Spectroscopy of Mesons Produced by Linearly Polarized Photons

Carlos W. Salgado
Norfolk State University, Norfolk, VA, USA
and
The Thomas Jefferson National Facility,
Newport News, VA, USA

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A formalism for the experimental analysis of mesons produced by a beam of linearly polarized photons is presented. This formalism introduces a more general use of the reflectivity operator. The goal is to recognize resonances in cross sections, their associated quantum numbers, and production mechanisms by performing partial wave analysis of multiple-meson final states.

I. INTRODUCTION

One of the standard experimental procedures to search for strongly interacting bounded states (of gluons and quarks) is to identify resonances in the production cross sections. However, physics information related to the production and decay of those states is contained in their complex amplitudes, therefore, resonances in the mass spectrum are only intricately related to the model parameters. Quantum Chromodynamics (QCD) is the current theory of strong interactions, however, we have not been able to analytically calculate the cross sections of bound states from fundamental QCD. Therefore, QCD-inspired models are necessary to relate the observables of the bound states to the amplitudes and study confinement in those bound states. These models are mainly based on the general properties of the S-matrix [1] as relativity, causality, and the conservation of probability. In recent years, numerical approximations to QCD (Lattice QCD) have been used to compute resonance properties [2]. This paper describes a method to include parity conservation in a phenomenological formalism. We specifically consider mesons produced by linearly polarized photons, and Parity is assumed to be conserved by all strong interactions. The definition of a reflectivity operator integrates parity conservation into the analysis formalism. We consider the information obtained by knowing the polarization of the incoming photon beam and how can relate to the production mechanisms. The properties of the photon beam are described by a spin density matrix. We discuss a new use of the reflectivity operator, where we also include information about beam polarization. As an example of the application of this formalism, we describe, in a specific simple model, how this formalism can be used in a mass-independent partial wave analysis (PWA) of multi-particle final states.

II. CROSS SECTIONS

We consider multiple final state mesons produced by linearly polarized photons diffractively colliding off a proton target at rest. The outline of the reaction is shown in Figure 1.

Let τ represent the complete set of variables needed to describe the decay of the resonance. In the case of two final state mesons, only two angles will be needed. We use the (θ, ϕ) angles of one meson in the Gottfried-Jackson (GJ) frame (see Appendix A in reference [3] for frame definitions). In the case of more than two mesons in the final state, at least two more angles for each extra meson will be required.

The cross section for the reaction $\gamma p \to Xp$, where $X \to (mesons)$ will be written as [3]

$$\frac{d\sigma}{dE_\gamma dt dM d\tau} = \frac{1}{BeamFlux(E_\gamma)} \sum_{ext.\ spins} |M|^2 d\rho \quad (1)$$

where M is the Lorentz-invariant (transition or scattering) amplitude and $d\rho$ is the Lorentz invariant phase-space element (LIPS). The spin's incoming and outgoing degrees of freedom are included in the sum over spins. The LIPS include the kinematical constraints and M include the spin and production/decay internal (transition) degrees of freedom. We can write $d\rho \propto \sum_i \frac{d^3p_i}{2E_i}$ where i runs over all the incoming and outgoing particles.

To measure cross-sections experimentally, we normally "bin" or divide data into small ranges of one variable such that the dependence of the cross-section on that variable is suppressed. For example, if this division is done with the mass, energy, and t-Mandelshtam, only the angular dependencies for two produced particles will remain (more are needed for more final state particles, i.e. perhaps isobar properties, see section 5.1.2). All the "external" (normalization) dependencies can be taken into an overall constant, κ (that we will just drop out afterward from the formulas as they will not affect the overall behavior). Therefore, in a data bin (E_{gamma}, t and M), we define an intensity

$$I(\tau) \equiv \frac{d\sigma}{d\tau} = \kappa \cdot \sum_{ext.\ spins} |M|^2 \quad (2)$$

M is a representation of the scattering operator or transition operator, T, it can be written as

$$M = \langle out|T|in\rangle \quad (3)$$

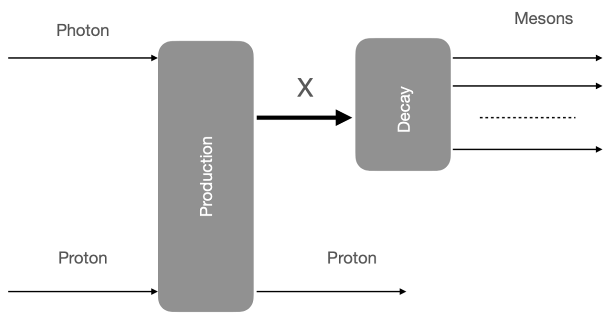

FIG. 1. Reaction Model.

and then

$$I(\tau) = \sum_{ext.\ spins} |M|^2 = \sum_{ext.\ spins} \langle out|T|in\rangle(\langle out|T|in\rangle)^* \tag{4}$$

and, further

$$\langle out|T|in\rangle(\langle out|T|in\rangle)^* = \langle out|T|in\rangle\langle in|T^*|out\rangle. \tag{5}$$

In the case that we have spin information from the incoming photon beam, we can include the beam polarization in our amplitude by defining the matrix $|in\rangle\langle in|$, the *photon spin density matrix operator*, ρ_{in}, (see reference [4] for details) as

$$\rho_{in} \equiv |in\rangle\langle in|. \tag{6}$$

Suppose that we prepare the polarization of the incoming photons or measure their states of polarization. The average over photon polarization will be completely described by this spin density matrix. In the case of a beam of linearly polarized photons, any polarized state can be written as a linear combination of two pure polarization states. Therefore, the general structure of this 2×2 matrix (for example in the helicity basis defined by $|\lambda\rangle$ and $|\lambda'\rangle$) will be

$$\rho_{in} = \rho^{\gamma}_{\lambda,\lambda'}. \tag{7}$$

$$I(\tau) = \sum_{ext.\ spins} \sum_{\lambda,\lambda'} \langle out|T^{\lambda}\rho^{\gamma}_{\lambda\lambda'}T^{\lambda'*}|out\rangle. \tag{8}$$

The "*ext. spins*" are now the target ($\lambda_1 = \pm$) and recoil ($\lambda_2 = \pm$) helicities. We will assume that the transition operator can be factorized into two parts: the production (of X) and the decay operators (of X) such that:

$$I(\tau) =$$

$$\sum_{\lambda_1\lambda_2} \sum_{\lambda,\lambda'} \langle out|T_{decay}\left[T^{\lambda,\lambda_1,\lambda_2}_{prod}\right]\rho^{\gamma}_{\lambda,\lambda'}\left[T^{\lambda',\lambda_1,\lambda_2}_{prod}\right]^* T^*_{decay}|out\rangle$$
(9)

