Single-spin asymmetry in deeply virtual Compton scattering: Fragmentation region of polarized lepton

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For the kinematical region when a hard photon is emitted predominantly close to the direction of motion of a longitudinally polarized initial electron and relatively small momentum is transferred to a proton we calculate the azimuthal asymmetry of photon emission. It arises from the interference of the Bethe-Heitler amplitude and amplitudes described by a heavy photon impact factor. Azimuthal asymmetry does not decrease in the limit of infinite c.m.s. energy. The lowest order expression for the impact factor of a heavy photon is presented.

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I. INTRODUCTION

At present the process of deeply virtual Compton scattering (DVCS) in lepton-nucleon collisions at high energies is of firsthand interest to theorists [1–8] as well as to experimenters [9] investigating the long-standing problem of the nucleon spin content. Among others it was realized that lepton production of real photons off nucleons could shed light on the so-called spin crisis problem by allowing for a direct measurement of off-forward parton distributions. Indeed a decomposition of the nonforward Compton scattering amplitude for the case in which one of the photons is on mass shell is stressed that we deal with a kinematical region of lepton fragmentation which means that the invariant mass of the scattered lepton and real photon is much less than the invariant variable \( s: (p_2+k_1)^2 < s \). In this region both Bethe-Heitler (BH) and IF amplitudes do not fall with \( s \) increasing which is ensured by a \( t \)-channel photon (BH case) or two-gluon (IF case) exchange, whereas the contribution corresponding to the so-called handbag diagram [5] falls with \( s \) and dominates in the region \((p_2+k_1)^2 \lesssim s\). The effect of azimuthal correlation appears as the interference of a real BH amplitude with a purely imaginary one of an IF mechanism of real photon creation. The interference is not zero due to the purely imaginary spin density matrix of a polarized lepton.

The azimuthal asymmetry has the simplest form \( A = \Delta |M|^2/|M_{BH}|^2 \sim \sin \phi \), where \( \phi \) is the azimuthal angle between the planes formed by the momenta of initial and scattered leptons and photons. It was shown that the higher harmonics in the Fourier decomposition of the asymmetry are related to the structure functions mentioned above. The contribution derived here is sensitive only to the gluon density \( z g(z,Q^2) \) inside a proton. For small values of the energy fraction \( z \) carried by gluons, one has \( z g(z,Q) \sim 6Q^2 \) [GeV\(^2\)], \( Q^2 \sim 1 \) GeV\(^2\) [10].

In what follows we study the case when the initial proton is unpolarized and the final states are a scattered lepton, a recoil proton, and a hard photon from the fragmentation region of initial leptons. Furthermore, the lowest order contribution \((\sim \alpha_s^2)\) to the asymmetry is investigated. The higher

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FIG. 1. Feynman diagrams corresponding to BH and IF amplitudes (the crossed diagram for the IF is implied).
PT effects were considered in Ref. [8] and took into account the Balitskii-Fadin-Kuraev-Lipatov (BFKL) ladder.

II. BETHE-HEITLER AMPLITUDE

Let us consider the radiative electron-proton scattering

$$e(p_1, \xi) + P(p) \rightarrow e(p_2) + P(p') + \gamma(k_1),$$

where we indicate in parentheses the four-momenta of particles, and $\xi$ is the degree of the longitudinal polarization of electron. We will restrict ourselves to the kinematics when the absolute magnitude of a square of the momentum transfer between initial and final state electrons is small with respect to the c.m.s. energy squared:

$$s = (p_1 + p)^2 \gg Q^2 = -(p_1 - p_2)^2,$$

$$-t = Q^2 = -q^2 \gg p_1^2 = p_2^2 = m_e^2,$$

$$p^2 - p'^2 = M^2, \quad q = p' - p. \quad (1)$$

Note that we use notation typical for calculations based on Sudakov parametrization of four-vectors. The invariants do not always coincide with those routinely used in data analysis. Thus our $x, \ Q_1^2 = p_1^2/x$, and $Q^2$ correspond to $1 - y, \ Q_1^2$, and $-t$, adopted by experimental collaborations; $y$ is a scaling variable.

