Introduction

Bjorken Sum Rule and Strong Coupling

Experimental Extraction of Bjorken Sum at Low-q
and (Effective) Strong Coupling

Summary

Acknowledgment: Thanks to Alexandre Deur and collaborators for the work in this talk and for providing slides
Introduction

Nucleon Spin Structure and Strong Interaction,
Nucleon Structure and Strong Interaction/QCD

- Nucleon Structure: discoveries
  - anomalous magnetic moment (1943 Nobel)
  - elastic: form factors (1961 Nobel)
  - DIS: parton distributions (1990 Nobel)

- Strong interaction, running coupling $\sim 1$
  - asymptotic freedom (2004 Nobel)
    - perturbation calculation works at high energy
  - interaction significant at intermediate energy, quark-gluon correlations
  - interaction strong at low energy confinement

- A major challenge in fundamental physics:
  - Understand QCD in all regions, including strong (confinement) region

- Nucleon: most convenient lab to study QCD

- Theoretical Tools:
  - pQCD, Lattice QCD, ChEFT, Sum Rules, …
UNPOLARIZED STRUCTURE FUNCTIONS

Q$^2$ evolution: the best test of QCD
Experiment – Theory Dialogue

• Theorist to experimentalist: (some time ago)
  give us spin structure functions in full phase space
  full range of $x$ [0-1], full range of $Q^2$: [0, $\infty$],
  we will take care of the rest (comparisons, understanding physics, …)
• Experimentalist: hmm…, we can only measure at limited region with some precision,
  and BTW, we also like to work with you to understand physics
• T: how about moments? we have some predictions at high $Q^2$ (PQCD) and low $Q^2$ (ChEFT)
• E: yes, we can measure moments in certain region
• T: we can make predictions on moments with LQCD and
  we are developing a method and might be able to predict $x$ dependence (recently)
• E: great, we are continuing to produce data, let’s find out how well data comparison with
  (PQCD, ChEFT, LQCD, …) predictions
  and how they can help us to understand QCD
POLARIZED STRUCTURE FUNCTIONS

**Proton**
- HERMES
- SMC
- E155
- E143

**Deuteron**
- COMPASS

**Neutron (^3He)**
- E142
- E154
- JLAB

\[ g_1(x, Q^2) + c_1 \]

**Graphs:**
- PDG (online 2023)
- Q^2 (GeV^2/c^2)

**Equation:**
- \( x = 0.0036 \) (i = 0)
- \( x = 0.0045 \)
- \( x = 0.0055 \)
- \( x = 0.007 \)
- \( x = 0.009 \)
- \( x = 0.012 \)
- \( x = 0.017 \)
- \( x = 0.024 \)
- \( x = 0.035 \)
- \( x = 0.049 \)
- \( x = 0.077 \) (i = 10)
- \( x = 0.12 \)
- \( x = 0.17 \)
- \( x = 0.22 \)
- \( x = 0.29 \)
- \( x = 0.41 \)
- \( x = 0.57 \)
- \( x = 0.74 \)
## Experiment Summary \((Q^2 > 0)\)

| Observable | H target | D target | \(^3\text{He} \text{ target} |
|------------|----------|----------|-----------------
| \(g_1, g_2, \Gamma_1 \& \Gamma_2\) at high \(Q^2\) | SLAC | SLAC | SLAC |
| \(\) | JLAB SANE | JLAB SANE | JLAB E97-117, JLAB E01-012, JLAB E06-014 |
| \(g_1 \& \Gamma_1\) at high \(Q^2\) | SMC | SMC | HERMES |
| \(\) | HERMES JLAB EG1 | HERMES JLAB EG1 | HERMES |
| \(\Gamma_1 \& \Gamma_2\) at low \(Q^2\) | JLab RSS | JLab RSS | JLab E94-010, JLab E97-103 |
| \(\Gamma_1\) at low \(Q^2\) | SLAC | SLAC | HERMES |
| \(\) | HERMES JLAB EG1 | HERMES JLAB EG1 | HERMES |
| \(\Gamma_1, Q^2 << 1 \text{ GeV}^2\) | JLab EG4 | JLab EG4 | JLab E97-110 |
| \(\Gamma_2, Q^2 << 1 \text{ GeV}^2\) | JLab E08-027 | JLab E08-027 | JLab E97-110 |
Bjorken Sum Rule and $Q^2$ dependence
Bjørken Sum Rule

\[ \Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \int \{g_1^p(x,Q^2) - g_1^n(x,Q^2)\} \, dx = \frac{1}{6} g_A C_{NS} \]

