

Light cone sum rules for $\gamma^* N \rightarrow \Delta$ transition form factorsV. M. Braun,¹ A. Lenz,¹ G. Peters,¹ and A. V. Radyushkin^{2,3,*}¹*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*²*Physics Department, Old Dominion University, Norfolk, Virginia 23529, USA*³*Theory Group, Jefferson Laboratory, Newport News, Virginia 23606, USA*

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A theoretical framework is suggested for the calculation of $\gamma^* N \rightarrow \Delta$ transition form factors using the light-cone sum rule approach. Leading-order sum rules are derived and compared with the existing experimental data. We find that the transition form factors in a several GeV region are dominated by the soft contributions that can be thought of as overlap integrals of the valence components of the hadron wave functions. The minus components of the quark fields contribute significantly to the result, which can be reinterpreted as large contributions of the quark orbital angular momentum.

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I. INTRODUCTION

The concept of form factors plays an extremely important role in the studies of the internal structure of composite particles. The nontrivial dependence of form factors on the momentum transfer Q^2 (i.e., its deviation from the constant behavior) is usually a signal of the nonelementary nature of the investigated particle. In particular, the pioneering study of the nucleon form factors by Hofstadter and collaborators [1] demonstrated that the nucleons have a finite size of the order of a Fermi. Later, it was observed that the behavior of the proton electromagnetic form factors, in a rather wide range of momentum transfers, is well described by the so-called dipole formula $G_p(Q^2)/G_p(0) \approx G_D(Q^2) \equiv 1/(1 + Q^2/0.71 \text{ GeV}^2)^2$, suggesting a simple $G_p(Q^2) \sim 1/(Q^2)^2$ power law for their large- Q^2 asymptotic behavior. At the same time, strong evidence was accumulated that the pion electromagnetic form factor is well described by the ρ -pole fit $F_\pi(Q^2) \approx 1/(1 + Q^2/m_\rho^2)$ indicating that, in the pion case, the asymptotic behavior looks more like $1/Q^2$. From the quark model point of view, the faster decrease of the proton form factor seems rather natural, since the proton contains more valence constituents than the pion. Furthermore, it was established that, if one can treat the hadrons at high momentum transfer as collinear beams of N valence quarks located at small transverse separations and exchanging intermediate gluing particles with which they interact via a dimensionless coupling constant, then the spin-averaged form factor behaves asymptotically as $1/(Q^2)^{N-1}$ [2]. This hard-exchange picture and the resulting dimensional power counting rules [2,3] can be formally extended onto other hard exclusive processes.

After the advent of quantum chromodynamics, this hard-gluon-exchange picture was formalized with the help of the QCD factorization approach to exclusive processes [4–6] that presents one of the highlights of perturbative QCD

(pQCD). Within this approach, the hard-gluon-exchange contribution proves to be dominant for sufficiently large momentum transfers Q^2 . An important ingredient of the asymptotic pQCD formalism for hard exclusive processes is the concept of hadron distribution amplitudes (DAs). They are fundamental nonperturbative functions describing the momentum distribution within rare parton configurations when the hadron is represented by a fixed number of Fock constituents. It was shown that in the $Q^2 \rightarrow \infty$ limit, form factors can be written in a factorized form, as a convolution of distribution amplitudes related to hadrons in the initial and final state times a “short-distance” coefficient function that is calculable in QCD perturbation theory. The leading contribution corresponds to DAs with minimal possible number of constituents, e.g., 3 for the proton and 2 for the pion.

The essential requirement for the applicability of the pQCD approach is a high virtuality of the exchanged gluons and also of the quarks inside the short distance subprocess. Since the quarks carry only some fractions $x_i P, y_j P'$ of the initial P and final P' momenta, the virtualities of the internal lines of the subprocess are generically given by $x_i y_j Q^2$, i.e., they may be essentially smaller than Q^2 , the nominal momentum transfer to the hadron. Assuming that $\langle x \rangle \sim 1/N$, one should expect the reduction factor of 0.1 for the proton and 0.2 for the pion. In the pion case, this expectation was confirmed by an explicit calculation of the one-loop pQCD radiative corrections [7]. Absorbing the terms proportional to the β -function coefficient β_0 into the effective coupling constant $\alpha_s(\mu^2)$ of the hard gluon exchange, one indeed obtains $\langle x \rangle \langle y \rangle Q^2$ as its argument. As a result, at accessible Q^2 , the bulk part of the hard pQCD contribution comes from the regions where the “hard” virtualities are much smaller than the typical hadronic scale of 1 GeV² [8–10]. According to the pQCD factorization recipe, contributions from such regions should not be included into the hard term, which is strongly reduced after such contributions are subtracted. In practice, the subtraction is never made, and pQCD estimates are based on the original expressions (which implies, in par-

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ticular, that the perturbative $\sim 1/k^2$ behavior of propagators is trusted even if $k^2 \rightarrow 0$). Despite this, in most cases pQCD results for hadronic form factors need special efforts to bring their magnitude close to experimental data. For example, assuming the ‘‘asymptotic’’ form $\varphi_\pi(x) = 6f_\pi x_1 x_2$ for the pion DA gives $Q^2 F_\pi^{\text{as}}(Q^2) = 8\pi f_\pi^2 \alpha_s \approx \alpha_s \times 0.44 \text{ GeV}^2$ for the pion form factor [5,6] that agrees with existing data only if one takes an uncomfortably large value $\alpha_s \approx 1$ for the hard gluon vertex. Switching to a wider Chernyak-Zhitnitsky (CZ) shape $\varphi_\pi^{\text{CZ}}(x) = 30f_\pi x_1 x_2 (x_1 - x_2)^2$ [11,12] gives $Q^2 F_\pi^{\text{CZ}}(Q^2) = \frac{200}{9} \pi f_\pi^2 \alpha_s$, which formally agrees with the data for $\alpha_s \approx 0.4$. However, at accessible Q^2 more than 90% of this contribution comes from the region of gluon virtualities below $(500 \text{ MeV})^2$ [9].

In the nucleon case, the situation is even worse. For the asymptotic $\sim x_1 x_2 x_3$ form of the leading twist three-quark distribution amplitude, the proton form factor G_M^p turns out to be zero to leading order [13,14], while the neutron form factor G_M^n is of opposite sign compared to the data. Assuming the equal sharing DA $\sim \delta(x_1 - 1/3)\delta(x_2 - 1/3)\delta(x_3 - 1/3)$ gives wrong signs both for the proton and neutron form factors [15]. Furthermore, if one takes the QCD sum rule estimate for the $\langle qq\bar{q}|N\rangle$ matrix element, the absolute magnitude of the form factors in the above examples is too small (by a factor of hundred) [16] compared to the data. Just like in the pion case, the magnitude of the formal pQCD result can be increased, and also the signs of the predicted proton and neutron magnetic form factors reversed to coincide with the experimental ones, by using CZ-type DAs [17–20] having peaks in a region where the momentum fraction of one of the quarks is close to 1. Since the average fractions of the nucleon momentum carried by the two other quarks are small, this formal result is strongly dominated for accessible Q^2 by regions of unacceptably small virtualities. It was argued [21] that higher order Sudakov-type corrections squeeze the size of the valence quark configuration participating in the pQCD subprocess. Indeed, in the pion case, there are negative terms in the one-loop correction to the short-distance amplitude that can be written as Sudakov double logarithms in the impact parameter b -space [22]. After resummation to all orders, they produce a factor like

$\exp[-\alpha_s \ln^2(Q^2 b^2)/3\pi]$ suppressing the contribution of large transverse separations. These effects increase the region of the x, y fractions where the leading-order pQCD expressions are formally applicable, though they are not strong enough to visibly suppress nonperturbative regions for accessible momentum transfers, see e.g. [23] for discussion of the nucleon form factor case.