The leading nonvanishing in the limit of large $s$ contribution arises from the two Feynman amplitudes. One of them, the so-called Bethe-Heitler amplitude, describing hard photon emission by an electron blob, has the following form:

$$M^{BH}_\lambda = \frac{(4\pi\alpha)^{3/2}}{q^2} \bar{u}(p_2) \gamma^\mu u(p_1, \xi) \gamma^\lambda(p')$$

$$\times V_{\lambda}\mu T(p) s^{\mu\nu} e^{\nu}(k_1)$$

$$= \frac{(4\pi\alpha)^{3/2}}{q^2} \left( -\frac{2x_1}{sdd_1} \right) \bar{u}(p_2) v_{\sigma} u(p_1, \xi)$$

$$\times e^{\sigma}(k_1) s N^\lambda, \quad (2)$$

with

$$V_{\lambda} = \gamma_\lambda F_1(Q^2) + \frac{[\gamma_{\sigma} \hat{q}]}{2M} F_2(Q^2),$$

$$N^\lambda = \frac{1}{s} u^\lambda(p') \left( \hat{p}_1 F_1(Q^2) + \frac{\hat{q} p_1}{M} F_2(Q^2) \right) u^\lambda(p).$$

$$\sum_\lambda |N^\lambda|^2 = 2F(Q^2),$$

$$F(Q^2) = F_1^2(Q^2) + \frac{Q^2}{M^2} F_2^2(Q^2). \quad (3)$$

Here $\lambda = \pm 1$ describes a proton chiral state. $F_{1,2}$ are the Dirac and Pauli form factors, $M$ is the proton mass, $e(k_1)$ is the photon polarization vector, and

$$v_{\sigma} = s x(d - d_1) \gamma_\sigma + x d_1 \gamma_\sigma \hat{p} \hat{q} + d \hat{p} \gamma_\sigma,$$

the effective vertex describing the Compton scattering [11]. The quantities

$$d = xx_1[(p_1 - q)^2 - m_e^2], \quad d_1 = -x_1[(p_1 - k_1)^2 - m_e^2],$$

$$q^2 = -q^2 = q^2,$$

where $k_1, p_2, q$ are the two-dimensional components of the photon, scattered electron, and recoil proton momenta in the plane transverse to the beam axis. They obey the conservation law $k_1 + p_2 + q = 0$. Here $x, x_1$ are the energy fractions of the scattered electron and real photon satisfying $x + x_1 = 1$. The squared matrix element summed over polarization states and the cross section can be brought to the form [11]

$$\sum_\lambda |M^{BH}_\lambda|^2 = 2^{11} \pi^3 \alpha^3 s x_1^2 x(1 + x^2) q^2 dd_1 F(Q^2),$$

$$d\sigma^{BH} = \frac{2}{\pi^2} \left( \frac{2}{M^2} q^2 \right) F(Q^2) d^2k_1 d^2q dx.$$

$$\quad (4)$$

It is important to note that the amplitude $M^{BH}$ is real.

III. ASYMMETRY EVALUATION

Consider now the two-loop level correction to the amplitude studied above, describing emission of a hard photon from the intermediate state of a pair of charged quarks created by the virtual photon and converted to the real one through the two gluons exchange. The corresponding amplitude differs from the QED one only by the factor $C = \Sigma Q_q^2$ ($Q_q$ is a quark charge in units of $e$) and the gluon density factor $G(z, k, Q) = z dg(z, k, Q)/d ln Q^2, \ k^2 - Q^2 - \Sigma s < s$ (see Ref. [10]). The amplitude of the IF mechanism is purely imaginary and may be expressed in terms of the photon IF:

$$M^{IF} = 4C \frac{s}{q_1^2} \left( \frac{4\pi\alpha}{q_1^2} \right)^{1/2} \bar{u}(p_2) \gamma_\mu u(p_1, \xi) N^\lambda$$

$$\times \int \frac{d^2 k G(z, k, Q)}{\pi k^2 (q - k)} \frac{d^2q_+ dx_+}{\pi x_+ x_-} I_{\mu\sigma}\sigma(k_1),$$

$$q_1^2 = - \frac{p_2^2}{x} \quad (5)$$

where the tensor $I_{\mu\sigma}$ is given through the tensor of elastic gluon-photon scattering.
\[ \Delta |M|^2 = \sum 2M^IF(M^{BH})^* \]

\[ = s^2 \xi^2 11 C \frac{x_1 \pi^2}{q'p'_2dd_1} \alpha_s^2 \alpha_s^2 \]

\[ \times \int dq + dx_+ \frac{k^2}{(q-k)^2} \int d^2 q_j F_1(Q^2), \]

\[ J = \frac{i}{s} I_{\mu \nu} l_{\mu \nu}, \quad L_{\mu \nu} = \frac{1}{4} \text{Tr} \{ \hat{p}_2 \hat{\mu}_1 \gamma_\mu \gamma_5 \}. \]