- \( g_A \): axial charge (from neutron \( \beta \)-decay)
- \( C_{NS} \): \( Q^2 \)-dependent QCD corrections (for flavor non-singlet)

- A fundamental relation relating an integration of spin structure functions to axial–vector coupling constant (axial charge)
- Based on Operator Product Expansion within QCD or Current Algebra
- Valid at large \( Q^2 \) (higher-twist effects negligible)
- Data are consistent with the Bjørken Sum Rule at 5–10 % level
(Generalized) Bjørken Sum Rule

\[
\Gamma_{I}^{p-n} = \frac{g_{A}}{6} \left[ 1 - \frac{\alpha_{s}}{\pi} - 3.58 \left( \frac{\alpha_{s}}{\pi} \right)^{2} - 20.21 \left( \frac{\alpha_{s}}{\pi} \right)^{3} + \cdots \right] + \sum_{i=2,3\ldots}^{\infty} \frac{\mu_{2i}^{p-n}(Q^{2})}{Q^{2i-2}},
\]

- A fundamental relation relating an integration of spin structure functions to axial-vector coupling constant (axial charge)
- Based on Operator Product Expansion within QCD or Current Algebra
- Valid at large \( Q^{2} \) (higher-twist effects negligible)
- Data are consistent with the Bjørken Sum Rule at 5–10 % level
Gerasimov-Drell-Hearn Sum Rule

Circularly polarized photon on longitudinally polarized nucleon

\[ \int_{\nu_{in}}^{\infty} \left( \sigma_{12}(\nu) - \sigma_{32}(\nu) \right) \frac{d\nu}{\nu} = -\frac{2\pi^2 \alpha_{EM}}{M^2} \kappa^2 \]

- A fundamental relation between the nucleon spin structure and its anomalous magnetic moment
- Based on general physics principles
  - Lorentz invariance, gauge invariance \(\rightarrow\) low energy theorem
  - unitarity \(\rightarrow\) optical theorem
  - causality \(\rightarrow\) unsubtracted dispersion relation
    applied to forward Compton amplitude
- Measurements on proton up to 800 MeV (Mainz) and up to 3 GeV (Bonn) agree with GDH with assumptions for contributions from un-measured regions
  New measurements on p, d and \(^3\)He from LEGS, MAMI(2), …
Generalized GDH Sum Rule

- Many approaches: Anselmino, Ioffe, Burkert, Drechsel, ...

  Forward Virtual-Virtual Compton Scattering Amplitudes: $S_1(Q^2,v), S_2(Q^2, v)$

  Same assumptions: no-subtraction dispersion relation
  optical theorem
  (low energy theorem)

- Generalized GDH Sum Rule

$$S_1(Q^2) = 4 \int_{el}^{\infty} G_1(Q^2, v) dv$$
Connecting GDH with Bjorken Sum Rules

- $Q^2$-evolution of GDH Sum Rule provides a bridge linking strong QCD to pQCD
  - Bjorken and GDH sum rules are two limiting cases
    - High $Q^2$, Operator Product Expansion: $S_1(p-n) \sim g_A$ $\rightarrow$ Bjorken
    - $Q^2 \rightarrow 0$, Low Energy Theorem: $S_1 \sim \kappa^2$ $\rightarrow$ GDH
  - High $Q^2$ ($> \sim 1$ GeV$^2$): Operator Product Expansion
  - Intermediate $Q^2$ region: Lattice QCD calculations?
  - Low $Q^2$ region ($< \sim 0.1$ GeV$^2$): Chiral Perturbation Theory

Calculations: $\chi$PT: Ji, Kao, ..., Vanderhaeghen, ...
- Lensky, Alarcon & Pascalutsa
- Bernard, Hemmert, Meissner
World data on $\Gamma_1$ for proton and neutron

Previous Publications and
New Low-Q data: talks on EG4 (A. Deur for M. Ripani on Tuesday
and E97-110 (A. Deur, next)
Bjorken Sum: $\Gamma_1$ of $p-n$ \textit{(before new low-Q data)}

