F^{(5)})^{JKZ}_{\mu e}, (a_F^{(5)})^{ZTJ}_{\mu e}, (a_F^{(5)})^{ZJZ}_{\mu e}, (b_F^{(5)})^{TTJ}_{\mu e}, (b_F^{(5)})^{TJZ}_{\mu e}, (b_F^{(5)})^{JTJ}_{\mu e}, (b_F^{(5)})^{JTK}_{\mu e}, (b_F^{(5)})^{JJZ}_{\mu e}, (b_F^{(5)})^{JKZ}_{\mu e}, (b_F^{(5)})^{ZTJ}_{\mu e}, (b_F^{(5)})^{ZJZ}_{\mu e}, (H_F^{(5)})^{TJTJ}_{\mu e}, (H_F^{(5)})^{TJTK}_{\mu e}, (H_F^{(5)})^{TJJZ}_{\mu e}, (H_F^{(5)})^{TJKZ}_{\mu e}, (H_F^{(5)})^{TZTJ}_{\mu e}, (H_F^{(5)})^{TZJZ}_{\mu e}, (H_F^{(5)})^{JKTJ}_{\mu e}, (H_F^{(5)})^{JKJZ}_{\mu e}, (H_F^{(5)})^{JZTJ}_{\mu e}, (H_F^{(5)})^{JZTK}_{\mu e}, (H_F^{(5)})^{JZJZ}_{\mu e}, (H_F^{(5)})^{JZKZ}_{\mu e}$	$< 6.0 \times 10^{-13}$
$(m_F^{(5)})^{TZ}_{\mu e}, (m_F^{(5)})^{JK}_{\mu e}, (m_{5F}^{(5)})^{TZ}_{\mu e}, (m_{5F}^{(5)})^{JK}_{\mu e}, (a_F^{(5)})^{TTZ}_{\mu e}, (a_F^{(5)})^{TJK}_{\mu e}, (a_F^{(5)})^{JTZ}_{\mu e}, (a_F^{(5)})^{JJK}_{\mu e}, (a_F^{(5)})^{ZTZ}_{\mu e}, (a_F^{(5)})^{ZJK}_{\mu e}, (b_F^{(5)})^{TTZ}_{\mu e}, (b_F^{(5)})^{TJK}_{\mu e}, (b_F^{(5)})^{JTZ}_{\mu e}, (b_F^{(5)})^{JJK}_{\mu e}, (b_F^{(5)})^{ZTZ}_{\mu e}, (b_F^{(5)})^{ZJK}_{\mu e}, (H_F^{(5)})^{TJTZ}_{\mu e}, (H_F^{(5)})^{TZTZ}_{\mu e}, (H_F^{(5)})^{JKTZ}_{\mu e}, (H_F^{(5)})^{JZTZ}_{\mu e}, (H_F^{(5)})^{TJJK}_{\mu e}, (H_F^{(5)})^{TZJK}_{\mu e}, (H_F^{(5)})^{JKJK}_{\mu e}, (H_F^{(5)})^{JZJK}_{\mu e}$	$< 6.4 \times 10^{-13}$

TABLE III. Constraints deduced from Ref. [30].