Furthermore, we can take a completely orthogonal set of states, $|X\rangle$, such that $\sum_X |X\rangle\langle X| = \mathbf{1}$, and include them in the previous relation such that

$$I(\tau) = \sum_{\lambda_1\lambda_2} \sum_{\lambda,\lambda'} \langle out|T_d \sum_X |X\rangle\langle X|T^{\lambda,\lambda_1,\lambda_2}_p \rho^{\gamma}_{\lambda,\lambda'}\left[T^{\lambda',\lambda_1,\lambda_2}_p\right]^*$$

$$\sum_{X'} |X'\rangle\langle X'|T^*_d|out\rangle \tag{10}$$

$$I(\tau) = \sum_{\lambda_1\lambda_2} \sum_{\lambda,\lambda'} \sum_{X,X'} \langle out|T_d|X\rangle\langle X|T^{\lambda,\lambda_1,\lambda_2}_p$$

$$\rho^{\gamma}_{\lambda,\lambda'}\left[T^{\lambda',\lambda_1,\lambda_2}_p\right]^*|X'\rangle\langle X'|T^*_d|out\rangle. \tag{11}$$

The set of states, $|X\rangle$, a full set of intermediate states, we call the *partial waves*. Each of these states can be described by a set of quantum numbers, for example $l, m, isobars(mass, width)$.... The total angular momentum by $J = l \oplus s$ ($|l - s < J < |l + s|$) and the total spin $s = s_1 \oplus s_2$. Where l = 0,1,2... (S,P,D...) and m ($-l < m < l$) will define the "waves" of the expansion. In practice, this expansion is truncated (to a very few states). We will refer to these quantum numbers as (l,m,I) where the I include all other parameters needed for a more extended model, for example the isobar model parameters (see section 5.1.2).

The production amplitudes describe the strong interaction production mechanism that we are not able to calculate (without a phenomenological model). In a mass-independent PWA fit, the production amplitudes, in a given bin, will be considered constant, independent of the decay properties (for example final particle angles), They function as a *weights* on each partial decay amplitude of the final mix, and will be extracted (fitted) from the data. We will rewrite

$$\langle X_{l,m,I}|T^{\lambda,\lambda_1,\lambda_2}_p \rho^{\gamma}_{\lambda\lambda'} T^{\lambda',\lambda_1,\lambda_2}_p|X_{l',m',I'}\rangle =$$

$$T^{\lambda,\lambda_1,\lambda_2}_{l,m,I}\rho^{\gamma}_{\lambda\lambda'}\left[T^{\lambda',\lambda_1,\lambda_2}_{l',m',I'}\right]^* \tag{12}$$

and calling,

$$\langle out|T_d|X_{l,m,I}\rangle \ = A_{l,m,I}(\tau) \tag{13}$$

being $T^{\lambda,\lambda_1,\lambda_2}_{l,m,I}$ the production amplitudes and $A_{l,m,I}(\tau)$ the decay amplitudes. Note that the A's and T's are both complex numbers and that A depends of the resonance quantum numbers and angles and T depends on the resonance and beam (photon) quantum numbers. The photon spin density matrix depends on the partial polarization P and the polarization angle Φ, (see reference [4] for definitions).

Therefore, in the helicity basis [5], λ being the helicities of the incoming photon, and λ_1, λ_2 the helicities of the target (outgoing) nucleons:

$$I(\tau,P,\Phi) = \sum_{\lambda_1\lambda_2}\sum_{\lambda,\lambda'}\sum_{(l,m,I,l',m',I')}$$

$$A_{l,m,I}(\tau)\left[T^{\lambda,\lambda_1,\lambda_2}_{l,m,I}\right]\rho^{\gamma}_{\lambda,\lambda'}(P,\Phi)\left[T^{\lambda',\lambda_1,\lambda_2}_{l',m',I'}\right]^* \ A^*_{l',m',I'}(\tau). \tag{14}$$

For example, in a two-mesons final state, we have 2(from λ_1)$\cdot$ 2(from λ_2)$\cdot$ 2(from λ) $\cdot\,(2l+1)$ unknown parameters $({}^{\lambda}T^{\lambda_1\lambda_2}_{l,m,I})$ for each wave $l = S, P, D, ...$ to be fitted to the data .

III. REFLECTIVITY

The effect of parity is defined as the inversion of the spatial coordinates with respect to the origin of coordinates. Most reactions in HEP are unchanged under this operation (as only weak interactions violate parity).

In our case, assuming vector-meson dominance for the photon and diffractive scattering from the nucleon, and since the strong interaction conserves parity, the parity operator commutes with the scattering matrix (or transition operator). Helicity states, however, are not eigenstates of the parity operator and therefore, they are not directly related to the parity exchanged in the reaction.

The parity operation is equivalent to a "mirror reflection" with respect to an arbitrary plane, followed by a π rotation with respect to an axis orthogonal to that plane. Let's call $\widehat{\Pi}$, the parity operator. Since the parity operation acting on rotations only changes the direction (sign), in the canonical representation (and in the rest frame of the particle), we have

$$\widehat{\Pi}|J,m\rangle = P|J,m\rangle \tag{15}$$

where $P = \pm 1$ are its eigenvalues. Let's consider a particle moving with momentum $\overrightarrow{p_z}$ in the z direction. We can get this state by boosting (L is a Lorentz transformation) the state at rest

$$|\overrightarrow{p_z}J,m\rangle = L(\overrightarrow{p_z})|0;J,m\rangle. \tag{16}$$

, Applying the parity operator

$$\widehat{\Pi}|\overrightarrow{p_z}J,m\rangle = \widehat{\Pi}L(\overrightarrow{p_z})|0;J,m\rangle \tag{17}$$

$$\widehat{\Pi}|\overrightarrow{p_z}J,m\rangle = PL(-\overrightarrow{p_z})|0;J,m\rangle. \tag{18}$$

To get back from $(-\overrightarrow{p_z})$ to $(\overrightarrow{p_z})$ we need a rotation of modulo π around the y axis

$$L(\overrightarrow{p_z}) = e^{-i\pi J_y}L(-\overrightarrow{p_z})e^{i\pi J_y} \tag{19}$$

and we know that

$$e^{-i\pi J_y}|\overrightarrow{p_z}J,m\rangle = (-1)^{J-m}|\overrightarrow{p_z}J,-m\rangle \tag{20}$$

Therefore, we finally have

$$\widehat{\Pi}|\overrightarrow{p_z}J,m\rangle = P(-1)^{J-m}e^{i\pi J_y}|\overrightarrow{p_z}J,-m\rangle. \tag{21}$$

