Counting linearly polarized gluons with lattice QCD

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We outline an approach to calculate the transverse-momentum-dependent distribution of linearly polarized gluons inside an unpolarized hadron on the lattice with the help of large momentum effective theory. To achieve this purpose, we propose calculating a Euclidean version of the degree of polarization for a fast-moving hadron on the lattice. It is ultraviolet finite, and no soft function subtraction is needed. In addition, the perturbative matching is trivial up to the one-loop level. It indicates a practical way to explore the content of the linearly polarized gluons in a proton on the lattice.

It has been widely accepted that hadrons are constructed by quarks and gluons. Due to the nonperturbative nature of strong interaction, it is hard to explore how those building blocks combine a hadron. In high-energy processes, the parton information is encoded in the Parton distribution functions (PDFs), which is a one-dimensional distribution function that describe the longitudinal momentum distribution of partons. If the partons are not collinear to the mother hadron but carry transverse momenta, then the parton structure should be described by the transverse momentum dependent distributions (TMDs) [1]. The TMDs can describe much richer partonic structures of a hadron.

Gluon plays an important role in a proton. Analog to a photon, a gluon can be unpolarized but also linearly polarized inside an unpolarized proton, if transverse motion of gluon is considered [2]. The TMD for unpolarized gluon is denoted as $f_1^g(x, \boldsymbol{k}_T^2)$, while the TMD of linearly polarized gluon is denoted as $h_1^{\perp g}(x, \boldsymbol{k}_T^2)$, which can be regarded as the gluonic analog of the Boer-Mulders function [3] for quark. The T-even function $h_1^{\perp g}$ describes how the $+1$ and -1 helicity gluon states are correlated in a hadron.

The linearly polarized gluon TMD has caused lots of attention recently. It has been pointed out that the linearly polarized gluons can modify the transverse spectrum of Higgs bosons [4] and heavy quarkonia [5] which are produced in collisions of unpolarized protons. Then, the essential question is how to determine it. In the past, the distribution of linearly polarized gluons inside an unpolarized hadron has been discussed in a model context in Ref. [6, 7], and many approaches based on experimental observables are proposed to extract the gluon TMDs, e.g., heavy quark pair or dijet [8–11], $\gamma\gamma$ [12], back-to-back quarkonium and photon productions [13], quarkonium and dilepton associated productions [14], single and double heavy quarkonium production at hadron colliders [5, 15–17], etc. The effect of linearly polarized gluon TMD can be found in azimuthal asymmetries, and also in total cross sections. Other previous studies are devoted to the linearly polarized gluons at small-x region [18–22]. Although the gluon TMDs can be probed at the high-energy electron-ion colliders, e.g., EICs in the US and China, however, the linearly polarized gluon TMD has never been extracted so far, either from experiment or from lattice QCD.

Measuring gluon TMDs inside a hadron now becomes possible due to the development of parton physics on the lattice in the past few years, which includes but not limited to quasi-PDFs and Large Momentum Effective Theory (LAMET) [23, 24], pseudo-PDFs [25, 26], lattice cross sections [27]. These approaches have made significant progress on PDFs, meson distribution amplitudes, generalized parton distributions, etc (see, e.g., [28, 29] for recent reviews of LAMET). Especially, the TMDs defined with staple-shaped Wilson line operators have been considered recently within the framework of LAMET, see [30–35] and references therein. The gluon-TMDs have also been considered very recently [36, 37].

In this work, we will show that the linearly polarized gluons inside an unpolarized hadron are feasible on the lattice with the help of LAMET. Our unambitious but practical idea is to calculate the ratio of $h_1^{\perp g}$ and the unpolarized gluon TMD f_1^g in coordinate space, so that future lattice simulations can help to reveal the scale of the linearly polarized gluons. To achieve this purpose, we define the Euclidean version of this ratio, which can be simulated on the lattice. We will show that this ratio is free of ultraviolet (UV) divergences so that one can take the continuum limit of the lattice data smoothly. The subtraction of the soft factor is not necessary, too. Furthermore, perturbative matching is not needed at least at the one-loop level, which further simplifies the calculation.