As already emphasized, according to the standard philosophy of separating large- and small-virtuality contributions underlying the pQCD factorization formulas, the low-virtuality contributions of gluon-exchange diagrams should be treated as a part of the soft contribution. More precisely, in the case of the nucleon form factors, the hard pQCD contribution is only the third term of the factorization expansion. Schematically, one can envisage the expansion of, say, the Dirac electromagnetic nucleon form factor $F_1(Q^2)$ of the form (see Fig. 1)

$$F_1(Q^2) \sim A(Q^2) + \left(\frac{\alpha_s(Q^2)}{\pi}\right) \frac{B(Q^2)}{Q^2} + \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 \frac{C}{Q^4} + \dots \tag{1.1}$$

where C is a constant determined by the nucleon DAs, while $A(Q^2)$ and $B(Q^2)$ are form-factor-type functions generated by soft contributions. Because of their nonperturbative nature, it is impossible to tell precisely what is their large- Q^2 behavior. On general grounds, one may expect that $A(Q^2)$ and $B(Q^2)/Q^2$ correspond to higher powers of $1/Q^2$ than the perturbative $1/Q^4$ term. Perturbative estimates suggest that $A(Q^2), B(Q^2)/Q^2 \lesssim 1/Q^6$. At very large Q^2 , one may also expect that they are further suppressed by the Sudakov form factor. The most important feature of the factorization expansion is a numerical suppression of each hard gluon exchange by the α_s/π factor, which is a standard perturbation theory penalty for each extra loop. If $\alpha_s \sim 0.3$, the pQCD contribution to baryon form factors is suppressed by a factor of 100 compared to the purely soft term. Thus, one may expect that the onset of the perturbative regime is postponed to very large momentum transfers since the factorizable pQCD contribution $O(1/Q^4)$ has to win over nonperturbative effects that are suppressed by extra powers of $1/Q^2$, but do not involve small coefficients. In the light cone formalism, the functions like $A(Q^2)$ and $B(Q^2)$ in the above

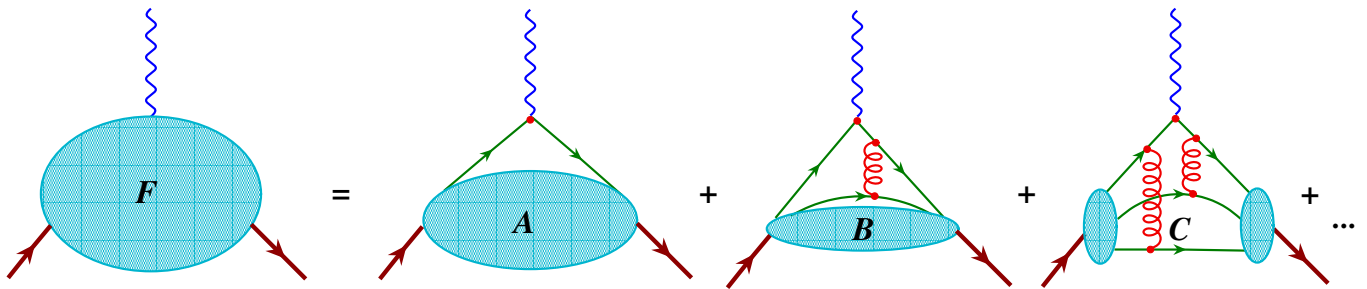


FIG. 1 (color online). Structure of QCD factorization for baryon form factors.

expansion are determined by overlap integrals of the soft parts of hadronic wave functions corresponding to large transverse separations. There is a growing consensus that such “soft” contributions play the dominant role at present energies. Indeed, it is known for a long time that the use of QCD-motivated models for the wave functions allows one to obtain, without much effort, soft contributions comparable in size to experimentally observed values (see, e.g. [9,24,25]). A new trend [26,27] is to use the concept of generalized parton distributions (GPDs, see [28–30] for recent extensive reviews on GPDs) to describe/parametrize soft contributions. The use of GPDs allows us to easily describe existing data by soft contributions alone (the latest attempts can be found in Refs. [31–33]). A subtle point for these semiphenomenological approaches is to avoid double counting of hard rescattering contributions “hidden” in the model-dependent hadron wave functions or GPD parametrizations.

The dominant role of the soft contribution for the pion form factor at moderate momentum transfers, up to $Q^2 \sim 2\text{--}3 \text{ GeV}^2$, is supported by its calculation [34,35] within the QCD sum rule approach [36] applied to the vacuum average $\langle 0|T\{\eta_2(0)j(z)\eta_1^*(y)\}|0\rangle$ of three currents, with j representing the electromagnetic probe and the two others η_1^* , η_2 having quantum numbers of the initial and final hadrons, respectively. The application of the method at higher Q^2 faces the problem that the inclusion of non-perturbative effects due to vacuum condensates through the expansion over inverse powers of the Borel parameter M^2 interferes with the large- Q^2 expansion of the form factors, producing a series of $(Q^2/M^2)^n$ type even for a decreasing function of Q^2 , like $\exp[-Q^2/M^2]$. For the nucleon form factors, the usual QCD sum rule approach works only in the region of small momentum transfers $Q^2 < 1 \text{ GeV}^2$ [37,38]. To extend the results to higher Q^2 , it was proposed [39] to sum back the $(Q^2/M^2)^n$ terms originating from the Taylor expansion of the same nonlocal condensate, using to this end simple models for it. Another approach [35,40] is to use the so-called local quark-hadron duality approximation, in which the values of the duality intervals are postulated to be Q^2 -independent and are taken from the two-point QCD sum rules. The parameter-free results for the pion and nucleon form factors obtained in this way are in a rather good agreement with existing data.

A less assumption-dependent approach allowing us to calculate hadronic form factors for moderately large Q^2 is based on light-cone sum rules (LCSR) [41,42]. The basic object of the LCSR approach is the matrix element $\langle 0|T\{\eta_2(0)j(z)\}|h_1\rangle$ in which the currents η_2 and j are the same as in the usual QCD sum rules, while the initial hadron is explicitly represented by its state vector $|h_1\rangle$, see a schematic representation in Fig. 2. When both the momentum transfer Q^2 and the Borel parameter M^2 of the η_2 channel are large, the asymptotics is governed by the operator product expansion $T\{\eta_2(0)j(z)\} \sim \sum C_i(y)\mathcal{O}_i(0)$

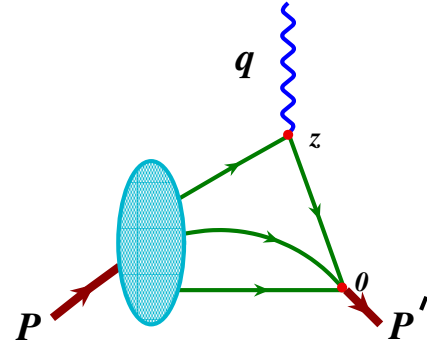


FIG. 2 (color online). Schematic structure of the light-cone sum rule for baryon form factors.

on the light-cone $z^2 = 0$. The z^2 -singularity of a particular perturbatively calculable short-distance factor $C_i(z)$ is determined by the twist of the relevant composite operator \mathcal{O}_i , whose matrix element $\langle 0|\mathcal{O}_i(0)|h_1\rangle$ accumulates non-perturbative information about the initial state parametrized in terms of a distribution amplitude. The lowest order $O(\alpha_s^0)$ terms of the OPE correspond to purely soft contributions ordered by twists of the relevant operators. The magnitude and details of their Q^2 -dependence are governed by the form of the corresponding DA. As shown by the LCSR calculation of the pion form factor [43], taking the CZ model for the pion DA, which enhances the hard contribution, one obtains a soft contribution that is too large. Hence one should take the DAs that are sufficiently narrower¹ in order to describe the experimental magnitude of the pion form factor at accessible Q^2 . The LCSR expansion also contains terms generating the asymptotic pQCD contributions. They appear at proper order in α_s , i.e., in the $O(\alpha_s)$ term for the pion form factor, at the α_s^2 order for the nucleon form factors, etc. In the pion case, it was explicitly demonstrated [49,56] that the contribution of hard rescattering is correctly reproduced in the LCSR approach as a part of the $O(\alpha_s)$ correction. It should be noted that the diagrams of LCSR that contain the hard pQCD contributions also possess soft parts, i.e., one should perform separation of hard and soft terms inside each diagram. As a result, the distinction between hard and soft contributions appears to be scale- and scheme-dependent [56]. During the last years there have been numerous applications of LCSRs to mesons, see [57,58] for a review. Nucleon electromagnetic form factors were first considered in [59,60] and the weak decay $\Lambda_b \rightarrow p\ell\nu_\ell$ in [61].

In this paper, we incorporate light-cone sum rules to develop an approach to the calculation of transition form

¹Recent studies of the pion DA [44–54] show a converging consensus that the integral $\int_0^1 dx x^{-1} \varphi_\pi(x)$ determining the size of the leading-order hard contribution has the value close to that corresponding to the asymptotic wave function, see Ref. [55] for an updated compilation.

factors for electroproduction of the Δ -resonance. All three possibilities for the virtual photon polarization are allowed in this case, hence the $\gamma^*N \rightarrow \Delta$ transition is described by three independent form factors.² Hence, the challenge is not only to fit the absolute magnitude of one of them, but also to explain the relations between different form factors. In particular, pQCD prediction [62,63] for the helicity amplitudes is $A_{1/2} \sim 1/Q^3$ and $A_{3/2} \sim 1/Q^5$. For multipole amplitudes $M1 = -\frac{1}{2}(A_{1/2} + \sqrt{3}A_{3/2})$ and $E2 = -\frac{1}{2}(A_{1/2} - A_{3/2}/\sqrt{3})$, the perturbative QCD approach predicts, hence, the same strength of the $E2$ and $M1$ transitions at asymptotically large Q^2 . Experimentally, their ratio is negative and extremely close to zero (within a few per cent) in the whole investigated region, i.e., up to $Q^2 \sim 4 \text{ GeV}^2$ [64–68]. To explain this phenomenon within the pQCD framework, it was suggested [69] that the observed small value of the $E2/M1$ ratio is due to cancellation of the $A_{1/2}$ and $A_{3/2}$ helicity amplitudes. It should be emphasized that there is no intrinsic reason inside perturbative QCD for such a cancellation to happen. Moreover, given the very strong sensitivity of the pQCD outcome on the form of the nucleon and Δ -isobar distribution amplitudes (e.g., by varying the CZ-type DAs without changing their moments within the limits specified by QCD sum rule estimates, one can change the result for $A_{1/2}$ by an order of magnitude and even reverse its sign,³ see Refs. [63,70,71]) such a fine-tuned cancellation looks like a miracle. Furthermore, since $A_{1/2}$ and $A_{3/2}$ are expected to behave differently with Q^2 , just a nearly constant value of $E2/M1$ ratio in a sufficiently wide region of Q^2 (say, about 1 GeV^2 wide) practically rules out the relevance of pQCD within such a region.