Using the gauge invariance conditions \( T_{a\bar{b}} \ldots k_a = T_{a\bar{b}} \ldots (q-k)_{\bar{b}} = 0 \) we can make the following replacement in the expression for \( I_{\mu \sigma} \):

\[ \frac{p_a p_\beta}{s^2} \rightarrow \frac{k_a(q-k)_{\bar{\beta}}}{ss' \sigma}, \]

\[ \bar{s} = \frac{1}{x_1} [(q_+ + q_-)^2 + (q_1 + k)^2], \]

\[ s' = \frac{1}{x_1} [(q_+ + q_-)^2 + (k_1 + k - q)^2]. \]

The next step is to perform the \( d^2 k \) integration. We suppose that small values of \( |k| \) dominate as this region is enhanced by the factor \( z g(z,|k|) \). Then the integration could be carried out as follows:

\[ \int \frac{d^2 k}{|k|^2} k'j/G(z,k,k') \]

\[ = \int dx \int d^2 k'$\frac{k(q-k)jG(z,k,q-k)}{\pi q'^2(1-x)D(x)} \]

\[ \approx \frac{1}{2} \delta^j' \int_0^1 dq dzg(z,xQ,(1-x)Q) \approx \frac{1}{2} \delta^j' z g(z/Q2). \]

Thus to the accuracy of approximately 10% (with \( Q^2 \) of a few GeV²) we may put \( |k| = |k'| = Q/2 \) in the nonsingular part of the integrand.

It should be noted that only the structure

\[ E = (p_1 p_2 q) = \epsilon_{\alpha \beta \gamma \delta} p^\alpha p^\beta p_2 q^\gamma \delta = \frac{1}{2} (p_2 \times q)_z. \]
matics higher order corrections acquire odderon features, unlike in DIS, and in our approach these are parametrized by a gluon density factor.

Above we applied this approach to give a rather rough estimate for the azimuthal asymmetry of a real photon emission induced by longitudinally polarized electron in its fragmentation region. It turns out that the asymmetry is enhanced by the gluon density $z_G(z,Q)$ which for $z\to0$ appears to be $\sim5–7 \times [Q/(\text{GeV})]^{2}$. Evidently the asymmetry results from the interference between the Born-level Bethe-Heitler amplitude and that of two-loop level containing a photon-gluon fusion block. The first amplitude is real and the last one is completely imaginary. Aiming at obtaining a definite analytical result for the asymmetry we study the problem within the kinematics region. It turns out that the asymmetry is enhanced like in DIS, and in our approach these are parametrized by a gluon density factor $z_G(z)$ which for $z\to0$ appears to be $\sim5–7 \times [Q/(\text{GeV})]^{2}$.

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APPENDIX A: EXPLICIT FORM OF $I_{\mu \nu}$

To find the contribution of the Feynman diagram containing the impact factor of a heavy photon we need to calculate the trace,

$$I_{\mu \nu} = \frac{1}{s^2} \text{Tr}((q' + m) B_{\mu}(q' + m) R_{\nu}),$$

$$B_{\mu} = \frac{1}{d_{+}} \gamma_{\mu}(k - \hat{q} + m) \hat{p}_2 + \frac{1}{d_{-}} \hat{p}_2 (\hat{q} - k + m) \gamma_{\mu},$$

$$R_{\nu} = \frac{1}{d_{-}} \gamma_{\nu}(\hat{q} + m) \hat{p}_2 + \frac{1}{d_{+}} \hat{p}_2 (\hat{q} - k' + m) \gamma_{\nu},$$

where $m$ is a quark mass and

$$d_{\pm} = k^2 - 2kq_{\pm}, \quad d'_{\pm} = k'^2 - 2k'q_{\pm}.$$ 

It is easy to show that the gauge conditions for the on-mass-shell quarks are satisfied:

$$\bar{u}(q_{-}) B_{\nu} v(q_{+}) q_{1}^{0} = 0, \quad \bar{v}(q_{+}) R_{\mu} u(q_{-}) k_{1}^{0} = 0. \quad (A2)$$