To start with, let us first review the gluon TMDs. In QCD, the TMDs of the gluon in an unpolarized hadron with momentum P are defined with the matrix element of the gluon field strength correlator [2]

$$\int \frac{d\xi d^2\boldsymbol{b}_T}{(2\pi)^3 P^+} e^{-ix\xi P^+ + i\boldsymbol{k}_T\cdot\boldsymbol{b}_T} \times \left\langle P \middle| F_a^{+\mu}\left(\frac{\xi n + \boldsymbol{b}_T}{2}\right) F_a^{+\nu}\left(-\frac{\xi n + \boldsymbol{b}_T}{2}\right) \middle| P \right\rangle$$

$$= -\frac{x}{2}\left[g_T^{\mu\nu} f_1^g(x,\boldsymbol{k}_T^2) - \left(\frac{k_T^\mu k_T^\nu}{\boldsymbol{k}_T^2} + \frac{1}{2}g_T^{\mu\nu}\right) \times h_1^{\perp g}(x,\boldsymbol{k}_T^2)\right], \quad (1)$$

where $F_a^{\mu\nu}$ is the gluon field strength tensor in adjoint representation with a being the color index, $\boldsymbol{k}_T$ is the transverse momentum of the gluon in the proton, ξ and $\boldsymbol{b}_T$ are the displacements of fields along the n and transverse directions, respectively. f_1^g is the TMD for unpolarized gluons, while $h_1^{\perp g}$ is the TMD for linearly polarized gluons, $\boldsymbol{k}_T^2 = -k_T^2$ and $g_T^{\mu\nu} = g^{\mu\nu} - n^\mu \bar{n}^\nu - n^\nu \bar{n}^\mu$ is the transverse metric, n and $\bar{n}$ are two unit light-cone vectors. For any vector a, $n \cdot a = a^+$ and $\bar{n} \cdot a = a^-$. We also note that the Wilson line in Eq. (2) which ensures the gauge invariance of the matrix element is omitted for the sake of simplicity. The direction of the Wilson line is along the light-cone, but process-dependent. $f_1^g(x,\boldsymbol{k}_T^2)$ is the TMD corresponding to the unpolarized gluon, while $h_1^{\perp g}(x,\boldsymbol{k}_T^2)$ is the linearly polarized gluon TMD. The upper bound for $h_1^{\perp g}$ is $|h_1^{\perp g}(x,\boldsymbol{k}_T^2)| \leq f_1^g(x,\boldsymbol{k}_T^2)$ [2]. There are rapidity singularities in TMDs, and they can be renormalized by introducing the soft factors [1].

On the lattice, it is more convenient to study the correlator itself, instead of its Fourier transform. In coordinate space, one can parameterize the correlator as

$$\left\langle P \middle| F_a^{+\mu}\left(\frac{\xi n + \boldsymbol{b}_T}{2}\right) F_a^{+\nu}\left(-\frac{\xi n + \boldsymbol{b}_T}{2}\right) \middle| P \right\rangle = -\frac{(P^+)^2}{2}\left[g_T^{\mu\nu} f_1^g(\xi,\boldsymbol{b}_T^2) - \left(\frac{b_T^\mu b_T^\nu}{\boldsymbol{b}_T^2} + \frac{1}{2}g_T^{\mu\nu}\right) \times h_1^{\perp g}(\xi,\boldsymbol{b}_T^2)\right]. \quad (2)$$

The TMDs in coordinate space can be converted into TMDs in momentum space through Fourier-Bessel transforms. The matrix element in Eq. (2) contains correlations along the light-cone. Note that for SIDIS (or Drell-Yan process) we should insert a staple-shaped Wilson line between the two gluon fields. The Wilson line is along the light-cone direction and has infinite length, which is hard to simulate on the Euclidean lattice.