On the other hand, the smallness of $E2/M1$ ratio is a famous prediction of the quark model. 40 years ago it was shown [72] that $E2$ is zero in the nonrelativistic SU(6) quark model, provided that quarks have zero orbital angular momentum. Small deviation of $E2$ from zero was later explained either by D -wave admixtures [73,74] or in terms of two-body exchange currents [75]. In the large N_c limit of QCD, it is possible to show [76] that the $E2/M1$ ratio has a smallness of order $1/N_c^2$ without making any assumptions about the angular momentum of the quarks. Small values for $E2/M1$ in the region $Q^2 < 4 \text{ GeV}^2$ were obtained in the relativistic quark model [77]. The $\gamma^*N \rightarrow \Delta$ transition form factors were also calculated [78] within the local quark-hadron duality approach motivated by QCD sum rules. It gives small values, within $\pm 20\%$, both for the ratios $E2/M1$ and $C2/M1$ ($C2$ being the electric quadru-

pole or Coulombic transition form factor). Recent lattice calculations [79–81] of the $N\Delta$ transition form factors up to 1.5 GeV^2 gives small negative values for both these ratios. All these results provide strong evidence that the observed small value of $E2/M1$ has a purely nonperturbative origin.

Another interesting feature of the $\gamma^*N \rightarrow \Delta$ reaction is that the leading $M1$ magnetic transition form factor $G_M(Q^2)$ decreases with Q^2 faster than the dipole fit (see, e.g., [82]). This was considered as a fact favoring pQCD since it usually gives for $G_M(Q^2)$ a result that is much smaller (by an order of magnitude) than that for the proton elastic form factor $G_M^p(Q^2)$. In the large N_c limit, assuming chiral and isospin symmetry, it was established [28,83] that the transition form factor $G_M(Q^2)$ is expressed through the isovector component of the GPD E related to the elastic spin-flip nucleon form factor $F_2(Q^2)$, which is also known to drop faster than the dipole fit. This observation was incorporated in Ref. [84] to describe both $F_2(Q^2)$ and $G_M(Q^2)$ using a model for the GPDs $E^{u,d}(x, Q^2)$ with a Gaussian $\sim \exp[-(1-x)Q^2/4x\lambda^2]$ plus a small power-law tail ansatz for the Q^2 -dependence. A Regge type ansatz $E^{u,d}(x, Q^2) = e^{u,d}(x)x^{\alpha^{(0)}(1-x)Q^2}$ was used in Ref. [33]. In both models, to get an accurate fit of the data, one needs to introduce a rescaling factor ~ 1.5 in the relation between elastic and transition GPDs.

The goal of the present study is to set up the LCSR-based framework for the calculation of the $\gamma^*N \rightarrow \Delta$ transition form factors. The light-cone sum rule formalism turns out to be considerably more cumbersome in this case because of the spin-3/2 of the Δ resonance. The local interpolating current with the Δ quantum numbers has also a nonzero projection on the spin-1/2 states with opposite parity, and the necessity to get rid of these contaminating contributions produces further complications. Still, as we show, it is possible to derive a general Lorentz decomposition and the twist expansion of sum rules for all three form factors in question. We explicitly calculate the leading α_s order sum rules and compare their consequences with existing experimental data.

Apart from resolving several technical issues, our main finding in this work is that the soft contribution to the $\gamma^*N \rightarrow \Delta$ form factors in the intermediate Q^2 range is strongly affected by the valence quark configurations involving “minus” components of the quark fields that do not have the simple interpretation in terms of the leading-twist amplitude but rather correspond to the contributions of the orbital angular momentum [85,86]. The same conclusion was reached in [59] for the nucleon electromagnetic form factor. In a more general context, large contributions of the orbital angular momentum can explain why helicity selection rules in perturbative QCD appear to be badly broken in hard exclusive processes at present energies. By construction, the LCSRs use nucleon distribution amplitudes of the leading and higher twist [87] as

²The supply of possible choices and definitions of the three form factors offered in the literature seems inexhaustible.

³In this connection, we would also like to remark that the model $A_{1/2}(Q^2) = A_{1/2}(0)/(1 + Q^2/\Lambda_1^2)$ proposed and used in Ref. [69], with experimental *negative* value of $A_{1/2}(0)$, is not able to reproduce the *positive* pQCD asymptotic results for $A_{1/2}(Q^2)$ quoted in Eqs. (5) and (6) of that paper.

the main input, and the results are very sensitive to their shape. Using the asymptotic distribution amplitudes we obtain a reasonable agreement with the data in the range $2 < Q^2 < 6 \text{ GeV}^2$ for the form factors. We believe that the accuracy can be improved significantly by the calculation of $O(\alpha_s)$ corrections to the sum rules and especially if lattice data on the moments of higher-twist distribution amplitudes become available. A long-term goal of our study is to determine leading twist nucleon distribution amplitudes from the combined fit to the experimental data on all of the existing form factors involving the nucleon. In this perspective, the present paper should be viewed as a step in this direction.

The presentation is organized as follows. In Sec. II we introduce the necessary notation and set up the general framework. Section III contains the derivation of sum rules including higher twist corrections, which is our main result. The numerical analysis of the LCSRs is carried out in Sec. IV, together with a summary and discussion. The paper has three appendices devoted to technical aspects of the calculation: In Appendix A we present a complete Lorentz-invariant decomposition of the correlation func-

tion, Appendix B contains the summary of asymptotic expressions for the nucleon distribution amplitudes, and in Appendix C the Belyaev-Ioffe sum rule for the Δ coupling constant is given and reanalyzed.

II. GENERAL FRAMEWORK

A. Definition of the form factors

The $\gamma^*N \rightarrow \Delta$ transition is described by the matrix element of the electromagnetic current

$$j_\nu = e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d \quad (2.1)$$

between the nucleon state with momentum P and the Δ -isobar state with momentum $P' = P - q$. It can be written as

$$\langle \Delta(P') | j_\nu(0) | N(P) \rangle = \bar{\Delta}_\beta(P') \Gamma_{\beta\nu} \gamma_5 N(P), \quad (2.2)$$

where $N(P)$ and $\Delta_\beta(P')$ are the nucleon spinor and the Rarita-Schwinger spinor for the Δ -isobar, respectively.⁴ The decomposition of the vertex function

$$\Gamma_{\beta\nu} = G_1(Q^2)[-q_\beta \gamma_\nu + \not{q} g_{\beta\nu}] + G_2(Q^2)[-q_\beta(P - q/2)_\nu + q(P - q/2)g_{\beta\nu}] + G_3(Q^2)[q_\beta q_\nu - q^2 g_{\beta\nu}]. \quad (2.3)$$

defines three scalar form factors $G_i(Q^2)$. As usual, $Q^2 = -q^2$. Following [88], one can also define the magnetic dipole G_M , electric quadrupole G_E , and Coulomb quadrupole G_C form factors instead of G_1, G_2, G_3 :

$$\begin{aligned} G_M(Q^2) &= \frac{m_P}{3(m_P + m_\Delta)} \left[((3m_\Delta + m_P)(m_\Delta + m_P) + Q^2) \frac{G_1(Q^2)}{m_\Delta} + (m_\Delta^2 - m_P^2)G_2(Q^2) - 2Q^2 G_3(Q^2) \right], \\ G_E(Q^2) &= \frac{m_P}{3(m_P + m_\Delta)} \left[(m_\Delta^2 - m_P^2 - Q^2) \frac{G_1(Q^2)}{m_\Delta} + (m_\Delta^2 - m_P^2)G_2(Q^2) - 2Q^2 G_3(Q^2) \right], \\ G_C(Q^2) &= \frac{2m_P}{3(m_\Delta + m_P)} \left[2m_\Delta G_1(Q^2) + \frac{1}{2}(3m_\Delta^2 + m_P^2 + Q^2)G_2(Q^2) + (m_\Delta^2 - m_P^2 - Q^2)G_3(Q^2) \right]. \end{aligned} \quad (2.4)$$