Since any other direction can be constructed by rotation, and the parity operator commutes with rotations (in the x-z plane), we can express the former formula, in the rest frame of the resonance , y perpendicular to the production plane (GJ/HEL frames) with the spin quantization in the z-axis given by m

$$\widehat{\Pi}|J,m\rangle = P(-1)^{J-m}e^{i\pi J_y}|J,-m\rangle. \tag{22}$$

It is useful to define the reflection operator [6]

$$\widehat{R}_y = \widehat{\Pi}e^{-i\pi J_y} \tag{23}$$

that involves parity and a π angular rotation around the y axis either in the GJ or HEL frames. It represents a mirror *reflection* through the production plane (x,z). This operator commute with the transition operator. The y axis in the GJ/HEL frame is perpendicular to the production plane, therefore the transition matrix is independent of y, and only the x, z coordinates participate in the parity transformation. Reflection commutes with the Hamiltonian. The reflection operator acting on the resonance states produces

$$\widehat{R}_y|J,m\rangle = e^{-i\pi J_y}\widehat{\Pi}|J,m\rangle = \tag{24}$$

$$e^{-i\pi J_y}P(-1)^{J-m}e^{i\pi J_y}|J,-m\rangle = P(-1)^{(J-m)}|J,-m\rangle \tag{25}$$

where P are the parity eigenvalues ($\pm$). We can build the following eigenstates of $\widehat{R}_y$ (since the reflection changes signs on the z-projection quantum numbers, m, we will create eingenstates that are a linear combination of both (m) signs states with adequate coefficients)

$$|\epsilon,J,m\rangle = \left[|J,m\rangle - \epsilon P(-1)^{(J-m)}|J,-m\rangle\right]\Theta(m) \tag{26}$$

The sign between both terms in equation (26) is arbitrary. We use the sign definition in reference [3] and defining:

$$\Theta(m) = \frac{1}{\sqrt{2}},\ if\ m>0\ ;\Theta(m) = \frac{1}{2},\ if\ m=0 \tag{27}$$

and

$$\Theta(m) = 0,\ if\ m<0 \tag{28}$$

It can be shown (see ref. [3]) that the ϵ's are the real (for mesons) eigenvalues of the reflectivity operator. We define a *resonance reflectivity* $= \epsilon_R$ as

$$|\epsilon_R, J, |m|\rangle = \left[|J,m\rangle - \epsilon_R P(-1)^{(J-m)}|J,-m\rangle\right]\Theta(m) \tag{29}$$

In our previous notation,

$$A^{\epsilon_R}_{J,|m|}(\tau) = \left[A_{J,m}(\tau) - \epsilon_R P(-1)^{(J-m)}A_{J,-m}(\tau)\right]\Theta(m) \tag{30}$$

Notice that since each state defined in the reflectivity basis includes a combination of m and $-m$, the projections of the spin on the quantization axis, m, is replaced by $|m|$ (a kind of "absolute value"). We can think as the reflectivity ϵ_R "carrying" the sign of m. When we sum on possible quantum numbers for each wave (l), we have (2l+1) terms in this sum; we have $2 \cdot l$ for two reflectivities for each $m > 0$ plus one $\epsilon_R = -1$ for m=0 [3].

In pion beam experiments [7] (spinless beam) or past photo production experiments (CLAS) [8], where no information on the beam polarization was available, the spin density matrix is (or is considered) a constant (see [3]) and, therefore it can be factor out from the intensity expression. The past CLAS formalism [3] includes the helicity of the photon in the rank of the matrices (in the external sum of spins). Invoking parity conservation we still reduced the number of degrees of freedom from eight to four, and the reflectivity was only defined for the resonance. Again, this was done for unpolarized photons or when no information about the photon polarization is available.

In the case we are considering, of having information about the photon polarization, it will be proper to also define a reflectivity state for the photon. For a real photon $P = -1$, $J = 1$ and $\lambda = +1, -1$,therefore we define a *photon reflectivity* $= \epsilon_\gamma$ from:

$$|\epsilon_\gamma, \lambda\rangle = \left[|\lambda\rangle - \epsilon_\gamma(-1)^\lambda|-\lambda\rangle\right]\Theta(\lambda) \tag{31}$$

then (the reflectivity eigenvalues for a photon are $\epsilon_\gamma = \pm 1$).

$$\begin{array}{l} |\epsilon_\gamma = +1, \lambda = +1\rangle = \frac{1}{\sqrt{2}}(|\lambda = +1\rangle + |\lambda = -1\rangle) \\ |\epsilon_\gamma = -1, \lambda = +1\rangle = \frac{1}{\sqrt{2}}(|\lambda = +1\rangle - |\lambda = -1\rangle) \end{array}. \tag{32}$$

Equation (14), in this new (two) reflectivity basis, is then:

$$I(\tau, P, \Phi) = \sum_{\lambda_1\lambda_2}\sum_{\epsilon_R\epsilon'_R,\epsilon_\gamma\epsilon'_\gamma}\sum_{J,|m|,J',|m'|}$$

$$A^{\epsilon_R}_{J,|m|}(\tau)T^{\epsilon_R\epsilon_\gamma,\lambda_1\lambda_2}_{J,|m|}\rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma}(P,\Phi)T^{\epsilon'_R\epsilon'_\gamma,\lambda_1\lambda_2 *}_{J',|m'|}A^{\epsilon'_R *}_{J',|m'|}(\tau) \tag{33}$$

The photon spin density matrix in the photon reflectivity basis has the form (see Appendix or reference [4]).

$$\rho_{\epsilon_\gamma,\epsilon'_\gamma}(P,\Phi) = 1/2\begin{pmatrix} 1 - P\cos 2\Phi & -iP\sin 2\Phi \\ iP\sin 2\Phi & 1 + P\cos 2\Phi \end{pmatrix} \tag{34}$$

We now write the expression for the intensity, where $|m|$ are defined positive in the reflectivity basis, and we include the "resonance" ϵ_R and "photon" ϵ_γ reflectivities.

There are only two degrees of freedom associated with λ_1, λ_2 target spins, we will call them $k = 1, 2$ (spin-flop and no spin-flop).

$$I(\tau, P, \Phi) = \sum_k\sum_{\epsilon_\gamma,\epsilon'_\gamma}\sum_{\epsilon_R,\epsilon'_R}\sum_{l|m|I,l'|m'|I'}$$

$$A^{\epsilon_R}_{l,|m|,I}(\tau)T^{\epsilon_R\epsilon_\gamma,k}_{l,|m|,I}\;\rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma}(P,\Phi)\left[T^{\epsilon'_R\epsilon'_\gamma,k}_{l',|m'|,I'}\right]^*\left[A^{\epsilon'_R}_{l',|m'|,I'}(\tau)\right]^*. \tag{35}$$