However, one can define a similar correlation matrix element but calculable on the lattice,

$$\left\langle P \middle| E_{\perp a}^i\left(\frac{\xi n_z + \boldsymbol{b}_T}{2}\right) E_{\perp a}^j\left(-\frac{\xi n_z + \boldsymbol{b}_T}{2}\right) \middle| P \right\rangle = -\frac{P_0^2}{2}\left[g_T^{ij} F_1^g(\xi,\boldsymbol{b}_T^2,P_3) - \left(\frac{b_T^i b_T^j}{\boldsymbol{b}_T^2} + \frac{1}{2}g_T^{ij}\right) \times H_1^{\perp g}(\xi,\boldsymbol{b}_T^2,P_3)\right], \quad (3)$$

where $n_z = (0,0,0,1)$ is the unit vector of the third Cartesian direction, $i,j = 1,2$ denote the transverse components and $E_\perp^i = F^{0i} (i = 1,2)$ is the color electric field along the transverse directions. The quasi-TMDs F_1^g and $H_1^{\perp g}$ are the Euclidean versions of f_1^g and $h_1^{\perp g}$, respectively. Note that the Wilson line between the two $E_\perp$'s which are omitted in Eq. (3) is also staple-shaped but along the third direction, which has been described in Ref. [37]. Because the lattice volume is finite, one can adopt a Wilson line with a finite size. Because it is an equal-time correlation matrix element, Eq. (3) can be simulated on the Euclidean lattice.

In the infinite momentum frame, i.e., $P_3 \to \infty$, the operator in Eq. (3) becomes a "light-cone" operator Eq. (2), in which the third direction dependence becomes a light-cone dependence, and the color electric field E^i becomes F^{+i}. According to LAMET, the two matrix elements can be connected by a perturbative matching, because $P_3 \gg \Lambda_{\text{QCD}}$ provides a hard scale.

Before moving on, we should add some remarks on quasi-TMDs. First, the choice of Euclidean correlation function is not unique. Any operator that approaches the operator in Eq. (2) under large Lorentz boost can be used to define quasidistributions. Second, there are UV divergences in Eq. (3), which may cause trouble for lattice calculations. The gluon quasi-TMDs have already been introduced in Refs. [36] and [37]. There are power-like UV divergences because of the space-like Wilson line, and such divergences have to be subtracted for practical calculations. This can be realized by subtracting the square root of a rectangular Wilson loop Z_E, see, e.g., Ref. [31]. But there are still UV divergences due to the interaction of the gluon field with the Wilson line. There is no rapidity divergence in quasi-TMDs; however, a reduced soft factor should also be subtracted for a correct perturbative matching between TMDs and quasi-TMDs [30, 31].

Although the perturbative matching has been discussed in Ref. [37], however, the renormalization is performed in $\overline{\text{MS}}$ scheme, which is good for perturbative calculations but not convenient for lattice. One may need some nonperturbative approaches to renormalize the UV singularities. In the case of quasi PDFs, DAs, and GPDs, several nonperturbative subtraction schemes have been employed, and have been applied to quark quasi-TMDs, such as RI/MOM scheme [38–40], ratio scheme [25, 35], hybrid scheme [41], etc. For the gluon TMD case, however, it is not clear to us that those are good choices. The large offshellness of the gluon in RI/MOM raises a high risk of gauge invariance violation. The ratio scheme may work [37], but calls for more nonperturbative inputs from the lattice.