A. Deur, \textit{et al.}

EG1b, PRD 78, 032001 (2008)
E94-010 + EG1a: PRL 93 (2004) 212001
Bjorken Sum \((p-n)\) (before new low-\(Q\))

- Low \(Q^2\): test of \(\chi pt\) calculations

Bernard et al., PRD 87 (2013)

Lensky, Alarcon & Pascalutsa
PRC 90 055202 (2014)
Effective $\alpha_s$ Extracted from Bjorken Sum (before new low-$Q$)

A. Deur, V. Burkert, J. P. Chen and W. Korsch
PLB 650, 244 (2007) and PLB 665, 349 (2008)
The strong coupling $\alpha_s$ at short distances (large $Q^2$)

$\alpha_s$ is not constant due to loops in gluon propagator, fermion self-energy, and vertex corrections:

$\alpha_s$ becomes small at short distances (large $Q^2$)

$\Rightarrow$ Asymptotic freedom:

perturbative treatment of QCD (pQCD). $\alpha_s(Q^2)$ is well defined within pQCD.

Figure adapted from Particle Data Group, 2020.
The strong coupling $\alpha_s$ at short distances (large $Q^2$)

$\alpha_s(Q^2) \Rightarrow$ needs data or non-perturbative methods to get $\alpha_s(Q^2)$.

Lattice calculations: currently most accurate determination of $\alpha_s(M_Z^2)$.

Otherwise, $\alpha_s(Q^2)$ is extracted from data, e.g. Bjorken sum rule:

$$\int (g_p^1 - g_n^1) dx = \frac{1}{6} g_A (1 - \frac{\alpha_s}{\pi} - 3.58(\frac{\alpha_s}{\pi})^2 - ...)$$

Figure adapted from Particle Data Group, 2020.
Projection of JLab22 (+ EIC) on Extraction of $\alpha_s$

JLab22 + EIC can make a significant improvement in the extraction of $\alpha_s$.

A. Deur, contribution to the JLab22 Whitepaper (to be published)

\[ \Gamma_{1}^{p-n} = \frac{1}{6} g_A \left[ 1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^3 - 175.7 \left( \frac{\alpha_s}{\pi} \right)^4 - \sim 893 \left( \frac{\alpha_s}{\pi} \right)^5 \right] + \frac{a}{Q^2}. \]
At $Q^2 \lesssim 1\text{GeV}^2$, pQCD cannot be used to define $\alpha_s$: if pQCD is trusted, $\alpha_s \to \infty$ when $Q \to \Lambda_{\text{QCD}}$.

- Contradict the perturbative hypothesis;
- The divergence (Landau pôle) is unphysical.

Definition and computation of $\alpha_s$ at long distance?
Bjorken Sum at Low-Q and Effective $\alpha_s$
Bjorken Sum: $\Gamma_1$ of $p-n$ (EG4 and E97-110)

A. Deur, et al.

Proton-neutron = Bjorken sum

$\Delta$-resonance contribution suppressed for the Bjorken sum

Fit $\Gamma_1 = bQ^2 + cQ^4$:

<table>
<thead>
<tr>
<th>Data set</th>
<th>$(b \pm \text{uncor} \pm \text{cor}) \ [\text{GeV}^{-2}]$</th>
<th>$c \pm \text{uncor} \pm \text{cor} \ [\text{GeV}^{-4}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>World data</td>
<td>0.182 $\pm$ 0.016 $\pm$ 0.034</td>
<td>$-0.117 \pm 0.091 \pm 0.095$</td>
</tr>
<tr>
<td>GDH Sum Rule</td>
<td>0.0618</td>
<td></td>
</tr>
<tr>
<td>$\chi$EFT Bernard et al.</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>$\chi$EFT Alarcón et al.</td>
<td>0.066(4)</td>
<td>0.3</td>
</tr>
<tr>
<td>Burkert-Ioffe</td>
<td>0.09</td>
<td>0.25(12)</td>
</tr>
<tr>
<td>Pasechnik et al.</td>
<td>0.09</td>
<td>0.4</td>
</tr>
<tr>
<td>LFHQCD</td>
<td>0.177</td>
<td>-0.067</td>
</tr>
</tbody>
</table>
\( \alpha_s \) at long distance (low Q)

**Prescription:** Define effective couplings from an observable’s perturbative series truncated to first order in \( \alpha_s \).