Coefficients	Constraint (GeV^{-1})
$(m_F^{(5)})^{TJ}_{\tau\mu}, (m_F^{(5)})^{TZ}_{\tau\mu}, (m_F^{(5)})^{JK}_{\tau\mu}, (m_F^{(5)})^{JZ}_{\tau\mu}$	$< 1.8 \times 10^{-9}$
$(m_{5F}^{(5)})^{TJ}_{\tau\mu}, (m_{5F}^{(5)})^{TZ}_{\tau\mu}, (m_{5F}^{(5)})^{JK}_{\tau\mu}, (m_{5F}^{(5)})^{JZ}_{\tau\mu}$	$< 2.0 \times 10^{-9}$
$(a_F^{(5)})^{TTJ}_{\tau\mu}, (a_F^{(5)})^{TTZ}_{\tau\mu}, (a_F^{(5)})^{TJK}_{\tau\mu}, (a_F^{(5)})^{TJZ}_{\tau\mu}, (b_F^{(5)})^{JTJ}_{\tau\mu}, (b_F^{(5)})^{JTK}_{\tau\mu}, (b_F^{(5)})^{JTZ}_{\tau\mu}, (b_F^{(5)})^{JJK}_{\tau\mu}, (b_F^{(5)})^{JJZ}_{\tau\mu}, (b_F^{(5)})^{JKZ}_{\tau\mu}, (b_F^{(5)})^{ZTJ}_{\tau\mu}, (b_F^{(5)})^{ZTZ}_{\tau\mu}, (b_F^{(5)})^{ZJK}_{\tau\mu}, (b_F^{(5)})^{ZJZ}_{\tau\mu}, (H_F^{(5)})^{JKTJ}_{\tau\mu}, (H_F^{(5)})^{JKJZ}_{\tau\mu}, (H_F^{(5)})^{JZTJ}_{\tau\mu}, (H_F^{(5)})^{JZTK}_{\tau\mu}, (H_F^{(5)})^{JZJZ}_{\tau\mu}, (H_F^{(5)})^{JZKZ}_{\tau\mu}, (H_F^{(5)})^{JZTZ}_{\tau\mu}, (H_F^{(5)})^{JZJK}_{\tau\mu}, (H_F^{(5)})^{JKTZ}_{\tau\mu}, (H_F^{(5)})^{JKJK}_{\tau\mu}$	$< 2.2 \times 10^{-9}$
$(a_F^{(5)})^{JTJ}_{\tau\mu}, (a_F^{(5)})^{JTK}_{\tau\mu}, (a_F^{(5)})^{JTZ}_{\tau\mu}, (a_F^{(5)})^{JJK}_{\tau\mu}, (a_F^{(5)})^{JJZ}_{\tau\mu}, (a_F^{(5)})^{JKZ}_{\tau\mu}, (a_F^{(5)})^{ZTJ}_{\tau\mu}, (a_F^{(5)})^{ZTZ}_{\tau\mu}, (a_F^{(5)})^{ZJK}_{\tau\mu}, (a_F^{(5)})^{ZJZ}_{\tau\mu}, (b_F^{(5)})^{TTJ}_{\tau\mu}, (b_F^{(5)})^{TTZ}_{\tau\mu}, (b_F^{(5)})^{TJK}_{\tau\mu}, (b_F^{(5)})^{TJZ}_{\tau\mu}, (H_F^{(5)})^{TJTJ}_{\tau\mu}, (H_F^{(5)})^{TJTK}_{\tau\mu}, (H_F^{(5)})^{TJJZ}_{\tau\mu}, (H_F^{(5)})^{TJKZ}_{\tau\mu}, (H_F^{(5)})^{TZTJ}_{\tau\mu}, (H_F^{(5)})^{TZJZ}_{\tau\mu}, (H_F^{(5)})^{TJTZ}_{\tau\mu}, (H_F^{(5)})^{TJJK}_{\tau\mu}, (H_F^{(5)})^{TZTZ}_{\tau\mu}, (H_F^{(5)})^{TZJK}_{\tau\mu}$	$< 2.5 \times 10^{-9}$
$(m_F^{(5)})^{TJ}_{\tau e}, (m_F^{(5)})^{TZ}_{\tau e}, (m_F^{(5)})^{JK}_{\tau e}, (m_F^{(5)})^{JZ}_{\tau e}, (m_{5F}^{(5)})^{TJ}_{\tau e}, (m_{5F}^{(5)})^{TZ}_{\tau e}, (m_{5F}^{(5)})^{JK}_{\tau e}, (m_{5F}^{(5)})^{JZ}_{\tau e}$	$< 1.6 \times 10^{-9}$
$(a_F^{(5)})^{TTJ}_{\tau e}, (a_F^{(5)})^{TTZ}_{\tau e}, (a_F^{(5)})^{TJK}_{\tau e}, (a_F^{(5)})^{TJZ}_{\tau e}$	$< 1.9 \times 10^{-9}$
$(a_F^{(5)})^{JTJ}_{\tau e}, (a_F^{(5)})^{JTK}_{\tau e}, (a_F^{(5)})^{JTZ}_{\tau e}, (a_F^{(5)})^{JJK}_{\tau e}, (a_F^{(5)})^{JJZ}_{\tau e}, (a_F^{(5)})^{JKZ}_{\tau e}, (a_F^{(5)})^{ZTJ}_{\tau e}, (a_F^{(5)})^{ZTZ}_{\tau e}, (a_F^{(5)})^{ZJK}_{\tau e}, (a_F^{(5)})^{ZJZ}_{\tau e}$	$< 2.1 \times 10^{-9}$
$(b_F^{(5)})^{TTJ}_{\tau e}, (b_F^{(5)})^{TTZ}_{\tau e}, (b_F^{(5)})^{TJK}_{\tau e}, (b_F^{(5)})^{TJZ}_{\tau e}, (b_F^{(5)})^{JTJ}_{\tau e}, (b_F^{(5)})^{JTK}_{\tau e}, (b_F^{(5)})^{JTZ}_{\tau e}, (b_F^{(5)})^{JJK}_{\tau e}, (b_F^{(5)})^{JJZ}_{\tau e}, (b_F^{(5)})^{JKZ}_{\tau e}, (b_F^{(5)})^{ZTJ}_{\tau e}, (b_F^{(5)})^{ZTZ}_{\tau e}, (b_F^{(5)})^{ZJK}_{\tau e}, (b_F^{(5)})^{ZJZ}_{\tau e}, (H_F^{(5)})^{TJTJ}_{\tau e}, (H_F^{(5)})^{TJTK}_{\tau e}, (H_F^{(5)})^{TJJZ}_{\tau e}, (H_F^{(5)})^{TJKZ}_{\tau e}, (H_F^{(5)})^{TZTJ}_{\tau e}, (H_F^{(5)})^{TZJZ}_{\tau e}, (H_F^{(5)})^{TJTZ}_{\tau e}, (H_F^{(5)})^{TJJK}_{\tau e}, (H_F^{(5)})^{TZTZ}_{\tau e}, (H_F^{(5)})^{TZJK}_{\tau e}, (H_F^{(5)})^{JKTJ}_{\tau e}, (H_F^{(5)})^{JKJZ}_{\tau e}, (H_F^{(5)})^{JZTJ}_{\tau e}, (H_F^{(5)})^{JZTK}_{\tau e}, (H_F^{(5)})^{JZJZ}_{\tau e}, (H_F^{(5)})^{JZKZ}_{\tau e}, (H_F^{(5)})^{JZTZ}_{\tau e}, (H_F^{(5)})^{JZJK}_{\tau e}, (H_F^{(5)})^{JKTZ}_{\tau e}, (H_F^{(5)})^{JKJK}_{\tau e}$	$< 2.0 \times 10^{-9}$