On the other hand, we will not be troubled by the renormalization and soft factor subtraction issues, when we are studying the ratio of $H_1^{\perp g}$ and F_1^g: $R \equiv H_1^{\perp g}/F_1^g$, as we will discuss below. Thus this quantity is more convenient for lattice simulations. We note that various ratios have been constructed on the lattice for quark TMDs [42–44] before. Our ratio $R(\xi,\boldsymbol{b}_T^2,P_3)$ can be expressed in terms of operator matrix elements as

$$\begin{aligned}&\frac{1}{2}+\frac{1}{4}R(\xi,\boldsymbol{b}_T^2,P_3)\\=&\frac{\langle P|\boldsymbol{b}_T\cdot\boldsymbol{E}_{\perp a}(\frac{\xi}{2}n_z+\frac{\boldsymbol{b}_T}{2})\boldsymbol{b}_T\cdot\boldsymbol{E}_{\perp a}(-\frac{\xi}{2}n_z-\frac{\boldsymbol{b}_T}{2})|P\rangle}{\boldsymbol{b}_T^2\langle P|\boldsymbol{E}_{\perp a}(\frac{\xi}{2}n_z+\frac{\boldsymbol{b}_T}{2})\cdot\boldsymbol{E}_{\perp a}(-\frac{\xi}{2}n_z-\frac{\boldsymbol{b}_T}{2})|P\rangle}\,.\end{aligned}\tag{4}$$

Its light-cone partner, $h_1^{\perp g}/f_1^g$, is the relative strength of the linearly polarized gluons over the unpolarized gluons, which is a reflection of the degree of polarization. In the infinite momentum limit, one can expect that $R(\xi,\boldsymbol{b}_T^2,P_3)\to h_1^{\perp g}(\xi,\boldsymbol{b}_T^2)/f_1^g(\xi,\boldsymbol{b}_T^2)+\mathcal{O}(\Lambda_{\rm QCD}/P_3)$. According to Eq. (4), R is a ratio of Euclidean correlation functions and there is no time-dependence, thus it can be simulated on the lattice.

For a practical calculation on the lattice, one has to renormalize the quantities properly, because the UV singularities prevent taking the continuum limit of the lattice data. Keep in mind that the operators in the numerator and denominator of Eq. (4) are "Wilson line operators": the two fields are connected with a Wilson line. Since the Wilson line structure of TMDs is generally process dependent, one may also adopt different shapes and directions of the Wilson line for quasi-TMDs. For example, one can choose the Wilson line as following: from $-\frac{\xi}{2}n_z-\frac{\boldsymbol{b}_T}{2}$ to $(L+\xi/2)n_z-\frac{\boldsymbol{b}_T}{2}$ along n_z direction, then from $(L+\xi/2)n_z-\frac{\boldsymbol{b}_T}{2}$ to $(L+\xi/2)n_z+\frac{\boldsymbol{b}_T}{2}$ along the transverse direction, then return to $\frac{\xi}{2}n_z+\frac{\boldsymbol{b}_T}{2}$ along $-n_z$. The renormalization of gluonic Wilson line operators has been studied a long time ago [45], and recently has been revisited in the context of quasi-PDF [46], by using the auxiliary field formalism. There is no essential difference between the "staple-shaped" operators here and the "straight line" operators in quasi-PDF on the renormalization of UV singularities. The UV singularities from the self-interaction of the Wilson line are multiplicatively renormalized, even if there are cusps in the Wilson line. The interaction between the Wilson line and the field located at the endpoint may lead to operator mixing; however, the operator is multiplicatively renormalizable if the field operator located at the endpoint is F^{0i}, F^{3i} or $F^{3\mu}$, where $i=1,2$ and $\mu=0,1,2$ [46]. In Eq. (4), the UV divergence in the denominator and numerator are multiplicative, and the renormalization factors are equal because the operators in both the denominator and numerator are of the $F^{0i}F^{0j}$ $(i,j=1,2)$ type and the Wilson line structures are the same. For the above reasons, although the matrix elements in the denominator and numerator in Eq. (4) are UV divergent, the ratio Eq. (4) is UV finite, because all UV singularities, including cusp and pinched pole singularities, as well as the endpoint UV singularities, are canceled in the ratio. So, the continuum limit of $R(\xi,\boldsymbol{b}_T,P_3)$ can be approached without a renormalization procedure on the lattice. We can go even further, that the ratio may be independent of, or at least not sensitive to the shape of the Wilson line.