For a comparison with the literature we also write down the form factors G_M^{Ash} [89], G_T [82] and the ratios R_{EM} [88] and R_{SM} (see e.g. [90–92]) that are used in experimental papers

$$\begin{aligned} G_M(Q^2) &= G_M^{\text{Ash}}(Q^2) \sqrt{1 + \frac{Q^2}{(m_\Delta + m_P)^2}}, & |G_M(Q^2)|^2 + 3|G_E(Q^2)|^2 &= \frac{Q^2}{Q^2 + \nu^2} \left(1 + \frac{Q^2}{(m_\Delta + m_P)^2} \right) |G_T(Q^2)|^2, \\ \nu &= \frac{m_\Delta^2 - m_P^2 + Q^2}{2m_P}, & R_{\text{EM}}(Q^2) &= \frac{E2(Q^2)}{M1(Q^2)} = \frac{E_{1+}}{M_{1+}} = -\frac{G_E(Q^2)}{G_M(Q^2)}, \\ R_{\text{SM}}(Q^2) &= \frac{C2(Q^2)}{M1(Q^2)} = \frac{S_{1+}}{M_{1+}} = -\sqrt{Q^2 + \frac{(m_\Delta^2 - m_P^2 - Q^2)^2}{4m_\Delta^2}} \frac{1}{2m_\Delta} \frac{G_C(Q^2)}{G_M(Q^2)}. \end{aligned} \quad (2.5)$$

Note that there is a disagreement in the literature on the overall sign in the relation between R_{SM} and the ratio of the quadrupole and the magnetic form factors $G_C(Q^2)/G_M(Q^2)$. We follow the definition from [92].

⁴We hope that denoting particle states by the same letters as the corresponding spinors will not create confusion.

B. Choice of the correlation function

To extract information about hadronic form factors within the light-cone sum rule approach we should analyze a matrix element in which one of the hadrons is represented by an interpolating field with proper quantum numbers, and another is described by its explicit state vector, cf. Figure 2. Building the sum rule, we would need distribution amplitudes of the latter hadron. The nucleon DAs for all three-quark operators were introduced and studied in Ref. [87], while in the case of the Δ -isobar such an analysis is yet to be performed. Thus, in the present paper, we consider the correlation function given by the matrix element

$$T_{\mu\nu}(P, q) = i \int d^4 z e^{iqz} \langle 0 | T \{ \eta_\mu(0) j_\nu(z) \} | N(P) \rangle \quad (2.6)$$

between the vacuum and a single-nucleon state $|N(P)\rangle$. The interpolating field for the Δ^+ -particle is taken in the form suggested in [93]

$$\begin{aligned} \eta_\mu(0) = & \epsilon^{abc} [2(u^a(0)C\gamma_\mu d^b(0))u^c(0) \\ & + (u^a(0)C\gamma_\mu u^b(0))d^c(0)], \end{aligned} \quad (2.7)$$

where a, b, c are color indices and C is the charge conjugation matrix. The contribution of Δ^+ to the correlation function in Eq. (2.6) is given by

$$\begin{aligned} T_{\mu\nu}(P, q) = & \frac{1}{m_\Delta^2 - (P')^2} \sum_s \langle 0 | \eta_\mu(0) | \Delta(P', s) \rangle \\ & \times \langle \Delta(P', s) | j_\nu(0) | N(P) \rangle. \end{aligned} \quad (2.8)$$

Parametrizing the matrix element

$$\langle 0 | \eta_\mu(0) | \Delta(P', s) \rangle = \frac{\lambda_\Delta}{(2\pi)^2} \Delta_\mu^{(s)}(P') \quad (2.9)$$

in terms of λ_Δ , the coupling constant of the Δ^+ -particle to the current η_μ , and using the standard spin summation formula for Rarita-Schwinger spinors

$$\begin{aligned} \sum_s \Delta_\mu^{(s)}(P') \bar{\Delta}_\beta^{(s)}(P') = & -(\not{P}' + m_\Delta) \left\{ g_{\mu\beta} - \frac{1}{3} \gamma_\mu \gamma_\beta \right. \\ & \left. - \frac{2P'_\mu P'_\beta}{3m_\Delta^2} + \frac{P'_\mu \gamma_\beta - P'_\beta \gamma_\mu}{3m_\Delta} \right\} \end{aligned} \quad (2.10)$$

we write this contribution as

$$\begin{aligned} T_{\mu\nu}^{(\Delta)}(P, q) = & -\frac{\lambda_\Delta}{(2\pi)^2} \frac{\not{P}' + m_\Delta}{m_\Delta^2 - (P')^2} \left[g_{\mu\beta} - \frac{1}{3} \gamma_\mu \gamma_\beta \right. \\ & \left. - \frac{2P'_\mu P'_\beta}{3m_\Delta^2} + \frac{P'_\mu \gamma_\beta - P'_\beta \gamma_\mu}{3m_\Delta} \right] \Gamma_{\beta\nu} \gamma_5 N(P). \end{aligned} \quad (2.11)$$

This expression provides us with one of the starting points of our analysis.

To extract the Δ -related information from the total correlator $T_{\mu\nu}(P, q)$, we should take into account a subtlety that the current η_μ couples not only to isospin $I = \frac{3}{2}$ spin-parity $\frac{3}{2}^+$ states, but also to isospin $I = \frac{3}{2}$ spin-parity $\frac{1}{2}^-$ states. It makes sense to eliminate their contribution by a clever choice of the Lorentz structures. For a generic $\frac{1}{2}^-$ resonance, call it Δ^* , we define

$$\langle 0 | \eta_\mu(0) | \Delta^*(P', s) \rangle = \frac{\lambda_*}{(2\pi)^2} (m_* \gamma_\mu - 4P'_\mu) \Delta^*(P', s) \quad (2.12)$$

with λ^* being the corresponding coupling and m_* the mass. The Δ^* spinor satisfies the usual Dirac equation $(\not{P}' - m_*)\Delta^*(P') = 0$, and the matrix element of the electromagnetic current between the nucleon and a Δ^* state takes the form

$$\begin{aligned} \langle \Delta^*(P') | j_\nu(0) | N(P) \rangle = & \bar{\Delta}^*(P') [(\gamma_\nu q^2 - \not{q} q_\nu) F_1^{N\Delta^*}(q^2) \\ & - i\sigma_{\nu\alpha} q^\alpha F_2^{N\Delta^*}(q^2)] \gamma_5 N(P). \end{aligned} \quad (2.13)$$

The corresponding contribution to the correlation function in Eq. (2.7) reads

$$\begin{aligned} T_{\mu\nu}^{(\Delta^*)}(P, q) = & \frac{\lambda_\Delta^*}{(2\pi)^2} [m_* \gamma_\mu - 4P'_\mu] \frac{\not{P}' + m_*}{m_*^2 - (P')^2} \\ & \times [(\gamma_\nu q^2 - \not{q} q_\nu) F_1^{N\Delta^*} \\ & - i\sigma_{\nu\alpha} q^\alpha F_2^{N\Delta^*}] \gamma_5 N(P), \end{aligned} \quad (2.14)$$

and the question is whether (and at which cost) the contributions in Eq. (2.12) and (2.15) can be separated. To achieve this, we have to understand the general decomposition of the correlation function in Eq. (2.7) into contributions of different Lorentz structures. To this end, it is convenient to go over to the twist decomposition in the infinite momentum frame.

C. Light cone expansion

1. Kinematics

Having in mind the practical construction of light-cone sum rules that involve nucleon distribution amplitudes, we define a lightlike vector n_μ by the condition

$$q \cdot n = 0, \quad n^2 = 0 \quad (2.15)$$

and introduce the second lightlike vector

$$p_\mu = P_\mu - \frac{1}{2} n_\mu \frac{m_P^2}{P \cdot n}, \quad p^2 = 0, \quad (2.16)$$

so that $P \rightarrow p$ if the nucleon mass can be neglected, $m_P \rightarrow 0$. The photon momentum then can be written as

$$q_\mu = q_{\perp\mu} + n_\mu \frac{P \cdot q}{P \cdot n}. \quad (2.17)$$

Assume for a moment that the nucleon moves in the

positive \mathbf{e}_z direction, then p^+ and n^- are the only non-vanishing components of p and n , respectively. The infinite momentum frame can be visualized as the limit $p^+ \sim Q \rightarrow \infty$ with fixed $P \cdot n = p \cdot n \sim 1$ where Q is the large scale in the process. Expanding the matrix element in powers of $1/p^+$ introduces the power counting in Q . In this language, twist counts the suppression in powers of p^+ . Similarly, the nucleon spinor $N_\gamma(P, \lambda)$ has to be decomposed in ‘‘large’’ and ‘‘small’’ components as

$$\begin{aligned} N_\gamma(P, \lambda) &= \frac{1}{2p \cdot n} (\not{p}\not{n} + \not{n}\not{p}) N_\gamma(P, \lambda) \\ &= N_\gamma^+(P, \lambda) + N_\gamma^-(P, \lambda), \end{aligned} \quad (2.18)$$

where we have introduced two projection operators

$$\Lambda^+ = \frac{\not{p}\not{n}}{2p \cdot n}, \quad \Lambda^- = \frac{\not{n}\not{p}}{2p \cdot n} \quad (2.19)$$

that project onto the ‘‘plus’’ and minus components of the spinor. Note the useful relations

$$\not{p}N(P) = m_P N^+(P), \quad \not{n}N(P) = \frac{2p \cdot n}{m_P} N^-(P) \quad (2.20)$$

that follow readily from the Dirac equation $\not{p}N(P) = MN(P)$. Using the explicit expressions for $N(P)$ it is easy to see that $\Lambda^+N = N^+ \sim \sqrt{p^+}$ while $\Lambda^-N = N^- \sim 1/\sqrt{p^+}$.