The canonical frame adopted in the literature is the Sun-centered frame (SCF) with right-handed cartesian coordinates (T, X, Y, Z), where T is zero at the 2000 vernal equinox, the Z axis is parallel to the Earth's rotation axis, and the X axis points from the Earth to the Sun at $T = 0$ [34]. The Earth's rotation makes all laboratory frames noninertial and so the coefficients expressed in the laboratory frame are time dependent, oscillating at harmonics of the Earth's sidereal frequency $\omega_\oplus \simeq 2\pi/(23\text{ h } 56\text{ min})$ [7]. Neglecting the Earth's boost, the transformation from the SCF to a standard laboratory frame with x axis pointing to the south, y axis to the east, and z axis to the local zenith is

$$\mathcal{R} = \begin{pmatrix} \cos\chi\cos\omega_\oplus T_\oplus & \cos\chi\sin\omega_\oplus T_\oplus & -\sin\chi \\ -\sin\omega_\oplus T_\oplus & \cos\omega_\oplus T_\oplus & 0 \\ \sin\chi\cos\omega_\oplus T_\oplus & \sin\chi\sin\omega_\oplus T_\oplus & \cos\chi \end{pmatrix}, \quad (6)$$

where the angle χ is the colatitude of the laboratory, which is $\chi \simeq 42.5^\circ$ for MEG and $\chi \simeq 52.6^\circ$ for *BaBar*. The laboratory sidereal time $T_\oplus \equiv T - T_0$ is shifted relative to T [35], with $T_0 \simeq 3.9$ h for MEG and $T_0 \simeq 12.5$ h for *BaBar*. Neither the MEG nor the *BaBar* detector frames coincide with the standard laboratory frame, so matching requires an extra rotation of ψ about the z axis followed by an improper rotation $(x, y, z) \to (-x, z, +y)$,

$$\mathcal{R}_{\text{detector}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & +1 & 0 \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (7)$$

where $\psi \simeq -30^\circ$ for MEG and $\psi \simeq -50^\circ$ for *BaBar*.