In LAMET, the Euclidean and light-cone quantities are linked by a matching relation, while the matching coefficient can be calculated in perturbation theory because it is associated with a hard scale P_3. It has been shown that the TMD matching in LAMET has the type of multiplication instead of a convolution. This is also confirmed in the case of gluon TMD [36, 37], where the matching for gluon TMD was derived as

$$\begin{aligned}&F_1^g\left(x,\boldsymbol{b}_T^2,\mu,\zeta_z\right)S_r^{\frac{1}{2}}\left(\boldsymbol{b}_T^2,\mu\right)\\=&H\left(\frac{\zeta_z}{\mu^2}\right)e^{\ln\frac{\zeta_z}{\zeta}K\left(\boldsymbol{b}_T^2,\mu\right)}f_1^g\left(x,\boldsymbol{b}_T^2,\mu,\zeta\right)\,,\end{aligned}\tag{5}$$

where S_r is the reduced soft factor, K is the Collins-Soper kernel and H is the hard function, $\zeta_z=(2xP_3)^2$ and ζ is the Collins-Soper scale. The matching relation for $H_1^\perp$ is the same but the hard function may be different. Note that this relation is in the $(x,\boldsymbol{b}_T)$-space. Since the matching is multiplicative and the hard coefficient is independent of x, by performing Fourier transform from x to ξ, one can see that the matching in $(\xi,\boldsymbol{b}_T)$-space is still multiplicative and the hard function is also $H\left(\zeta_z/\mu^2\right)$. This is also confirmed in the analysis of factorization in coordinate space [44]. Therefore, for the degree of polarization, we have the matching relation

$$R(\xi,\boldsymbol{b}_T^2,P_3)=\frac{H_h\left(\frac{\zeta_z}{\mu^2}\right)}{H_f\left(\frac{\zeta_z}{\mu^2}\right)}\frac{f_1^g(\xi,\boldsymbol{b}_T^2,\mu,\zeta)}{h_1^{\perp g}(\xi,\boldsymbol{b}_T^2,\mu,\zeta)}\,,\tag{6}$$

where H_h and H_f are matching coefficients for f_1^g and $h_1^{\perp g}$, respectively. The S_r and K terms cancel in the matching formula. Thus we do not need to worry about the reduced soft factors, which makes the evaluation simpler.

The matching for the denominator in Eq. (4) has already been studied and H_f has been calculated at the one-loop level. Now we will derive the matching relation for the numerator. To perform the matching calculation in perturbation theory, one can replace the hadron state with a parton state because the hard function is independent of external states. In previous works, the external states are always chosen as unpolarized gluons. It is shown in [15] that $h_1^{\perp g}$ in unpolarized gluon target is $xh_1^{\perp g}(x,\boldsymbol{p}_T)=2\alpha_sC_A(1-x)/(\pi^2\boldsymbol{p}_T^2)+\mathcal{O}(\alpha_s^2)$, in which the nonzero result starts at one-loop level, and only box diagram (see Fig. 1(a)) has nonzero contribution. So, if the external gluon is unpolarized, one can only work out the matching coefficient by calculating at least two-loop diagrams, which will be a rather tough task.

Instead, we assume that the external gluons are emitted from an unpolarized hadron and they are polarized, then extract $h_1^{\perp g}$ and $H_1^{\perp g}$ by calculating the helicity-flip matrix element, i.e., $\langle p,-|\cdots|p,+\rangle-\langle p,+|\cdots|p,-\rangle$, where $+/-$ denotes the gluon helicity $+1$ or -1. The

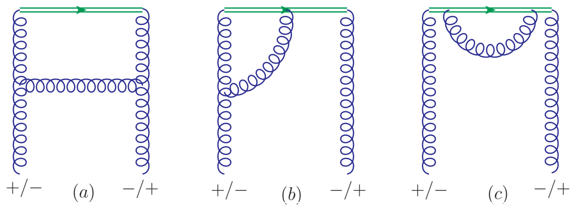

FIG. 1. The typical Feynman diagrams for one-loop correction of the operators in Eq. (2) and Eq. (3) in Feynman gauge.