The correlation function $T_{\mu\nu}$ in Eq. (2.7) can be expanded in contributions with increasing twist. The leading twist contribution corresponds to the projection

$$\Lambda_+ T_{++} \quad (2.21)$$

and contains the maximum power of the large momentum p^+ . There exist three projections of the next-to-leading twist that are suppressed by one power of the large momentum compared to the leading twist, namely,

$$\Lambda_- T_{++}, \quad \Lambda_+ T_{\perp+}, \quad \Lambda_+ T_{+\perp}, \quad (2.22)$$

and more contributions with loosing two powers of the momentum, etc.

Each light-cone projection in a general situation gives rise to several invariant Lorentz structures, in particular,

$$\begin{aligned} \Lambda_+ n^\mu n^\nu T_{\mu\nu} &= (p \cdot n)^2 \{ \mathcal{A}(Q^2, Pq) \\ &\quad + \not{q}_\perp \mathcal{B}(Q^2, Pq) \} \gamma_5 N^+(P), \\ \Lambda_- n^\mu n^\nu T_{\mu\nu} &= (p \cdot n)^2 \{ C(Q^2, Pq) \\ &\quad + \not{q}_\perp \mathcal{D}(Q^2, Pq) \} \gamma_5 N^-(P), \\ \Lambda_+ n^\mu e_\perp^\nu T_{\mu\nu} &= (p \cdot n) \{ \not{e}_\perp \mathcal{E}(Q^2, Pq) \\ &\quad + \not{e}_\perp \not{q}_\perp \mathcal{F}(Q^2, Pq) \} \gamma_5 N^+(P), \\ \Lambda_+ e_\perp^\mu n^\nu T_{\mu\nu} &= (p \cdot n) \{ \not{e}_\perp \mathcal{G}(Q^2, Pq) \\ &\quad + \not{e}_\perp \not{q}_\perp \mathcal{H}(Q^2, Pq) \} \gamma_5 N^+(P), \\ \Lambda_+ q_\perp^\mu n^\nu T_{\mu\nu} &= (p \cdot n) \{ I(Q^2, Pq) \\ &\quad + \not{q}_\perp \mathcal{J}(Q^2, Pq) \} \gamma_5 N^+(P), \end{aligned} \quad (2.23)$$

where e_\perp is a two-component vector in the transverse plane that is orthogonal to q_\perp :

$$e_\perp \cdot q_\perp = 0, \quad (e_\perp)^2 = 1. \quad (2.24)$$

The invariant amplitudes $\mathcal{A} - \mathcal{J}$ are not all independent. First of all, note that the $n^\mu q_\perp^\nu T_{\mu\nu}$ -projection does not lead to new independent amplitudes because of the transversality condition $q^\nu T_{\mu\nu} = 0$. Second, the Rarita-Schwinger constraint $\gamma^\mu T_{\mu\nu} = 0$ results in two relations

$$\mathcal{G} + \mathcal{J} - m_P \mathcal{C} = 0, \quad \mathcal{H} - \frac{1}{Q^2} I + m_P \mathcal{D} = 0. \quad (2.25)$$

Finally, more relations follow from the Lorentz symmetry. In order to find them, we write a Lorentz-invariant decomposition of the correlation function $T_{\mu\nu}$ which involves 10 independent amplitudes, see Appendix A. Taking the necessary light-cone projections, we obtain two new relations that can be chosen as

$$\begin{aligned} 2\mathcal{F} - m_P \mathcal{D} + m_P \mathcal{B} + 2\mathcal{H} &= 0, \\ m_P \mathcal{A} - 2(Pq)\mathcal{B} - m_P \mathcal{C} - 2\mathcal{E} - 2\mathcal{G} &= 0. \end{aligned} \quad (2.26)$$

The remaining 6 independent invariant functions produce an overcomplete set of sum rules of the leading and next-to-leading twist for the three form factors of the $\gamma^*N \rightarrow \Delta$ transition.

2. The $J^P = \frac{3}{2} +$ contributions

Now, the contribution of the Δ^+ to the invariant functions defined above can readily be calculated using Eq. (2.12). The result has the following form:

$$\begin{aligned}
\mathcal{A}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{Q^2}{3m_\Delta^2} [2G_1 + (2m_\Delta - m_P)G_2 + 2(m_P - m_\Delta)G_3], \\
\mathcal{B}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{1}{3m_\Delta^2} [-2m_P G_1 + [m_\Delta(m_\Delta - m_P) - Q^2]G_2 + 2Q^2 G_3], \\
\mathcal{C}^\Delta &= \frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{Q^2}{3m_\Delta^2 m_P} [2(m_P + m_\Delta)G_1 + (m_\Delta^2 + Q^2)G_2 + 2[m_\Delta(m_P - m_\Delta) - Q^2]G_3], \\
\mathcal{D}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{1}{3m_\Delta^2 m_P} [2(Q^2 - m_\Delta m_P)G_1 + (m_\Delta - m_P)(m_\Delta^2 + Q^2)G_2 + 2m_P Q^2 G_3], \\
\mathcal{E}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{1}{6m_\Delta^2} [2[m_P(m_P^2 - m_\Delta^2) + Q^2(2m_\Delta + m_P)]G_1 + m_\Delta(m_\Delta - m_P)^2(m_\Delta + m_P)G_2 \\
&\quad + 2Q^2 m_\Delta(m_P - m_\Delta)G_3], \\
\mathcal{F}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{1}{6m_\Delta^2} [2[Q^2 - 4m_\Delta^2 + (m_\Delta - m_P)^2]G_1 + m_\Delta(m_P^2 - m_\Delta^2)G_2 + 2m_\Delta Q^2 G_3], \\
\mathcal{G}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{Q^2}{6m_\Delta} [-2G_1 + (m_P + m_\Delta)G_2 + 2(m_\Delta - m_P)G_3], \\
\mathcal{H}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{1}{6m_\Delta} [2(3m_\Delta + m_P)G_1 + (2m_\Delta(m_\Delta - m_P) + Q^2)G_2 - 2Q^2 G_3], \\
\mathcal{I}^\Delta &= \frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{Q^2}{6m_\Delta^2} [2(m_\Delta m_P - 3m_\Delta^2 - 2Q^2)]G_1 + (4m_\Delta^2(m_P - m_\Delta) + Q^2(2m_P - 3m_\Delta))G_2 + 2Q^2(m_\Delta - 2m_P)G_3], \\
\mathcal{J}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \frac{Q^2}{6m_\Delta^2} [-2(2m_P + m_\Delta)G_1 - (m_\Delta(m_P + 3m_\Delta) + 2Q^2)G_2 + 2(m_\Delta(m_\Delta - m_P) + 2Q^2)G_3]. \quad (2.27)
\end{aligned}$$

3. The $J^P = \frac{1}{2}^-$ contributions

For completeness, we present here the (unwanted) contributions to the same invariant functions $\mathcal{A} \cdots \mathcal{J}$ of the negative parity spin-1/2 Δ -resonances, cf. (2.15):

$$\begin{aligned}
\mathcal{A}^{\Delta^*} &= \frac{8\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} Q^2 F_1, & \mathcal{B}^{\Delta^*} &= -\frac{8\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} F_2, & \mathcal{C}^{\Delta^*} &= -\frac{4\lambda_*/(2\pi)^2 Q^2}{(m_*^2 - P^2) m_P} [m_* F_1 + 2F_2], \\
\mathcal{D}^{\Delta^*} &= \frac{4\lambda_*/(2\pi)^2}{(m_*^2 - P^2) m_P} [2Q^2 F_1 - m_* F_2], & \mathcal{E}^{\Delta^*} &= \frac{4\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} [Q^2(m_* + m_P)F_1 + (m_P^2 - m_*^2)F_2], \\
\mathcal{F}^{\Delta^*} &= \frac{4\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} [Q^2 F_1 + (m_P - m_*)F_2], & \mathcal{G}^{\Delta^*} &= -\frac{2\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} Q^2 m_* F_1, & \mathcal{H}^{\Delta^*} &= \frac{2\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} m_* F_2, \\
\mathcal{I}^{\Delta^*} &= \frac{2\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} Q^2 [4Q^2 F_1 - m_* F_2], & \mathcal{J}^{\Delta^*} &= -\frac{2\lambda_*/(2\pi)^2}{(m_*^2 - P^2)} Q^2 [m_* F_1 + 4F_2].
\end{aligned} \quad (2.28)$$