**Ex: Bjorken sum rule:**

\[
\int (g_p^p - g_n^p) \, dx \equiv \Gamma_{f_p-n} = \frac{1}{6} g_A \left( 1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 \right) + \frac{M^2}{9Q^2} [a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)] + \ldots
\]

\( \int (g_p^p - g_n^p) \, dx \equiv \Gamma_{f_p-n} = \frac{1}{6} g_A \left( 1 - \frac{\alpha_s}{\pi} \right) \)

**Nucleon axial charge.**

**pQCD corrections (gluon bremsstrahlung)**

\[
\Rightarrow \quad \Gamma_{f_p-n} \equiv \frac{1}{6} g_A \left( 1 - \frac{\alpha_s}{\pi} \right)
\]

This means that additional short distance effects, and long distance confinement force and parton distribution correlations are now folded into the definition of \( \alpha_s \).

Analogy with the original coupling constant becoming an effective coupling when short distance quantum loops are folded into its definition.
\( \alpha_{g1} \) Extracted from the Bjorken Sum data

**Bjorken sum** \( \Gamma_1^{p-n} \) measurements

\[ \Gamma_1^{p-n} \propto \frac{g_A}{6} g_A (1 - \frac{\alpha_{g1}}{\pi}) \]
At $Q^2 = 0$, a sum rule related to the Bjorken sum rule exists: the Gerasimov-Drell-Hearn (GDH) sum rule:

At $Q^2 = 0$, GDH sum rule:

$$\Gamma_1 = \frac{-\kappa^2 Q^2}{8M^2}$$

⇒ $Q^2 = 0$ constraints:

$$\alpha_{g1} = \pi$$

⇒

$$\frac{da_{g1}}{dQ^2} = \frac{3\pi}{4g_A} \left( \frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2} \right)$$

First experimental evidence of nearly *conformal behavior* (i.e. no $Q^2$-dependence) of QCD at low $Q^2$. 

**Low Q limit**

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$\alpha_{g1}/\pi$ DESY HERMES
$\alpha_{g1}/\pi$ CERN COMPASS
$\alpha_{g1}/\pi$ SLAC E142/E143
$\alpha_{g1}/\pi$ SLAC E154/E155
$\alpha_{g1}/\pi$ JLab RSS
$\alpha_{g1}/\pi$ CERN SMC
$\alpha_{g1(\tau)}/\pi$ OPAL
$\alpha_{F3}/\pi$
Comparisons with SDE and LFHQCD Calculations

Binosi et al. PRD 96, 054026 (2017)
Brodsky, de Téramond, Dosch, Lorcé. PLB 759, 171 (2016)

⇒ SDE, LFHQCD and data agree very well.
Effective Coupling and Impact

Featured as Cover
Featured in JLab News
Featured in YouTube
https://www.youtube.com/watch?v=8BTZOz850GI&t=497s
hailed as
“accidental discovery”
“pretty major breakthrough”

Base for understanding of emergence of hadron properties, can have impact on:

- hadron spectroscopy
- PDFs and GPDs
- quark mass functions
- pion decay constant
- scale of QCD, \( \Lambda_{\text{QCD}} \)
- QCD Phase/Hot QCD

\[
\alpha_{g1}(Q) \pi
\]

A. Deur, V. Burkert, J. P. Chen and W. Korsch
Particles, 5-171 (2022)
Summary

• Bjorken Sum Rule: Link flavor non-singleton (isovector) part of the nucleon spin structure moment with the axial charge

• Generalized Bjorken/GDH Sum Rules provide a tool to study QCD in full $Q^2$ range
  • Extractions of (effective) strong coupling $\alpha_s (\alpha_{g1})$

• Experimental Data on Bjorken Sum Over a Wide $Q^2$ range
  • High $Q^2$: PQCD, extraction of strong coupling $\alpha_s$, potential of JLab22 + EIC
  • Intermediate $Q^2$: Transition from PQCD to Strong QCD region
  • Low $Q^2$: Strong QCD region, 1st extraction of effective strong coupling $\alpha_{g1}$