amplitude for general gluon helicities can be expressed as $\mathcal{M}_{ij}\epsilon_1^i\epsilon_2^{*j}$, then the hecility-flip contribution we needed is $\mathcal{M}_{ij}(\epsilon_+^i\epsilon_-^{*j} - \epsilon_-^j\epsilon_+^{*i})$. We prefer to calculate in $(x, \boldsymbol{b}_T)$-space instead of in $(\xi, \boldsymbol{b}_T)$-space. One can replace the gluon density matrix $\epsilon_1^i\epsilon_2^{*j}$ with $\frac{1}{2}(b_T^i b_T^j/\boldsymbol{b}_T^2 + \frac{1}{2}g_T^{ij})$ to simplify the calculation. The tree-level result is no longer zero but $\delta(1-x)$.

Although the denominator and numerator are UV divergent, their ratio is UV finite, thus independent of the UV regulator. One can perform a one-loop calculation in dimensional regularization, in which the dimensions of spacetime are $d = 4-2\epsilon$. The decomposition of correlator in Eqs. (2)(3) in d-dimensions then becomes

$$-\frac{1}{d-2}\left[g_T^{ij} f_1^g - \left(\frac{b_T^i b_T^j}{\boldsymbol{b}_T^2} + \frac{1}{d-2}g_T^{ij}\right) h_1^{\perp g}\right].$$

In Fig. 1, we list three typical Feynman diagrams in the Feynman gauge at the one-loop level. Here we adopt the procedure in Ref. [37]. All the Feynman diagrams are categorized into three classes: (a): No Wilson line interaction; (b): Involving gluon-Wilson line interactions and (c): Wilson line self-interaction. For (a), we find that the TMD and quasi-TMD have the same results, and thus have no contribution to the matching coefficient; (c) has no contribution if the soft factor is subtracted. Because we are discussing the ratio, we do not need to consider the soft factor and hand because they are canceled in the ratio. (b) involves rapidity singularities and contributes to the matching. After some tedious but straightforward calculation, we find that the total result for both $H_1^{\perp g}$ at large P_3 and $h_1^{\perp g}$ have the structure

$$\begin{aligned}-\frac{\alpha_s}{2\pi}C_A&\left[\frac{2x}{(1-x)_+} + \frac{\beta_0}{2C_A}\delta(1-x)\right]\\ &\times\left(\frac{1}{\epsilon_{\rm IR}} + \ln\frac{\mu^2\boldsymbol{b}_T^2 e^{2\gamma_E}}{4}\right) + \delta(1-x)\mathcal{C}\,,\end{aligned} \tag{7}$$

where "+" denotes the plus distribution, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$. Note that the above expression is defined in the support $[0,1]$. The values of constant $\mathcal{C}$ are

$$\begin{aligned}\mathcal{C}_H = \frac{\alpha_s}{2\pi}C_A&\left[\left(\frac{1}{\epsilon_{\rm UV}} + \ln\frac{\mu^2\boldsymbol{b}_T^2 e^{2\gamma_E}}{4}\right)\left(\frac{\beta_0}{2C_A} - 1\right)\right.\\ &\left. -\frac{1}{2}\ln^2(p_3^2\boldsymbol{b}_T^2 e^{2\gamma_E}) + 2\ln(p_3^2\boldsymbol{b}_T^2 e^{2\gamma_E}) - 4\right],\end{aligned} \tag{8a}$$

$$\begin{aligned}\mathcal{C}_h = \frac{\alpha_s}{2\pi}C_A&\left[\frac{1}{\epsilon_{\rm UV}^2} + \left(\frac{1}{\epsilon_{\rm UV}} + \ln\frac{\mu^2\boldsymbol{b}_T^2 e^{2\gamma_E}}{4}\right)\right.\\ &\left.\times\left(\frac{\beta_0}{2C_A} + \ln\frac{\mu^2}{\zeta}\right) - \frac{1}{2}\ln^2\frac{\mu^2\boldsymbol{b}_T^2 e^{2\gamma_E}}{4} - \frac{\pi^2}{12}\right]\end{aligned} \tag{8b}$$