We have organized the indices such that k are the external or non-interfering indices. Expanding the sum over the photon reflectivities (using the photon spin density matrix) we have (just for clarity we drop the k, I indexes in the next expression):

$$I(\tau, P, \Phi) = \sum_{\epsilon_R,\epsilon'_R}\sum_{l|m|,l'|m'|}\Bigg[$$

$$(1 - P\cos 2\Phi)A^{\epsilon_R}_{l,|m|}(\tau)T^{\epsilon_R,+}_{l,|m|}\;\left[T^{\epsilon'_R,+}_{l',|m'|}\right]^*\;\left[A^{\epsilon'_R}_{l',|m'|}(\tau)\right]^*$$

$$+(-iP\sin 2\Phi)A^{\epsilon_R}_{l,|m|}(\tau)T^{\epsilon_R,+}_{l,|m|}\;\left[T^{\epsilon'_R,-}_{l',|m'|}\right]^*\;\left[A^{\epsilon'_R}_{l',|m'|}(\tau)\right]^*$$

$$+(iP\sin 2\Phi)A^{\epsilon_R}_{l,|m|}(\tau)T^{\epsilon_R,-}_{l,|m|}\;\left[T^{\epsilon'_R,+}_{l',|m'|}\right]^*\;A^{\epsilon'_R}_{l',|m'|}(\tau)\big]^*$$

$$+(1 + P\cos 2\Phi)A^{\epsilon_R}_{l,|m|}(\tau)T^{\epsilon_R,-}_{l,|m|}\;\left[T^{\epsilon'_R,-}_{l',|m'|}\right]^*\;\left[A^{\epsilon'_R *}_{l',|m'|}(\tau)\right]^*\Bigg] \tag{36}$$

We define the *resonance spin density matrices* as

$${}^{\epsilon_R\epsilon'_R}\rho^{l,l'}_{|m|,|m'|} = \sum_k\sum_{\epsilon_\gamma\epsilon'_\gamma}T^{\epsilon_\gamma\epsilon_R,k}_{l,m}\;\rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma}\left[T^{\epsilon'_\gamma\epsilon'_R,k}_{l',m'}\right]^* \tag{37}$$

Then

$$I(\phi, \theta, P, \Phi) = \sum_{\epsilon_R\epsilon'_R}\sum_{l,|m|,l',|m'|}$$

$${}^{\epsilon_R\epsilon'_R}\rho^{ll'}_{|m||m'|}\;{}^{\epsilon_R}Y^{|m|}_l(\phi,\theta)\;{}^{\epsilon'_R}Y^{|m'|*}_{l'}(\phi,\theta). \tag{38}$$

We can write (see reference [4])

$$\rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma} = \frac{1}{2}\left(\mathbf{1} + \textstyle\sum_{j=1,2,3}P^j_\gamma\sigma_j\right) \tag{39}$$

where σ_j are the Pauli matrices and P_γ^j the photon polarization vector. Therefore

$$ {}^{\epsilon_R\epsilon'_R}\rho^{l,l'}_{|m|,|m'|} = \frac{1}{2}\Big[\sum_k \sum_{\epsilon_\gamma\epsilon'_\gamma} T^{\epsilon_\gamma\epsilon_R,k}_{l,|m|}\big[T^{\epsilon'_\gamma\epsilon'_R,k}_{l',|m'|}\big]^* + $$

$$ \sum_{j=1,2,3} P_\gamma^j \sum_k \sum_{\epsilon_\gamma\epsilon'_\gamma} T^{\epsilon_\gamma\epsilon_R,k}_{l,|m|}\sigma_j\big[T^{\epsilon'_\gamma\epsilon'_R,k}_{l',|m'|}\big]^*\Big] \quad (40) $$

that can be written as:

$$ {}^{\epsilon_R\epsilon'_R}\rho^{l,l'}_{|m|,|m'|} = {}^{\epsilon_R\epsilon'_R}\rho^{(0),l,l'}_{|m|,|m'|} + \sum_{j=1,2,3} P_\gamma^j\big[{}^{\epsilon_R\epsilon'_R}\rho^{(j),l,l'}_{|m|,|m'|}\big] \quad (41) $$

.

with ${}^{\epsilon_R\epsilon'_R}\rho^{l,l'}_{|m|,|m'|}$ being the polarized SDME (Spin Density Matrix Elements).

$$ {}^{\epsilon_R\epsilon'_R}\rho^{(0),l,l'}_{|m|,|m'|} = \sum_k \sum_{\epsilon_\gamma} T^{\epsilon_\gamma\epsilon_R,k}_{l,|m|}\big[T^{\epsilon_\gamma\epsilon'_R,k}_{l',|m'|}\big]^* \quad (42) $$

.

$$ {}^{\epsilon_R\epsilon'_R}\rho^{(1),l,l'}_{|m|,|m'|} = \sum_k \sum_{\epsilon_\gamma} T^{-\epsilon_\gamma\epsilon_R,k}_{l,|m|}\big[T^{\epsilon_\gamma\epsilon'_R,k}_{l',|m'|}\big]^* \quad (43) $$

.

$$ {}^{\epsilon_R\epsilon'_R}\rho^{(2),l,l'}_{|m|,|m'|} = i\cdot\sum_k \sum_{\epsilon_\gamma} \epsilon_\gamma\cdot T^{-\epsilon_\gamma\epsilon_R,k}_{l,|m|}\big[T^{\epsilon_\gamma\epsilon'_R,k}_{l',|m'|}\big]^* \quad (44) $$

.

$$ {}^{\epsilon_R\epsilon'_R}\rho^{(3),l,l'}_{|m|,|m'|} = \sum_k \sum_{\epsilon_\gamma} \epsilon_\gamma\cdot T^{\epsilon_\gamma\epsilon_R,k}_{l,|m|}\big[T^{\epsilon_\gamma\epsilon'_R,k}_{l',|m'|}\big]^* \quad (45) $$

IV. NATURALITY AND REFLECTIVITY

A state is said to have natural parity if $P = (-1)^J$, while is said to have unnatural parity if $P = -(-1)^J$. We can recast this definition by introducing the *naturality* of the particle, $\mathcal{N}$, as

$$ \mathcal{N} = P \times (-1)^J. \quad (46) $$

Naturality is $\mathcal{N} = +1$ (natural) for $J^P = 0^+, 1^-, 2^+, \cdots$ (i.e., $\rho, \omega\cdots$) and $\mathcal{N} = -1$ (unnatural) for $J^P = 0^-, 1^+, 2^-, \cdots$ (i.e., $\pi, \eta\cdots$). Determining (or constraining) the naturality of the production (exchange particle) will give us extra information on the produced resonances.