Taking into account an enhancement due to the large gluon density factor $z_G(z,Q) \gg 1$ one can restrict further consideration to the kinematics $p_{2}^{2} \gg k^2 \sim q^2$ which is thus preferable. Then it could be verified that

$$d_{\pm} = d'_{\pm} = -x_{\pm} \bar{s}, \quad \bar{s} = \frac{1}{x_{1}} [s_{1} + q_{1}^2] = \frac{\sigma}{s_{1} y_{+} y_{-}},$$

$$s_{1} = (q_{+} + q_{-}) = \frac{1}{y_{+} y_{-}} [m^2 + q_{1}^2],$$

$$\sigma = m^2 + q_{1}^2, \quad m^2 = m^2 + y_{+} y_{-} p_{2}^2, \quad q_{1} = q_{+} y_{+} p_{2} = -q_{-} y_{-} p_{2}, \quad y_{\pm} = \frac{x_{\pm}}{x_{1}}. \quad (A3)$$

Here $x_{\pm}$ are the energy fractions of the pair, $q_{\pm}$ their components of momentum transverse to the beam axis. They obey the conservation laws

$$y_{+} + y_{-} = 1 \quad \text{and} \quad q_{+} + q_{-} + p_{2} = 0.$$ 

With the substitution $p_{2} \mu \rightarrow -s k_{1}^{\mu}/s$ in the quantity $B$ and, respectively, $p_{2} \mu \rightarrow -s k_{1}^{\mu}/s$ in $R$ the tensor $I_{\mu \nu}$ can be transformed to take the following form:

$$I_{\mu \nu} = \frac{1}{4s^2} \text{Tr}((\hat{q}_+ + m) B_{1\mu}(\hat{q}_+ + m) R_{1\nu}).$$
where structures \(k\) are satisfied up to terms of order \(k^2/|p_z|^2\).

Here the vectors \(k, k' = k - q\) are pure two-dimensional ones transverse to the beam axis.

Once again one can check that the gauge conditions,

\[ \bar{u}(q_-) B_{1\mu} v(q_+) q^\mu = 0, \]

\[ \bar{v}(q_+) R_{1\mu} u(q_-) k^\mu = 0 \]

are satisfied up to terms of order \(k^2/|p_z|^2\).

\[ \frac{s^4}{|p_z|^2 k^2} J = \frac{1}{2} (1 + P_z) [xB + C], \]

\[ C = \left( \frac{s_l^2}{x_1} - \frac{4m^2r^2}{z^2} \right) + 2(p, p_2, q_-, q) \left( - \frac{s_f}{x_1 x_+ z} q_r + \frac{s_f}{x_1 z} r^2 + \frac{2a_+}{z} r^2 - \frac{s_f}{x_1 x_+} \right) - 2(p, p_2, q, r) s^a_+ / x_1 z, \]

\[ B = 2(p, p_1, q_-, q) \left( - \frac{s_t}{x_1 z} r p_2 + \frac{x s_t}{r^2} q r + \frac{x s_t}{x_+ x_1 z} q r \right) \left( - \frac{s_t}{2 x_1 z} q r - \frac{s_t}{2 z} r^2 s_r + \frac{s_t}{2 x_1 z} q r \right) (p, p_1, q, r) s^a_+ / x_1 z + (p, p_1, p_2, r) s^a_+ / x_1 z q - q r \]

\[ + 2(p, p_1, p_2, q_-) \left( \frac{s_t}{x_1 z} q r - \frac{s_t}{2 z} r^2 s_r + \frac{s_t}{2 x_1 z} q r \right) \left( - \frac{s_t}{2 z} r^2 s_r - \frac{s_t}{2 z} r^2 (s_r - p_2^2) \right) 2(p, p_2, q, r) s^a_+ / x_1 z + \]

\[ \times \left( \frac{s_t}{x_1 x_+ z} q r - \frac{s_t}{2 x_1 z} q r - \frac{s_t}{2 z} r^2 + \frac{s_t}{2 x_1 z} q r \right) (p, p_2, q, r) s^a_+ / x_1 z + \frac{2 (p, p_2, q_-, q) x_+ r^2}{z^2} + (p, p_2, q, r) x_+ q / x_1 z, \]

where \(s_t = x_1 s, z = x_+ x_\) and \(P_z\) is the permutation operator. In deriving these formulas it has been assumed that \(k^2 \approx q^2\). The structures \((\cdots)\) entering Eq. \((B3)\) can be rewritten as follows:¹

\[ (p, p_2, q_-, q) = x(p, p_1, q_-, q) - x_- E, \]