Here $F_{1,2} = F_{1,2}^{\Delta^*}(Q^2)$ and $m_* = m_{\Delta^*}$ is the mass of the Δ^* resonance. Taking into account the relations in (2.26) and (2.27) we find that there exist two independent combinations of the invariant functions that are free from contributions of $J^P = \frac{1}{2}^-$ resonances with arbitrary mass m_* :

$$\begin{aligned}
\mathcal{G}^{\Delta^*} - \mathcal{J}^{\Delta^*} + Q^2 \mathcal{B}^{\Delta^*} &= 0, \\
\mathcal{I}^{\Delta^*} + Q^2 \mathcal{H}^{\Delta^*} - Q^2 \mathcal{A}^{\Delta^*} &= 0.
\end{aligned} \quad (2.29)$$

On the other hand, using (2.27) one obtains very simple expressions for the Δ -isobar contributions to the same combinations:

$$\begin{aligned}
\mathcal{G}^\Delta - \mathcal{J}^\Delta + Q^2 \mathcal{B}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} Q^2 G_2(Q^2), \\
\mathcal{I}^\Delta + Q^2 \mathcal{H}^\Delta - Q^2 \mathcal{A}^\Delta &= -\frac{\lambda_\Delta/(2\pi)^2}{(m_\Delta^2 - P^2)} \\
&\quad \times Q^2 [2G_1(Q^2) \\
&\quad + (m_\Delta - m_P)G_2(Q^2)]. \quad (2.30)
\end{aligned}$$

This implies that one can construct light-cone sum rules for the $\gamma^* N \rightarrow \Delta$ form factors G_1 and G_2 that are free from contamination by negative parity spin-1/2 resonances. For

the form factor G_3 this separation is not possible, unless one goes over to Lorentz structures of yet higher twist. Since the accuracy of light-cone sum rules is expected to decrease with twist, the gain of excluding spin-1/2 resonances in this case is probably not worth the effort. Note that the difference between the magnetic G_M and electric G_E form factors only involves G_1 :

$$G_M(Q^2) - G_E(Q^2) = \frac{2m_P}{3m_\Delta} \frac{[(m_\Delta + m_P)^2 + Q^2]}{(m_\Delta + m_P)} G_1(Q^2) \quad (2.31)$$

III. CALCULATION OF CORRELATION FUNCTIONS

On the other side, one can calculate the invariant functions $\mathcal{A} \cdots \mathcal{J}$ for Euclidean virtuality $(P')^2 = (P - q)^2$ in

$$\begin{aligned} 4\langle 0 | \varepsilon^{ijk} u_\alpha^i(0) u_\beta^j(0) d_\gamma^k(z) | N(P) \rangle = & \left(\mathcal{V}_1 + \frac{z^2 m_P^2}{4} \mathcal{V}_1^M \right) (\not{P} C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{V}_2 m_P (\not{P} C)_{\alpha\beta} (\not{z} \gamma_5 N)_\gamma \\ & + \mathcal{V}_3 m_P (\gamma_\mu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{V}_4 m_P^2 (\not{z} C)_{\alpha\beta} (\gamma_5 N)_\gamma \\ & + \mathcal{V}_5 m_P^2 (\gamma_\mu C)_{\alpha\beta} (i\sigma^{\mu\nu} z_\nu \gamma_5 N)_\gamma + \mathcal{V}_6 m_P^3 (\not{z} C)_{\alpha\beta} (\not{z} \gamma_5 N)_\gamma + \left(\mathcal{T}_1 + \frac{z^2 m_P^2}{4} \mathcal{T}_1^M \right) \\ & \times (P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_2 m_P (z^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma_5 N)_\gamma \\ & + \mathcal{T}_3 m_P (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \gamma_5 N)_\gamma + \mathcal{T}_4 m_P (P^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\varrho} z_\varrho \gamma_5 N)_\gamma \\ & + \mathcal{T}_5 m_P^2 (z^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_6 m_P^2 (z^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\not{z} \gamma_5 N)_\gamma \\ & + \mathcal{T}_7 m_P^2 (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} z \gamma_5 N)_\gamma + \mathcal{T}_8 m_P^3 (z^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\varrho} z_\varrho \gamma_5 N)_\gamma. \end{aligned} \quad (3.1)$$

The expansion in (3.1) should be viewed as an operator product expansion to the leading order in the strong coupling. Each of the functions \mathcal{V}_i and \mathcal{T}_i is a function of the scalar product $(P \cdot z)$ and also depends on the deviation from the light-cone z^2 at most logarithmically. In addition, we take into account the $O(z^2)$ corrections to the leading-twist-3 structures, denoted by \mathcal{V}_1^M and \mathcal{T}_1^M . The invariant functions $\mathcal{V}_1(Pz), \dots, \mathcal{T}_8(Pz)$ can be expressed in terms of the nucleon distribution amplitudes $V_1(x_i), \dots, T_8(x_i)$ with increasing twist, introduced in Ref. [87]:

$$\begin{aligned} \mathcal{V}_1 = V_1, \quad 2(pz)\mathcal{V}_2 = V_1 - V_2 - V_3, \quad 2\mathcal{V}_3 = V_3, \quad 4(pz)\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5, \\ 4(pz)\mathcal{V}_5 = V_4 - V_3, \quad 4(pz)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6, \quad \mathcal{T}_1 = T_1, \quad 2\mathcal{T}_2 = T_1 + T_2 - 2T_3, \\ 2\mathcal{T}_3 = T_7, \quad 2(pz)\mathcal{T}_4 = T_1 - T_2 - 2T_7, \quad 2(pz)\mathcal{T}_5 = -T_1 + T_5 + 2T_8, \\ 4(pz)^2\mathcal{T}_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \quad 4(pz)\mathcal{T}_7 = T_7 - T_8, \\ 4(pz)^2\mathcal{T}_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8. \end{aligned} \quad (3.2)$$

Each distribution amplitude $F = V_i, T_i$ can be represented by a Fourier integral

$$F(pz) = \int_0^1 dx_3 \int_0^{1-x_3} dx_1 e^{-ix_3(pz)} F(x_1, 1 - x_1 - x_3, x_3), \quad (3.3)$$

where the functions $F(x_i)$ depend on the dimensionless variables $x_1, x_2 = 1 - x_1 - x_3, x_3, 0 < x_i < 1$ which correspond to the longitudinal momentum fractions carried by the quarks inside the nucleon. In difference to the ‘‘calligraphic’’ functions $\mathcal{V}_i(x_i), \mathcal{T}_i(x_i)$ each of the distribution

terms of the nucleon (proton) DAs of increasing twist. The calculation can be simplified considerably by observing that only the isospin-one part of the electromagnetic current can initiate $N\Delta$ -transitions. Hence, all correlation functions must be proportional to $e_u - e_d$ and a (simpler) calculation of the d -quark contribution suffices to obtain the complete result. An additional simplification is due to the fact that, to the leading-order accuracy in QCD coupling, the two u -quarks in the Δ -current (2.7) remain spectators and retain their position at the origin. It is, therefore, sufficient to retain only those terms in the general Lorentz decomposition of the three-quark matrix element between vacuum and the proton state, that are symmetric with respect to the interchange of the momentum fractions of the two u -quarks [87]:

amplitudes $V_i(x_i), T_i(x_i)$ has definite twist, see Table I, and corresponds to the matrix element of a (renormalized) three-quark operator with exactly lightlike separations $z^2 \rightarrow 0$, see Table 2 and Appendix C in Ref. [87] for the details. The higher-twist distribution amplitudes $V_2(x_i), \dots, V_6(x_i)$ correspond to ‘‘wrong’’ components of the quark spinors and have different helicity structure compared to the leading twist amplitude. For baryons these ‘‘bad’’ components cannot all be traded for gluons as in the case of mesons [94,95]. They are not all independent, but related to each other by the exact QCD equations of

TABLE I. Twist classification of the distribution amplitudes in (3.2).

twist-3	twist-4	twist-5	twist-6
V_1	V_2, V_3	V_4, V_5	V_6
T_1	T_2, T_3, T_7	T_4, T_5, T_8	T_6

motion. As a result, to the leading conformal spin accuracy, the five functions $V_2(x_i), \dots, V_6(x_i)$ involve only a single nonperturbative higher twist parameter. In the calculations presented below we use the conformal expansions of higher twist distribution amplitudes to the next-to-leading order (include “ P -wave”). This accuracy is consistent with neglecting multiparton components with extra gluons (quark-antiquark pairs) that are of yet higher spin. Finally, the invariant functions $\mathcal{V}_1^M(Pz), \mathcal{T}_1^M(Pz)$ are twist-5 and can be calculated using equations of motion in terms of the nucleon distribution amplitudes [59].