The reflectivities are defined for the resonance decay *and* for the incoming photon. Reflection is a conserved quantum number since both rotation and parity are conserved. Therefore, the product of the initial photon reflectivity and the exchange particle reflectivity must equal the reflectivity of the resonance:

$$ \epsilon_\gamma \times \epsilon_{ex} = \epsilon_R. \quad (47) $$

or

$$ \epsilon_\gamma \times \epsilon_R = \epsilon_{exchange}. \quad (48) $$

And since $\epsilon_{exchange} = P(-1)^J$, then

$$ \epsilon_\gamma \times \epsilon_R = \mathcal{N}. \quad (49) $$

The photon spin density matrix, in the photon reflectivity basis, represents a mix of photon states as seen in equation (34). We can see that the spin density matrix of the reaction (equation 37) will not be diagonal in this formalism. Only for full polarization $P = 1$, there are two configurations that contribute to the reaction and correspond to uniquely defined photon reflectivities, these are for $\Phi = 0$ only ($\epsilon_\gamma = -1$) contribute and for $\Phi = \frac{\pi}{2}$ only ($\epsilon_\gamma = +1$) contribute. Using linearly polarized photons at those explicit configurations we could constrain the naturality of the exchange and particles produced. For example, in the case of pion exchange (or other Regge unnatural trajectory particles) the reflectivity of the resonance (ϵ_R) will be opposite to that of the photon (ϵ_γ). In the case of ρ exchange (or other Regge natural trajectory particles), the reflectivity of the resonance and the photon will be the same. For unpolarized beams, the reflectivity is only defined for the resonance and the spin density matrix of the reaction becomes diagonal. For polarized beams, we can still use similar methods if we include the beam polarization in the rank of the sum (added to the external spin). The JPAC collaboration ([9]) defined a reflectivity for the case of two pseudo-scalars final states taken into account combined photon-resonance parity conservation. In that case, there is only one reflectivity and the spin density matrix becomes diagonal. In the JPAC definition:

$$ \epsilon = P(-1)^J \quad (50) $$

or

$$ \epsilon = \mathcal{N} \quad (51) $$

the reflectivity coincides directly (by construction) with the naturality of the resonance. It has been shown ([10]) that the JPAC definition and the two reflectivity scenarios defined in this paper are equivalent for the case of two final state pseudo-scalars.

V. SEARCH FOR RESONANCES

A. Decay Amplitudes

To obtain the decay amplitudes we will consider two cases: first, the resonance decaying into two particles, and second, the resonance decaying into three or more particles. In this latter case, we will use the *isobar model* [3, 7]. The isobar model assumes a series of sequential two-body decays. We consider the resonance decaying into an intermediate unstable particle (isobar) plus a stable particle (bachelor), and all bachelors will be among the final states. The isobar will decay subsequently into other particles (children), which may also be isobars, and continue the process. We assume that there are no interactions after the particles are produced through this sequential process and that all final (observed) particles are spinless. We calculate amplitudes in the spin formalism of Jacob and Wick [5, 11].

1. *Two-Body Decays*

Let's consider the case of a resonance X decaying into two particles labeled as 1 and 2 (see figure 2 for notation). We describe the decay of X in its rest frame, that is

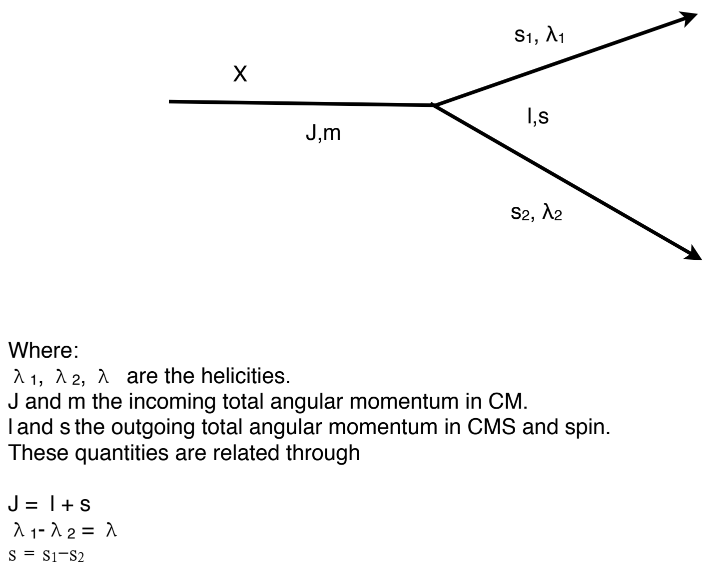

FIG. 2. Two-body Decay.

$p_1 + p_2 = 0$, with the z in the direction of the beam; this is the Gottfried-Jackson (GJ) frame. We can thus describe the kinematics with just one momentum $q(\phi,\theta) = p_1 = -p_2$. In this case, what we called τ to describe the final particles, will be just given by two angles. We use the helicity basis to represent amplitudes.

$$\tau = \{\phi_{GJ}, \theta_{GJ}\} \tag{52}$$

where ϕ_{GJ}, θ_{GJ} are the angles of one of the decay products in the Gottfried-Jackson frame. For a given mass M and transfer momentum t, the decay amplitudes will depend only on τ (angles).

It can be shown that the decay amplitude is (see references [11],[3])

$$A_{l,m}(\tau) = \sqrt{\frac{2l+1}{4\pi}} F_l(p) \cdot$$

$$\sum_{\lambda_1 \lambda_2} D^{J*}_{m\lambda}(\Omega_{GJ})(l0s\lambda|J\lambda)(s_1\lambda_1 s_2 - \lambda_2|s\lambda)a_{ls}. \tag{53}$$

The expressions in parenthesis represent Clebsch-Gordan coefficients. We introduced the factor $F_l(p)$, the Blatt-Weisskopf centrifugal-barrier factor, (see reference [3]). This factor takes into account the *centrifugal-barrier effects* caused by the angular factors on the potential. The factor is close to one and in many cases can be ignored. The sum on λ_1 and λ_2 is over all possible helicities of the daughters' particles.

The "unknown" factor a_{ls} will be included in the fitting parameters of our model ("T's") and will not be carried over to our next formulas.

Consider the decay of a resonance into two spinless final particles. Experimentally, we normally detect spinless particles, therefore this is a very common case (kaons, etas, or pions). In this case, $\lambda = \lambda_1 = \lambda_2 = 0$, $s = s_1 = s_2 = 0$ and $J = l$. We will take $F_l(p) \sim 1$. The angular dependences, τ, will be given by the (ϕ, θ) angles of one the decay particles in the GJ frame. Therefore, $(l0s0|J0) = 1$ and $(s_1 0 s_2 0|s0) = 1$.

Then

$$A_{lm}(\tau) = \sqrt{\frac{2l+1}{4\pi}} D^{l\ *}_{m0}(\phi,\theta,0) \tag{54}$$

and since

$$D^{l\ *}_{m0}(\phi,\theta,0) = e^{im\phi}\sqrt{\frac{(l-m)!}{(l+m)!}} P^m_l(cos\theta) \tag{55}$$

where $P^m_l(cos\theta)$ are the Associated Legendre functions [12]. Therefore,

$$A_{lm}(\phi,\theta) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P^m_l(cos\theta)e^{im\phi} = Y^m_l(\phi,\theta) \tag{56}$$

where $Y^m_l(\phi,\theta)$ are the spherical harmonic functions.