for quasi-TMD and TMD, respectively. The result for TMD here is subtracted by the soft factor; however, subtracting the soft factor or not does not affect the matching of the ratio. Because the IR structure of the normal and quasi-TMDs are the same, their differences are only UV related and the matching coefficients read

$$\begin{aligned}&H_h\left(\frac{\zeta_z}{\mu^2}\right) = H_f\left(\frac{\zeta_z}{\mu^2}\right)\\ &=1 + \frac{\alpha_s}{2\pi}C_A\left(-\frac{1}{2}\ln^2\frac{\zeta_z}{\mu^2} + 2\ln\frac{\zeta_z}{\mu^2} + \frac{\pi^2}{12} - 4\right)\\ &\quad + \mathcal{O}(\alpha_s^2)\,,\end{aligned} \tag{9}$$

where $\zeta_z = (2xP_3)^2$. The matching coefficients for $f_1^g(x, \boldsymbol{b}_T^2)$ and $h_1^{\perp g}(x, \boldsymbol{b}_T^2)$ are equal at one-loop accuracy. We are not sure that this relation holds to all orders of perturbation theory; on the other hand, it is likely that the matching coefficient for all of the eight gluons TMDs of twist-2 and their Euclidean partners are the same, at least at one-loop level. We also note that the matching coefficient is independent of x, as indicated in the matching formula Eq. (5). Therefore, the matching coefficient for the coordinate space TMDs $h_1^{\perp}(\xi, \boldsymbol{b}_T^2)$ and $H_1^{\perp g}(\xi, \boldsymbol{b}_T^2)$ should be the same one. Then, according to Eq. (6) and Eq. (9), one can conclude that $H_h/H_f = 1 + \mathcal{O}(\alpha_s^2)$, and

$$\begin{aligned}&f_1^g(\xi, \boldsymbol{b}_T^2, \mu, \zeta)/h_1^{\perp g}(\xi, \boldsymbol{b}_T^2, \mu, \zeta)\\ &\quad\simeq R(\xi, \boldsymbol{b}_T^2, P_3) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\Lambda_{\rm QCD}/P_3\right)\,.\end{aligned} \tag{10}$$

This indicates that the lattice results for $R(\xi, \boldsymbol{b}_T^2, P_3)$ is a very nice approximation of the degree of polarization because the perturbative matching effect is suppressed by α_s^2.

We note that the matching coefficient in Eq. (9) is derived in $\overline{\rm MS}$ scheme. In a practical lattice calculation, the commonly used schemes include the ratio scheme, RI/MOM, hybrid scheme, etc. In our proposal, we do not need those schemes because the ratio is confirmed to be UV finite, and although different results for hard functions H_h and H_f may be derived in different schemes, their ratio should be equal.

To summarize, we have explored the feasibility of calculating the TMD of linearly polarized gluons in an unpolarized hadron on the lattice, in the framework of large momentum effective theory. We propose to calculate the ratio of linearly polarized gluon TMD over the unpolarized gluon TMD, which characterizes the degree of gluon polarization. We define a Euclidean version of this ratio, which is UV finite. Therefore, no renormalization and soft factor subtraction are necessary. Furthermore, we evaluate the perturbative matching that connects the ratio and its light-cone partner and find that the perturbative matching coefficient is zero at one-loop. Thus the ratio discussed in this work is a good approximation of the ratio of $h_1^\perp$ and f_1^g. At last, the proposal in this work is not limited to the linearly polarized gluon TMD, but can also be extended to other TMDs when weighing their relative strength. Future lattice simulations will shed light on the distribution of linearly polarized gluons as well as other gluon TMDs in a hadron.

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