Explicit expressions for the distribution amplitudes are collected in Appendix B.

In the expressions given below we use the following shorthand notations:

$$\begin{aligned} \tilde{F}(x_3) &= \int_1^{x_3} dx'_3 \int_0^{1-x'_3} dx_1 F(x_1, 1-x_1-x'_3, x'_3), \\ \tilde{\tilde{F}}(x_3) &= \int_1^{x_3} dx'_3 \int_1^{x'_3} dx''_3 \int_0^{1-x''_3} dx_1 F(x_1, 1-x_1-x''_3, x''_3), \end{aligned} \quad (3.4)$$

where $F = V_i, T_i$. These functions result from partial integration in x_3 which is done in order to eliminate the $1/pz$ factors that appear in the definition of nucleon distribution amplitudes (3.2). After this, the $\int d^4z$ integration becomes trivial. The surface terms sum up to zero.

After a straightforward but tedious calculation we obtain the desired expansions:

$$\begin{aligned} \mathcal{A}^{\text{QCD}} &= 4(e_d - e_u) \int_0^1 dx_3 \left\{ \frac{x_3}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (V_1 - T_1)(x_i) + \frac{x_3 m_P^2}{(x_3 P - q)^4} (V_1^{M(d)} - T_1^{M(d)}) \right. \\ &\quad + \frac{x_3^2 m_P^2}{(x_3 P - q)^4} (-2\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 + 2\tilde{T}_1 - \tilde{T}_2 - \tilde{T}_5 - 2\tilde{T}_7 - 2\tilde{T}_8) \\ &\quad + \frac{2x_3^3 m_P^4}{(x_3 P - q)^6} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6 - \tilde{T}_1 + \tilde{T}_2 + \tilde{T}_5 - \tilde{T}_6 + 2\tilde{T}_7 + 2\tilde{T}_8) \\ &\quad \left. + \left(\frac{x_3 m_P^2}{(x_3 P - q)^4} + \frac{2x_3 m_P^2 Q^2}{(x_3 P - q)^6} \right) (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \right\}, \end{aligned} \quad (3.5)$$

$$\begin{aligned} \mathcal{B}^{\text{QCD}} &= 4(e_d - e_u) \int_0^1 dx_3 \left\{ \frac{x_3 m_P}{(x_3 P - q)^4} (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{T}_1 - \tilde{T}_3 - \tilde{T}_7) \right. \\ &\quad \left. + \frac{2x_3^2 m_P^3}{(x_3 P - q)^6} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6 - \tilde{T}_1 + \tilde{T}_3 + \tilde{T}_4 - \tilde{T}_6 + \tilde{T}_7 + \tilde{T}_8) \right\}, \end{aligned} \quad (3.6)$$

$$\begin{aligned} \mathcal{C}^{\text{QCD}} &= 2(e_d - e_u) \int_0^1 dx_3 \left\{ \frac{x_3}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (-T_1 + V_3)(x_i) - \frac{x_3 m_P^2}{(x_3 P - q)^4} T_1^{M(d)} - \frac{1}{(x_3 P - q)^2} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3) \right. \\ &\quad + \frac{Q^2}{(x_3 P - q)^4} (\tilde{T}_1 - \tilde{T}_3 - \tilde{T}_7 - \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3) + \frac{x_3^2 m_P^2}{(x_3 P - q)^4} (\tilde{V}_4 - \tilde{V}_3 + \tilde{T}_1 - \tilde{T}_2 + \tilde{T}_3 - \tilde{T}_5 - \tilde{T}_7 - 2\tilde{T}_8) \\ &\quad + \left(\frac{2x_3 m_P^2}{(x_3 P - q)^4} + \frac{4x_3 m_P^2 Q^2}{(x_3 P - q)^6} \right) (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6) \\ &\quad + \left(\frac{x_3 m_P^2}{(x_3 P - q)^4} + \frac{4x_3 m_P^2 Q^2}{(x_3 P - q)^6} \right) (-\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_5 - \tilde{T}_6 + 2\tilde{T}_7 + 2\tilde{T}_8) \\ &\quad \left. - \frac{2x_3 m_P^2 [Q^2 - x_3^2 m_P^2]}{(x_3 P - q)^6} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \right\}, \end{aligned} \quad (3.7)$$

$$\begin{aligned}
 \mathcal{D}^{\text{QCD}} = & 2 \frac{(e_d - e_u)}{m_P} \int_0^1 dx_3 \left\{ \frac{1}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 V_1(x_i) + \frac{m_P^2}{(x_3 P - q)^4} V_1^{M(d)} \right. \\
 & + \frac{x_3 m_P^2}{(x_3 P - q)^4} (-3\tilde{V}_1 + \tilde{V}_2 + 2\tilde{V}_3 + \tilde{V}_4 + 2\tilde{V}_5 + 2\tilde{T}_1 - \tilde{T}_2 - \tilde{T}_5 - 2\tilde{T}_7 - 2\tilde{T}_8) \\
 & + \frac{4x_3^2 m_P^4}{(x_3 P - q)^6} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6 - \tilde{T}_1 + \tilde{T}_2 + \tilde{T}_5 - \tilde{T}_6 + 2\tilde{T}_7 + 2\tilde{T}_8) \\
 & \left. + \left(\frac{m_P^2}{(x_3 P - q)^4} + \frac{2m_P^2 [Q^2 - x_3^2 m_P^2]}{(x_3 P - q)^6} \right) (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \right\}, \quad (3.8)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{E}^{\text{QCD}} = & (e_d - e_u) \int_0^1 dx_3 \left\{ \frac{2m_P x_3}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (V_1 - V_3) + \frac{2m_P}{(x_3 P - q)^2} (\tilde{T}_1 - \tilde{T}_3 - \tilde{T}_7) + \frac{2x_3 m_P^3}{(x_3 P - q)^4} V_1^{M(d)} \right. \\
 & \left. + \frac{2x_3^2 m_P^3}{(x_3 P - q)^4} (-\tilde{V}_1 + \tilde{V}_3 + \tilde{V}_5) + \frac{2m_P Q^2}{(x_3 P - q)^4} (\tilde{T}_1 - \tilde{T}_3 - \tilde{T}_7) + \frac{2x_3 m_P^3}{(x_3 P - q)^4} (-\tilde{T}_1 + \tilde{T}_3 + \tilde{T}_4 - \tilde{T}_6 + \tilde{T}_7 + \tilde{T}_8) \right\}, \quad (3.9)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}^{\text{QCD}} = & (e_d - e_u) \int_0^1 dx_3 \left\{ \frac{2}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (T_1 + V_1) + \frac{2m_P^2}{(x_3 P - q)^4} (V_1^{M(d)} + T_1^{M(d)}) \right. \\
 & \left. - \frac{2x_3 m_P^2}{(x_3 P - q)^4} (\tilde{T}_1 - \tilde{T}_3 - \tilde{T}_7 + \tilde{V}_1 - \tilde{V}_3 - \tilde{V}_5) - \frac{2m_P^2}{(x_3 P - q)^4} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \right\}, \quad (3.10)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{G}^{\text{QCD}} = & (e_d - e_u) \int_0^1 dx_3 \left\{ \frac{x_3 m_P}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (V_3 - T_1)(x_i) + \frac{m_P}{(x_3 P - q)^2} (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3) - \frac{x_3 m_P^3}{(x_3 P - q)^4} T_1^{M(d)} \right. \\
 & + \frac{m_P Q^2}{(x_3 P - q)^4} (-\tilde{T}_1 + \tilde{T}_3 + \tilde{T}_7 - \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3) + \frac{x_3^2 m_P^3}{(x_3 P - q)^4} (\tilde{T}_1 - \tilde{T}_2 + \tilde{T}_3 - \tilde{T}_5 - \tilde{T}_7 - 2\tilde{T}_8 - \tilde{V}_3 + \tilde{V}_4) \\
 & + \frac{x_3 m_P^3}{(x_3 P - q)^4} (-\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_5 - \tilde{T}_6 + 2\tilde{T}_7 + 2\tilde{T}_8) + \frac{2x_3 m_P^3}{(x_3 P - q)^4} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6) \\
 & \left. + \frac{2x_3 m_P^3 [Q^2 + x_3^2 m_P^2]}{(x_3 P - q)^6} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \right\}, \quad (3.11)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{H}^{\text{QCD}} = & (e_d - e_u) \int_0^1 dx_3 \left\{ \frac{-1}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (V_1 + 2T_1)(x_i) - \frac{m_P^2}{(x_3 P - q)^4} (V_1^{M(d)} + 2T_1^{M(d)}) \right. \\
 & + \frac{x_3 m_P^2}{(x_3 P - q)^4} (2\tilde{T}_1 - \tilde{T}_2 - \tilde{T}_5 - 2\tilde{T}_7 - 2\tilde{T}_8 + \tilde{V}_1 - \tilde{V}_2 - 2\tilde{V}_3 + \tilde{V}_4) \\
 & + \frac{3m_P^2}{(x_3 P - q)^4} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) + \frac{2x_3^2 m_P^4}{(x_3 P - q)^6} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \\
 & \left. + \frac{2m_P^2 Q^2}{(x_3 P - q)^6} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \right\}, \quad (3.12)
 \end{aligned}$$