The amplitudes in the reflectivity basis are then:

$${}^{\epsilon_R}Y^{|m|}_l(\phi,\theta) = Y^m_l(\phi,\theta) - \epsilon_R P(-1)^{(l-m)} Y^{-m}_l(\phi,\theta) \tag{57}$$

Since $P = P_1 P_2 (-1)^l = (-1)^l$,

$${}^{\epsilon_R}Y^{|m|}_l(\phi,\theta) = Y^m_l(\phi,\theta) - \epsilon_R(-1)^m Y^{-m}_l(\phi,\theta) \tag{58}$$

$$I(\phi,\theta,P,\Phi) = \sum_k \sum_{\epsilon_\gamma,\epsilon'_\gamma} \sum_{\epsilon_R,\epsilon'_R} \sum_{l|m|,l'|m'|}$$

$$^{\epsilon_R}Y_l^{|m|}(\phi,\theta)T_{l,|m|}^{\epsilon_R\epsilon_\gamma,k}\ \rho^{\gamma}_{\epsilon_\gamma,\epsilon'_\gamma}(P,\Phi)T_{l',|m'|}^{\epsilon'_R\epsilon'_\gamma,k*}\ ^{\epsilon'_R}Y_{l'}^{|m'|*}(\phi,\theta). \quad (59)$$

The same formalism can be used for the decay of resonance into a vector ($s_1 = 1$) and a spinless ($s_1 = 0$) particle. Experimentally, we normally detect spinless particles, therefore is common that the vector particle can be treated as an isobar. In this case, $\lambda = \lambda_1 = \pm 1$, and $\lambda_2 = 0$, $s = s_1 = 1$ and $s_2 = 0$ and $J = l \oplus s$.

Therefore $(l0s\lambda|J\lambda) = (l01\lambda_1|J\lambda_1)$ and $(s_1\lambda_1 s_2 - \lambda_2|s\lambda) = (1\lambda_1 00|1\lambda_1) = 1$.

Then

$$A_{l,m}(\phi,\theta) = \sqrt{\frac{2l+1}{4\pi}} \sum_{\lambda_1=\pm 1} D^{J*}_{m\lambda_1}(\phi,\theta,0)(l01\lambda_1|J\lambda_1) \quad (60)$$

and in the reflectivity basis

$$A^{\epsilon_R}_{l,|m|}(\phi,\theta) = \sqrt{\frac{2l+1}{4\pi}} \sum_{\lambda_1=\pm 1}$$

$$\left[D^{J*}_{m\lambda_1}(l01\lambda_1|J\lambda_1) - \epsilon_R P(-1)^{(l-m)} D^{J*}_{-m\lambda_1}(l01\lambda_1|J\lambda_1)\right] \quad (61)$$

2. Three$^+$-Body Decays - Isobar Model Formalism

Let's consider now the case where the final particles are three or more. In the isobar formalism, we will treat the decay amplitude of the resonance as the product of successive isobar plus bachelor decay amplitudes [13].

$$A_{lm}(\tau) = A_{l'm'}(\tau')A_{l''m''}(\tau'')A_{l'''m'''}(\tau''')... \quad (62)$$

For example, let's consider a resonance decaying into a "di-particle" (isobar $\rightarrow$ two-daughters) and a particle "bachelor". The isobar will decay into two children (to consider more particles the process is repeated).

The degrees of freedom (uncorrelated variables describing the kinematics) will include the mass of the isobar, w, and the angles of its decay products

$$\tau = \left\{\Omega_{GJ}, \Omega_h, w\right\} \quad (63)$$

where $\Omega_{GJ} = (\phi_{GJ}, \theta_{GJ})$ and $\Omega_h = (\phi_h, \theta_h)$ are the angular descriptions in the Gottfried-Jackson and helicity frames (see Appendix A of reference [3]) of the isobar and its decay products respectively. Let l be the angular momentum between the bachelor and isobar and s the spin of the isobar (we will consider a spinless bachelor). Therefore $J = l \oplus s$. The amplitude is then written [14] as

$$A_{l,m,s}(\tau) = E^{Jls*}_m(\Omega_{GJ}, \Omega_h)Q_{ls}(w). \quad (64)$$

We factorize the amplitude with a factor that depends only on the angles, and a factor that only depends on mass. The mass factor comes from the propagator of the isobar. The angular factor can be written, in the isobar model, as

$$E^{Jls*}_m(\Omega,\Omega_h) = \langle\Omega_h; 0|\widehat{T}^I_{decay}|s\lambda\rangle\ \langle\Omega_{GJ}; s\lambda|\widehat{T}^R_{decay}|Jm\rangle \quad (65)$$

where $R \rightarrow IB$ describes the decay of the resonance (R) into the isobar (I) and the bachelor (B), and $I \rightarrow D_1D_2$ is the decay of the isobar. Using our previous result, equation (60), for each two-body decay we have

$$\sqrt{\frac{2l+1}{4\pi}} \sum_{\lambda_1\lambda_2} D^{J*}_{m\lambda}(\Omega_{GJ})(l0s\lambda|J\lambda)(s_1\lambda_1 s_2{-}\lambda_2|s\lambda). \quad (66)$$

For the bachelor $\lambda_2 = 0$ and for the isobar $\lambda_1 = \lambda$, therefore

$$(s_1\lambda_1 00|s\lambda) = 1 \quad (67)$$

then

$$\langle\Omega_{GJ}; s\lambda|\widehat{T}^R_{decay}|Jm\rangle\ = \sqrt{\frac{2l+1}{4\pi}} D^{J}_{m\lambda}{}^*(\phi_{GJ},\theta_{GJ},0)(l0s\lambda|J\lambda). \quad (68)$$

And for the isobar,

$$\langle\Omega_h; 0|\widehat{T}^I_{decay}|s\lambda\rangle\ = \sqrt{\frac{2s+1}{4\pi}} D^{s}_{\lambda 0}{}^*(\phi_h,\theta_h,0) \quad (69)$$

therefore

$$E^{Jls*}_m(\Omega_{GJ},\Omega_h) = \sqrt{(2l+1)}\sqrt{2s+1} \quad (70)$$

$$\sum_\lambda D^{J*}_{m\lambda}(\phi_{GJ},\theta_{GJ},0)D^{s*}_{\lambda 0}(\phi_h,\theta_h,0)\langle l0s\lambda|J\lambda\rangle. \quad (71)$$

Since

$$D^{s*}_{\lambda 0}(\phi_h,\theta_h,0) = e^{i\lambda\phi_h} d^s_{\lambda 0}(\theta_h) \quad (72)$$

and

$$D^{J*}_{m\lambda}(\phi_{GJ},\theta_{GJ},-\phi_h) = D^{J*}_{m\lambda}(\phi_{GJ},\theta_{GJ},0)e^{-i\lambda\phi_h} \quad (73)$$

the angular amplitude can, then, be written as

$$E^{Jls*}_m(\Omega_{GJ},\Omega_h) = \sqrt{(2l+1)}\sqrt{2s+1}$$

$$\sum_\lambda D^{J*}_{m\lambda}(\phi_{GJ},\theta_{GJ},\phi_h)d^s_{\lambda 0}(\theta_h)\langle l0s\lambda|J\lambda\rangle. \quad (74)$$