¹At this point one should be aware that only the transverse components of the four-vector \(q\) have to be taken into account.
\[
\begin{align*}
s(p_1, p_2, q_-, q) &= -\frac{p_2^2}{x} (p, p_1, q_-, q) + a_+ E, \\
\frac{s(p_1, p_2, q_-, r)}{x} &= \frac{p_2^2}{x} (p, p_1, r, q), \\
(p, p_2, q_-, r) &= -x(p, p_1, r, q).
\end{align*}
\]

Having all the above at hand we turn to the \(d^2q_+\) integration. A set of relevant integrals reads

\[
\int \frac{d^2q_+}{\pi} \left\{ \frac{1}{\sigma^2} \frac{p^2 r^2}{\sigma^3} \frac{q_+^2}{\sigma^3} \frac{q_+^2}{\sigma^4} \frac{(r q_+) q_+}{\sigma^4} \frac{r^2 a_+ q_+}{\sigma^4} \frac{r^2 a_+ q_+}{\sigma^4} \frac{a_+ r_+ a_+ r_+}{\sigma^4} \frac{a_+ r_+ a_+ r_+}{\sigma^4} \right\}
\]

\[
= \frac{1}{m^2} \left[ \frac{x_1^2}{6m^2} - \delta_{ij} x_1 \frac{y_{-} y_{+} y_{-} p_2^2}{2m^2} + \frac{x_1^2}{3m^2} \frac{y_{-} y_{+} y_{-} p_2^2}{2m^2} + \frac{x_1^2}{3m^2} \frac{y_{-} y_{+} y_{-} p_2^2}{2m^2} \right].
\]

Above we have discarded terms that give contributions of order \(m^2/p_2^2\) as compared with unity. The integration over \(y_{\pm}\) becomes almost trivial in the limit \(Q_1^2 \gg m^2\):

\[
\int_{0}^{1} \frac{dy_{+}}{m^2} \left\{ 1, y_{+} y_{+} y_{+} y_{-} \frac{m^2}{y_{-} y_{+}} \right\} = \frac{1}{p_2^2} \{2L; L - 1, 1, 0\},
\]

where \(L = \ln(p_2^2/m^2)\).

**APPENDIX C: HEAVY PHOTON IMPACT FACTOR**

To obtain the heavy photon IF one has to consider the \(s\)-channel discontinuity of the heavy photon amplitude in an external field,

\[
\gamma_\mu(P_1) + A(p) \to q(q_+) + q(q_-) + A(p')
\]

\[
\to \gamma_\mu(P_2) + A(p'),
\]

\[
P_1^2 = -Q^2, \quad P_2^2 = -Q'^2.
\]

which is described by the tensor

\[
\Delta A_{\mu\nu}(s, t) = \frac{(4\pi\alpha)^3}{k^2 k'^2} \left( \frac{2}{s} \right)^2 N_\lambda s^4 I_{\mu\nu} d\Gamma_3,
\]

with

\[
d\Gamma_3 = \frac{1}{(2\pi)^5} \frac{d^3q_+}{2\epsilon_+} \frac{d^3q_-}{2\epsilon_-} \frac{d^3p_\gamma}{2E_\gamma} \delta^4(P_1 + p - q_+ - q_- - p') = \frac{d^2k^2 d\xi_+ dx_+}{4s(2\pi)^5 x_+ x_-},
\]