Experiments searching for LV aim to measure the coefficients for LV in the SCF. The above transformations show that the experimental observables in the detector frame are functions of $\chi, \psi, T_\oplus$, and the SCF coefficients and that a given coefficient with n Lorentz indices is generically accompanied by oscillations in $T_\oplus$ involving from zero to n harmonics. As a result, data taken with time stamps can be binned in sidereal time and used to extract the amplitudes and phases of the various harmonics, yielding a series of independent constraints on the SCF coefficients. The dependence on colatitude and longitude implies that different experiments are sensitive to distinct coefficient combinations.

For the MEG and *BABAR* experiments, the published limits on the branching ratios can be viewed as time-averaged measurements. The time averages of the results (2) and (3) involve only rotation-invariant combinations of the SCF coefficients appearing in Table I, although they can still depend on the experiment colatitude χ and detector orientation ψ. Note also that the integration (3) over the full final-state phase space means that the decay rate for *BABAR* is unaffected by the rotation to the SCF. The three published limits [29, 30] on the decays yield three constraints on the combinations of SME coefficients in the SCF given in Eqs. (2) and (3). They are compatible with the three recent bounds obtained [16] on combinations of trace components of $(a_F^{(5)})_{AB}^{\mu\alpha\beta}$ and $(b_F^{(5)})_{AB}^{\mu\alpha\beta}$, provided the propagating eigenstates are taken to match the physical ones appropriate for experiments. Calculation reveals that all types of independent coefficients in Table I contribute to the time-averaged signals. Following standard procedure in the field [6], we can transform the three experimental constraints into limits on independent coefficient components taken one at a time. This procedure yields the 576 constraints on LV presented in Tables II and III. Of these, the entries involving the coefficients $(a_F^{(5)})_{AB}^{\mu\alpha\beta}$ and $(b_F^{(5)})_{AB}^{\mu\alpha\beta}$ also are bounds on CPT violation. Each entry is a constraint at the 90% confidence level on the modulus of the real and imaginary parts of a coefficient component in the SCF, with indices J and $K \neq J$ taking the values X or Y.

To summarize, an analysis of published data from the MEG and *BABAR* experiments places constraints on 576 independent coefficients for electromagnetic flavor-changing Lorentz and CPT violation in the charged-lepton sector. The results are consistent with no flavor-changing Lorentz violation in the range of parts in 10^{-13} GeV^{-1} to 10^{-9} GeV^{-1}, and they establish a bar excluding flavor-changing LV effects on these scales. Excellent prospects exist for future improvements on these results, both via analyses incorporating sidereal and annual time variations and via increased sensitivities to $\ell_A \to \ell_B + \gamma$ and related decays in upcoming experiments [36–40].

This work is supported in part by the U.S. Department of Energy under grants DE-SC0010120 and DE-AC05-06OR23177, by the U.S. National Science Foundation under grants PHY-1748958 and PHY-2013184, by the U.K. Science and Technology Facilities Council under grant ST/T006048/1, and by the Indiana University Center for Spacetime Symmetries.