The mass term depends on the isobar mass, and is given by

$$Q_{ls}(w) = F_l(p)F_s(q)\Psi(w) \quad (75)$$

where the Ψ-function is the standard relativistic Breit-Wigner form for the isobar mass distribution, p is the momentum of the isobar in the GJ frame, and q the momentum of the leading isobar's decay particle in the helicity frame

$$\Psi(w) = \frac{w_0\Gamma_0}{w_0^2 - w^2 - iw_0\Gamma(w)} \tag{76}$$

with

$$\Gamma(w) = \Gamma_0 \frac{w_0 q F_s^2(q)}{w q_0 F_s^2(q_0)} \tag{77}$$

w_0 and Γ_0 are the mass and width of the isobar, and q_0 is found such that $\Gamma(w_0) = \Gamma_0$ and then $|\Psi(w_0)| = 1$.

The $F_l(p)$ and $F_s(q)$ functions are the Blatt-Weisskopf centrifugal-barrier factors. These factors take into account the *centrifugal-barrier effects* caused by the angular (spin) factors on the potentials.

Adding all these components into our final form for the amplitude for three (spinless) particles in the final state, we obtain [13]:

$$A_{J,l,m,s}(\Omega_{GJ}, \Omega_h, w) = \sqrt{(2l+1)}\sqrt{2s+1} \tag{78}$$

$$F_l(p)F_s(q)\frac{w_0\Gamma_0}{w_0^2 - w^2 - iw_0\Gamma(w)}$$

$$\times\sum_\lambda D^{J*}_{m\lambda}(\phi_{GJ}, \theta_{GJ}, \phi_h) d^s_{\lambda 0}(\theta_h)\langle l0s\lambda|J\lambda\rangle.$$

B. Mass-independent Fit

The probability to observe an event i with properties τ_i in the $\Delta E\Delta M\Delta t$ bin is

$$p_i = \frac{I(\tau_i)\eta(\tau_i)}{N} = \frac{I(\tau_i)\eta(\tau_i)}{\int I(\tau)\eta(\tau)d\tau}. \tag{79}$$

where $\eta(\tau)$ is the detector acceptance. The value of N, the average number of events expected to be observed in the total phase-space defined by $\Delta E\Delta M\Delta t$, is calculated numerically through a Monte Carlo (MC) full simulation of the detector and a (flat) phase-space generator of the reaction. In many cases, due to limited statistics, the binning is done only in M, therefore a model for the t cross-section dependence is introduced in the MC. The numerical (MC) value of N is then

$$N = \frac{1}{N_g}\sum_i^{N_g} I(\tau_i)\eta(\tau_i) \tag{80}$$

N_g is the total number of events generated in the MC. The function $\eta(\tau)$ represents the acceptance (resolution is taken to be perfect, only acceptance is considered here - no inter-bin crosstalk). A Monte CarloC simulation of the detector will provide the values of this function that are $\eta(\tau) = 1$ if the event is accepted and $\eta(\tau) = 0$ if the event is not accepted , then

$$N = \frac{1}{N_g}\sum_i^{N_a} I(\tau_i) \tag{81}$$

where N_a is the total number of accepted events. Introducing $\eta_x = \frac{N_a}{N_g}$, as the total fraction of events accepted, or total acceptance, then

$$N = \eta_x \frac{1}{N_a}\sum_i^{N_a} I(\tau_i) \tag{82}$$

therefore

$$N = \eta_x \frac{1}{N_a}\sum_i^{N_a}\Big[\sum_k \sum_{\epsilon_\gamma\epsilon_R, \epsilon'_\gamma\epsilon'_R} \sum_{lm, l'm'} A^{\epsilon_R}_{l,m}(\tau_i) \tag{83}$$

$$T^{\epsilon_\gamma\epsilon_R,k}_{l,m}\ \rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma} T^{\epsilon'_\gamma\epsilon'_R,k*}_{l',m'}\ \ A^{\epsilon'_R,*}_{l',m'}(\tau_i)\Big]. \tag{84}$$

The extended likelihood is defined as including the probability of observing N events by

$$L = Prob(N)\prod_{i=1}^N p(\vec{x}_i, \vec{a}). \tag{85}$$

Assuming a Poisson distribution for the probability of observing N events, with an expected value of N

$$Prob(N) = \frac{N^N}{N!}e^{-N} \tag{86}$$

the extended likelihood is then

$$L = \left[\tfrac{N^N}{N!}e^{-N}\right]\prod_{i=1}^N p(\vec{x}_i, \vec{a}) \tag{87}$$

and taking the log

$$lnL = -ln\left[N!\right] - N + \sum_{i=1}^N ln\left[I(\vec{x}_i, \vec{a})\right]. \tag{88}$$

Therefore,

$$lnL \propto \sum_{i=1}^N ln\left[I(\vec{x}_i, \vec{a})\right] - N \tag{89}$$

Including the expression for the intensity into the likelihood function, we have

$$-lnL \propto$$

$$-\sum_{i=1}^{N} ln\left[\sum_k \sum_{\alpha,\alpha'} A_\alpha(\tau_i) T_\alpha^k \rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma} T_{\alpha'}^{k*}\ A^*_{\alpha'}(\tau_i)\right] + N \tag{90}$$

Where we included all quantum numbers in α and the external target spins in k. This is the function to be minimized to obtain the T_α^k values [3]. To find the *true* number of events in the $\Delta E \Delta M \Delta t$ bin, which we will call N_{true}, we take

$$N_{true} = \frac{1}{N_g}\sum_i^{N_g} I(\tau_i) \tag{91}$$

where we will use the fitted T_α^k values. Then

$$N_{true} = \frac{1}{N_g}\sum_i^{N_g}\Big[\sum_k \sum_{\alpha,\alpha'} A_\alpha(\tau_i)\ T_\alpha^k \rho^\gamma_{\epsilon_\gamma,\epsilon'_\gamma} T_{\alpha'}^{k*}\ A^*_{\alpha'}(\tau_i)\Big] \tag{92}$$

and the yield for each partial wave (α, for a given k) is

$$N_{\alpha,true} = \frac{1}{N_g}\sum_i^{N_g} \rho^\gamma_{\epsilon_\gamma,\epsilon_\gamma}\left|T_\alpha A_\alpha(\tau_i)\right|^2. \tag{93}$$

After we obtain the T_α^k values, we are able to generate MC events through our partial wave model and *predicted* many distributions of data properties (i.e., angular distributions, t-distributions, etc.) to compare directly with data and check our model accuracy. We can also use the phase of the production amplitudes to obtain information about the resonant behavior of a particular wave. A single wave phase is arbitrary but the difference of phases between two waves contains physical information. We use the phase difference between the wave under study versus a well establish resonant wave (see reference [3] for details).