and the quantity \(N_\lambda\) given in Eq. (3). The tensor \(I_{\mu\nu}\) has the form [see Eq. (A1)].
\[ \frac{1}{x_+ x_-} l_{\mu\nu} = (1 + P_z) \left\{ \frac{x_+}{a_+ a'_+} \left[ 2q_\mu q_\nu + (2 + 4x_) q_\mu q_\mu - 8x_- q_\mu q_\nu \right] + \frac{1}{a_- a'_-} \left[ 2x_+ q_\mu q_\nu + (\mu q_\nu + (-2x_+ + 4x_+ x_-) q_\nu q_\mu \right] \right. \]
\[ - 2q_\mu q_\mu + (2 - 8x_+ x_-) q_\mu q_\nu + g_{\mu\nu} \left\{ \frac{1}{a_- a'_-} \left[ -x_- k^2 - x_+ k'^2 + x_+ x_- (q^2 - Q^2 - Q'^2) \right] \right. \]
\[ + \left. \frac{x_+}{a_+ a'_+} (x_- (Q^2 + Q'^2) + x_+ q^2) \right\} , \]
(C4)

with

\[ a_\pm = a + q_\pm^2 , \quad a'_\pm = b + (q_\pm - x_- q) , \]
\[ a = m^2 + x_+ x_- Q^2 , \quad b = m^2 + x_+ x_- Q'^2 . \]

One can argue that the gauge condition \( l_{\mu\nu} = 0 \) for \( k = 0, k' = 0 \) is satisfied. Joining the denominators with the use of the Feynman trick and performing an integration over the components of the quark pair momenta transverse to the beam axis we get

\[ \int \frac{d^2 q_+}{\pi a_+ a'_+} = \int_0^1 dy \frac{D_{++}}{D_{++}} , \quad \int \frac{d^2 q_+}{\pi a_+ a'_-} = \int_0^1 dy \frac{D_{--}}{D_{--}} , \]
\[ D_{++} = A + q^2 x^2 y (1 - y) , \quad D_{--} = A + y (1 - y) b^2 , \]
\[ b = k - x_+ q , \]
\[ A = m^2 + x_+ x_- [y Q'^2 + (1 - y) Q^2] . \]
(C5)

The result for the IF takes the following form (we choose only the transverse polarizations of photons \( \mu = \rho, \nu = \rho' \)):

\[ \tau_{ij}^f = 2 \alpha^2 \int_0^1 dx_+ dx_- \delta(x_+ + x_- - 1) \int dy \left[ \frac{x^2}{D_{++}} \right] \]
\[ \times \left[ 8x_+ x_- y (1 - y) q_\mu q_\mu - q^2 \delta_{ij} (1 + 4x_+ x_- y (1 - 2y) - \right] \]
\[ - \frac{1}{D_{--}} \left[ 8x_+ x_- y (1 - y) b_\rho b_{\rho'} - b^2 \delta_{ij} (1 + 4x_+ x_- y \right) \]
\[ \times (1 - 2y) \left] + 4x_+ x_- y (x_+ - x_-) q_\rho \right] \left[ \frac{x_+ q_\rho}{D_{++}} + \frac{b_{\rho'}}{D_{++}} \right] \]
\[ + \delta_{ij} x_+ x_- (Q^2 + Q'^2 + 4x_+ x_- y (Q'^2 - Q^2)) \]
\[ \times \left( \frac{1}{D_{--}} - \frac{1}{D_{++}} \right) , \] (C6)

with \( D_{++}, D_{--}, \) and \( b \) defined in the same way as in the Eq. (C5). Once again it is clearly seen that the gauge conditions are satisfied:

\[ \tau_{ij}^{k=0} = \tau_{ij}^{k=q} = 0 . \]

It is important to note that even for the on-mass-shell photons \( Q^2 = Q'^2 = 0 \) this expression differs from the one derived by Cheng and Wu [13]. The difference is found to be

\[ \Delta \tau_{ij}^{f} = \tau - \tau_{CW} \]
\[ = 4 \alpha^2 \int dx_+ dy x_+ x_- (1 - 2x_+) q_\rho \]
\[ \times \left[ \frac{x_+ q_\rho}{D_{++}^0} + \frac{b_{\rho'}}{D_{++}^0} \right] , \]
\[ D_{++}^0 = m^2 + x_+ x_- y (1 - y) q^2 , \]
\[ D_{--}^0 = m^2 + y (1 - y) b^2 . \] (C7)

The reason for this discrepancy is the different definition of the initial and final photon’s four-momenta. Similar results were obtained in Ref. [14].