[1] C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes, and R.P. Hudson, Phys. Rev. **105**, 1413 (1957).
[2] J.H. Christenson, J.W. Cronin, V.L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).
[3] R. Davis, Jr., D.S. Harmer, and K.C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968); Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001).
[4] Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
[5] V.A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); V.A. Kostelecký and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).
[6] V.A. Kostelecký and N. Russell, *Data Tables for Lorentz and CPT Violation*, Rev. Mod. Phys. **83**, 11 (2011), arXiv:0801.0287v15 (2022).
[7] V.A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998).
[8] W.J. Marciano and A.I. Sanda, Phys. Lett. B **67**, 303 (1977); S.M. Bilenky, S.T. Petcov, and B. Pontecorvo, Phys. Lett. B **67**, 309 (1977); B.W. Lee, S. Pakvasa, R.E. Schrock, and H. Sugawara, Phys. Rev. Lett. **38**, 937 (1977).
[9] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); Phys. Rev. D **58**, 116002 (1998).
[10] V.A. Kostelecký, Phys. Rev. D **69**, 105009 (2004).
[11] For reviews see, for example, A. Hees, Q.G. Bailey, A. Bourgoin, H. Pihan-Le Bars, C. Guerlin, and C. Le Poncin-Lafitte, Universe **2**, 30 (2016); J.D. Tasson, Rep. Prog. Phys. **77**, 062901 (2014); C.M. Will, Liv. Rev. Rel. **17**, 4 (2014); R. Bluhm, Lect. Notes Phys. **702**, 191 (2006).
[12] O.W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002).
[13] V.A. Kostelecký and J.D. Tasson, Phys. Rev. Lett. **102**, 010402 (2009).
[14] S.R. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1998).
[15] E.O. Iltan, JHEP **06**, 016 (2003).
[16] Y.M.P. Gomes and J.A. Helayel-Neto, Eur. Phys. J. C **80**, 287 (2020).
[17] A. Crivellin, F. Kirk and M. Schreck, JHEP **04**, 082 (2021).
[18] S. Hollenberg and P.B. Pal, Phys. Lett. B **701**, 89 (2011).
[19] J.P. Noordmans, C.J.G. Onderwater, H.W. Wilschut, and R.G.E. Timmermans, Phys. Rev. D **93**, 116001 (2016).
[20] M.A. López-Osorio, E. Martínez-Pascual, and J.J. Toscano, J. Phys. G **43**, 025003 (2016).
[21] V.E. Mouchrek-Santos and M.M. Ferreira, Jr., Phys. Rev. D **95**, 071701 (2017).
[22] Y.M.P. Gomes, P.C. Malta, and M.J. Neves, Phys. Rev. D **101**, 075001 (2020).
[23] V.A. Kostelecký and Z. Li, Phys. Rev. D **99**, 056016 (2019); Phys. Rev. D **103**, 024059 (2021).
[24] J.S. Bell, Proc. Roy. Soc. (London) **A231**, 479 (1955); W. Pauli, in W. Pauli, ed., *Niels Bohr and the Development of Physics*, McGraw-Hill, New York, 1955; G. Lüders, Ann. Phys. (N.Y.) **2**, 1 (1957).
[25] W. Dekens, E.E. Jenkins, A.V. Manohar and P. Stoffer, JHEP **01**, 088 (2019).
[26] A. Celis, V. Cirigliano, and E. Passemar, Phys. Rev. D **89**, 095014 (2014).
[27] E.E. Jenkins, A.V. Manohar and P. Stoffer, JHEP **3**, 016 (2018).
[28] V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001).
[29] A.M. Baldini *et al.*, Eur. Phys. J. C **76**, 434 (2016).
[30] B. Aubert *et al.*, Phys. Rev. Lett. **104**, 021802 (2010).
[31] J. Adam *et al.*, Eur. Phys. J. C **73**, 2365 (2013).
[32] D. Boutigny *et al.*, SLAC-R-0457 (1995).

[33] G. Ciezarek, M. Franco Sevilla, B. Hamilton, R. Kowalewski, T. Kuhr, V. Lüth, and Y. Sato, Nature **546**, 227 (2017).

[34] R. Bluhm, V.A. Kostelecký, C.D. Lane, and N. Russell, Phys. Rev. D **68**, 125008 (2003); Phys. Rev. Lett. **88**, 090801 (2002); V.A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).

[35] V.A. Kostelecký, A.C. Melissinos, and M. Mewes, Phys. Lett. B **761**, 1 (2016); Y. Ding and V.A. Kostelecký, Phys. Rev. D **94**, 056008 (2016).

[36] A.M. Baldini *et al.*, Eur. Phys. J. C **78**, 380 (2018).

[37] E. Kou *et al.*, PTEP **2019**, 123C01 (2019).

[38] L. Bartoszek *et al.*, arXiv:1501.05241.

[39] R. Abramishvili *et al.*, PTEP **2020**, 033C01 (2020).

[40] A. Blondel *et al.*, arXiv:1301.6113.