  Extracted effective strong coupling from the new JLab low-$Q$ data
  → conformal behavior,
  providing a potential base for understanding strong QCD
  significant impact
Proton-neutron = Bjorken sum

\[ \Delta \text{-resonance contribution} \text{ suppressed for the Bjorken sum} \]

\[ \Gamma_1 = bQ^2 + cQ^4 : \]

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<tr>
<th>Data set</th>
<th>( (a \pm \text{uncor} \pm \text{cor}) ) [GeV(^{-2})]</th>
<th>( (b \pm \text{uncor} \pm \text{cor}) ) [GeV(^{-2})]</th>
<th>( c \pm \text{uncor} \pm \text{cor} ) [GeV(^{-4})]</th>
<th>( d \pm \text{uncor} \pm \text{cor} ) [GeV(^{-6})]</th>
<th>( \chi^2/n.d.f. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EG4, no low-x</td>
<td>NA</td>
<td>0.093 ± 0.032 ± 0.000</td>
<td>(-0.137 \pm 0.191 \pm 0.000)</td>
<td>NA</td>
<td>1.24</td>
</tr>
<tr>
<td>EG4/E97110, no low-x</td>
<td>NA</td>
<td>0.112 ± 0.022 ± 0.028</td>
<td>(-0.123 \pm 0.118 \pm 0.078)</td>
<td>NA</td>
<td>1.00</td>
</tr>
<tr>
<td>EG4</td>
<td>NA</td>
<td>0.170 ± 0.032 ± 0.000</td>
<td>(-0.046 \pm 0.191 \pm 0.000)</td>
<td>NA</td>
<td>1.04</td>
</tr>
<tr>
<td>EG4/E97110</td>
<td>NA</td>
<td>0.185 ± 0.023 ± 0.027</td>
<td>(-0.144 \pm 0.123 \pm 0.075)</td>
<td>NA</td>
<td>1.00</td>
</tr>
<tr>
<td>World data</td>
<td>NA</td>
<td>( b^{\text{GDH}} \equiv 0.0618 )</td>
<td>(1.41 \pm 0.17 \pm 0.39)</td>
<td>(-4.30 \pm 0.80 \pm 1.48)</td>
<td>1.97</td>
</tr>
<tr>
<td>World data</td>
<td>((4.3 \pm 1.8 \pm 0.1) \times 10^{-3})</td>
<td>0.092 ± 0.042 ± 0.031</td>
<td>(0.213 \pm 0.167 \pm 0.086)</td>
<td>NA</td>
<td>0.82</td>
</tr>
</tbody>
</table>

\[ \chi_{EFT} \text{ prediction} \]

\[ \Gamma_1 = bQ^2 \]

\[ \chi_{EFT} \text{ prediction} \]

Fit \( \Gamma_1 = a + bQ^2 + cQ^4 + dQ^6 : \)
When charges are quantized: \( (\text{coupling constant})^{1/2} \) normalizes the unit charge to 1 (e.g. \( \alpha \))
\[ \Rightarrow \text{set the magnitude of the force (classical domain) or the probability amplitude to emit a quantum force vector (QFT).} \]

**Force** = \( \text{coupling constant} \times \text{charge}_1 \times \text{charge}_2 \times f(r) \)

(static case)

\( \alpha (\text{QED}), \alpha_s (\text{QCD}), G_F (\text{Weak Force}), G_N (\text{gravity}). \)

Quantum effects induce an energy dependence.

(effective couplings: the couplings are “running”)
The effective coupling is then:
  • Extractable at any $Q^2$;
  • Free of divergence;
  • Renormalization scheme independent.
But it is:
  • Process dependent.

⇒ There is *a priori* a different $\alpha_s$ for each different process.

However these $\alpha_s$ can be related (Commensurate Scale Relations).


⇒ pQCD retains it predictive power.

Such definition of $\alpha_s$ using a particular process is equivalent to a particular choice of renormalization scheme.

$$\alpha_{g_1} = \alpha_s$$ in the “$g_1$ scheme”.

Relations between $g_1$ scheme and other schemes are known in pQCD domain, e.g.

$$\Lambda_{g_1} = 2.70 \Lambda_{\overline{\text{MS}}} = 1.48 \Lambda_{\text{MOM}} = 1.92 \Lambda_V = 0.84 \Lambda_\tau.$$