C. Phenomenological Models

After performing mass-independent fits in each bin of M (or M and t) for a given ΔE, we obtained the predicted mass distribution of $N_{true}(M)$ for each partial wave included in the fit. In the past, the mass-dependence of those partial waves has been described by a coherent sum of Breit–Wigner amplitudes and, if needed, a phenomenological model of the background or other effects (i.e. Deck mechanisms,...). Such a procedure can produce a good fit to the data, however (especially using the isobar approximation), it violates fundamental principles such as probability conservation and causality. Therefore, in order to obtain more physically grounded amplitudes, models which fulfill the principles of unitarity and analyticity (which originate from probability conservation and causality) are to be used. Unitarity is especially important when we deal with resonances since it controls resonance widths and pole positions in the complex energy plane. One will first look for regions of enhancement (peaks or valleys) in the distributions and fit a theoretical-based distribution to obtain the resonance properties (mass and width), however, interference and overlapping can greatly disturb the appearance of the spectrum. The properties (and positions) of the resonances should be obtained from the poles on the complex amplitudes of the S-Matrix expansion [15]. These poles (and thresholds) had been studied using the Regge treatment of the S-matrix [16]. Resonances are poles in the complex plane (Riemann surfaces) and only their projected real axis values can then be evaluated experimentally. In the case of multiple poles with the same quantum numbers and/or poles far from the real axis, the axis projections can deviate considerably from the BW distribution. The shape of these distributions is also influenced by the QCD dynamics. Effective field theories, i.e Chiral Perturbation Theory has been combined with the dispersion relations to obtain better parametrization of the mass distributions [17]. Recent studies (i.e. reference [18, 19]) had used those approaches to obtain mass and width values for several resonances.

VI. SUMMARY

We described a formalism that introduces parity conservation into the transition amplitudes for the description of photo production of mesons. Two reflectivities are defined by applying, independently, the reflectivity operator to the resonance decay amplitude and to the incoming photon (beam) state. These are two (reflectivity) quantum numbers, the product of which, at least at higher energies, coincides with the naturally of the exchange particle in the t-production channel. Notice that the definition of two reflectivities is suited for any number of particles into which the resonance can finally decay in the final state. This two-reflectivity formalism might also be used in more refined models of the S-matrix phenomenology, i.e. Regge inspired models respecting unitary and analyticity. In this paper, as a simpler example of its application in PWA, we showed the formalism for a mass-independent analysis, and in the case of more than three particles in the final state, we used the isobar model approximation.

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Appendix A

1. Reflectivity Photon Spin Density Matrix

For a general discussion on the photon spin density matrix see reference [4]. The spin density matrix of the photon in the helicity basis is (see reference [20]).

$$\rho_{\lambda\lambda'}(P,\Phi) = {}^1\!/_2 \begin{pmatrix} 1 & -Pe^{-2i\Phi} \\ -Pe^{2i\Phi} & 1 \end{pmatrix} \tag{A1}$$

This corresponds to:

$$\rho_{+,-}(P,\Phi) = \begin{pmatrix} \langle +|+\rangle & \langle +|-\rangle \\ \langle -|+\rangle & \langle -|-\rangle \end{pmatrix}. \tag{A2}$$

On the reflectivity basis we will have:

$$\rho_{\epsilon,\epsilon'}(P,\Phi) = \begin{pmatrix} \langle \epsilon=+|\epsilon=+\rangle & \langle \epsilon=+|\epsilon=-\rangle \\ \langle \epsilon=-|\epsilon=+\rangle & \langle \epsilon=-|\epsilon=-\rangle \end{pmatrix}. \tag{A3}$$

-

To calculate the spin density matrix in the reflectivity basis, we turn to the relations of the reflectivity basis with the helicity basis [6].

We have that

$$|\epsilon a\lambda\rangle = \left[|a\lambda\rangle - \epsilon P(-1)^{j-\lambda}|a-\lambda\rangle\right]\Theta(\lambda) \tag{A4}$$

where P is the parity of particle "a", and

$$\Theta(\lambda) = \frac{1}{\sqrt{2}} \text{ for } \lambda > 0 \tag{A5}$$

$$\Theta(\lambda) = \frac{1}{2} \text{ for } \lambda = 0 \tag{A6}$$

$$\Theta(\lambda) = 0 \quad \text{for} \quad \lambda < 0 \tag{A7}$$

the eigenvalue of reflectivity for λ=0 is $P(-1)^J$.

For a real photon $P = -1$, $J = 1$ and $\lambda = +1, -1$,therefore

$$|\epsilon\rangle = \frac{1}{\sqrt{2}}\left[|\lambda\rangle - \epsilon(-1)^{\lambda}|-\lambda\rangle\right] \tag{A8}$$

then (the reflectivity eigenvalues for a photon are $\epsilon = \pm$).

$$\begin{aligned} |\epsilon=+\rangle &= \tfrac{1}{\sqrt{2}}(|\lambda=+\rangle + |\lambda=-\rangle) \\ |\epsilon=-\rangle &= \tfrac{1}{\sqrt{2}}(|\lambda=+\rangle - |\lambda=-\rangle) \end{aligned}. \tag{A9}$$

Therefore, we find that,

$$\langle \epsilon=-|\epsilon=-\rangle = \langle +|+\rangle - \langle -|+\rangle - \langle +|-\rangle + \langle -|-\rangle = \tag{A10}$$

$${}^1\!/_2\,(1 + P\cos 2\Phi) \tag{A11}$$

$$\langle \epsilon=+|\epsilon=+\rangle = \langle +|+\rangle + \langle -|+\rangle + \langle +|-\rangle + \langle -|-\rangle = \tag{A12}$$

$${}^1\!/_2\,(1 - P\cos 2\Phi) \tag{A13}$$

$$\langle \epsilon=+|\epsilon=-\rangle = \langle +|+\rangle - \langle -|+\rangle + \langle +|-\rangle - \langle -|-\rangle = \tag{A14}$$

$$-{}^1\!/_2 i\,(P\sin 2\Phi) \tag{A15}$$

$$\langle \epsilon=-|\epsilon=+\rangle = \langle +|+\rangle + \langle -|+\rangle - \langle +|-\rangle - \langle -|-\rangle = \tag{A16}$$

$${}^1\!/_2 i\,(P\sin 2\Phi) \tag{A17}$$

We obtain the spin density matrix of the photon on the reflectivity basis:

$$\rho_{\epsilon\epsilon'}(P,\Phi) = {}^1\!/_2 \begin{pmatrix} 1 - P\cos 2\Phi & -iP\sin 2\Phi \\ iP\sin 2\Phi & 1 + P\cos 2\Phi \end{pmatrix}. \tag{A18}$$

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