$$\begin{aligned}
J^{\text{QCD}} = & (e_d - e_u) Q^2 \int_0^1 dx_3 \left\{ \frac{1}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (V_1 - 2T_1)(x_i) + \frac{m_P^2}{(x_3 P - q)^4} (V_1^{M(d)} - 2T_1^{M(d)}) \right. \\
& - \frac{x_3 m_P^2}{(x_3 P - q)^4} (-6\tilde{T}_1 + 3\tilde{T}_2 + 3\tilde{T}_5 + 6\tilde{T}_7 + 6\tilde{T}_8 + 5\tilde{V}_1 - \tilde{V}_2 - 2\tilde{V}_3 - 3\tilde{V}_4 - 4\tilde{V}_5) \\
& + \frac{5m_P^2}{(x_3 P - q)^4} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) - \frac{2x_3^2 m_P^4}{(x_3 P - q)^6} (4\tilde{T}_1 - 3\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 - 3\tilde{T}_5 + 4\tilde{T}_6 - 7\tilde{T}_7 - 7\tilde{T}_8) \\
& \left. - \frac{8x_3^2 m_P^4}{(x_3 P - q)^6} (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 - \tilde{V}_6) - \frac{6m_P^2 Q^2}{(x_3 P - q)^6} (-\tilde{T}_2 + \tilde{T}_3 + \tilde{T}_4 - \tilde{T}_5 - \tilde{T}_7 - \tilde{T}_8) \right\}, \quad (3.13)
\end{aligned}$$

$$\begin{aligned}
\mathcal{J}^{\text{QCD}} = & (e_d - e_u) \int_0^1 dx_3 \left\{ \frac{x_3 m_P}{(x_3 P - q)^2} \int_0^{1-x_3} dx_1 (V_3 - T_1)(x_i) + \frac{m_P}{(x_3 P - q)^2} (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3) - \frac{x_3 m_P^3}{(x_3 P - q)^4} T_1^{M(d)} \right. \\
& + \frac{m_P Q^2}{(x_3 P - q)^4} (3\tilde{T}_1 - 3\tilde{T}_3 - 3\tilde{T}_7 - \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3) + \frac{x_3 m_P^3}{(x_3 P - q)^4} (-\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_5 - \tilde{T}_6 + 2\tilde{T}_7 + 2\tilde{T}_8) \\
& + \frac{2x_3 m_P^3}{(x_3 P - q)^4} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6) + \frac{x_3^2 m_P^3}{(x_3 P - q)^4} (\tilde{T}_1 - \tilde{T}_2 + \tilde{T}_3 - \tilde{T}_5 - \tilde{T}_7 - 2\tilde{T}_8 - \tilde{V}_3 + \tilde{V}_4) \\
& + \frac{2x_3^3 m_P^5}{(x_3 P - q)^6} (\tilde{T}_2 - \tilde{T}_3 - \tilde{T}_4 + \tilde{T}_5 + \tilde{T}_7 + \tilde{T}_8) \\
& + \frac{x_3 m_P^3 Q^2}{(x_3 P - q)^6} (-8\tilde{T}_1 + 2\tilde{T}_2 + 6\tilde{T}_3 + 6\tilde{T}_4 + 2\tilde{T}_5 - 8\tilde{T}_6 + 10\tilde{T}_7 + 10\tilde{T}_8) \\
& \left. + \frac{8x_3 m_P^3 Q^2}{(x_3 P - q)^6} (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5 + \tilde{V}_6) \right\}. \quad (3.14)
\end{aligned}$$

The Borel transformation and the continuum subtraction are performed by using the following substitution rules:

$$\begin{aligned}
\int dx \frac{\varrho(x)}{(q - xP)^2} &= - \int_0^1 dx \frac{\varrho(x)}{x (s - P^2)} \rightarrow - \int_{x_0}^1 dx \frac{\varrho(x)}{x} \exp\left(-\frac{\bar{x}Q^2}{xM_B^2} - \frac{\bar{x}m_\Delta^2}{M_B^2}\right), \\
\int dx \frac{\varrho(x)}{(q - xP)^4} &= \int_0^1 dx \frac{\varrho(x)}{x^2 (s - P^2)^2} \rightarrow \frac{1}{M_B^2} \int_{x_0}^1 dx \frac{\varrho(x)}{x^2} \exp\left(-\frac{\bar{x}Q^2}{xM_B^2} - \frac{\bar{x}m_\Delta^2}{M_B^2}\right) + \frac{\varrho(x_0)e^{-s_0/M_B^2}}{Q^2 + x_0^2 m_\Delta^2}, \\
\int dx \frac{\varrho(x)}{(q - xP)^6} &= - \int_0^1 dx \frac{\varrho(x)}{x^3 (s - P^2)^3} \rightarrow - \frac{1}{2M_B^4} \int_{x_0}^1 dx \frac{\varrho(x)}{x^3} \exp\left(-\frac{\bar{x}Q^2}{xM_B^2} - \frac{\bar{x}m_\Delta^2}{M_B^2}\right) - \frac{1}{2} \frac{\varrho(x_0)e^{-s_0/M_B^2}}{x_0(Q^2 + x_0^2 m_\Delta^2)M_B^2} \\
&+ \frac{1}{2} \frac{x_0^2}{Q^2 + x_0^2 m_\Delta^2} \left[\frac{d}{dx_0} \frac{\varrho(x_0)}{x_0(Q^2 + x_0^2 m_\Delta^2)} \right] e^{-s_0/M_B^2} \quad (3.15)
\end{aligned}$$

where M_B is the Borel parameter, $s = \frac{1-x}{x} Q^2 + (1-x)m_\Delta^2$ and x_0 is the solution of the corresponding quadratic equation for $s = s_0$:

$$\begin{aligned}
x_0 = & \left[\sqrt{(Q^2 + s_0 - m_\Delta^2)^2 + 4m_\Delta^2 Q^2} \right. \\
& \left. - (Q^2 + s_0 - m_\Delta^2) \right] / (2m_\Delta^2). \quad (3.16)
\end{aligned}$$

The contributions $\sim e^{-s_0/M_B^2}$ in Eq. (3.15) correspond to the ‘‘surface terms’’ arising from successive partial integrations to reduce the power in the denominators $(q - xP)^{2N} = (s - P^2)^{2N} (-x)^{2N}$ with $N > 1$ to the usual dispersion representation with the denominator $\sim (s - P^2)$. Without continuum subtraction, i.e. in the limit $s_0 \rightarrow \infty$ these terms vanish.

In addition, in the hadronic representation for the same correlation functions (2.27) one has to make the substitution

$$\frac{1}{m_\Delta^2 - P^2} \rightarrow e^{-m_\Delta^2/M_B^2}. \quad (3.17)$$

IV. LIGHT-CONE SUM RULES

Our general strategy is as follows. At the first step, we analyze the two light-cone sum rules that are obtained from the combinations of the correlation functions that are free from contributions of negative parity spin-1/2 resonances, cf. (2.29):

$$\mathbb{B}[\mathcal{G}^{\text{QCD}} - \mathcal{J}^{\text{QCD}} + Q^2 \mathcal{B}^{\text{QCD}}](M_B^2) = -\frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2} Q^2 G_2(Q^2), \quad (4.1)$$

$$\mathbb{B}[\mathcal{I}^{\text{QCD}} + Q^2 \mathcal{H}^{\text{QCD}} - Q^2 \mathcal{A}^{\text{QCD}}](M_B^2) = -\frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2} Q^2 [2G_1(Q^2) + (m_\Delta - m_P)G_2(Q^2)].$$

Here $\mathbb{B}[\dots](M_B^2)$ stands for the Borel transform with respect to P^2 . From these sum rules we extract G_1 and G_2 form factors and also $G_M - G_E \sim G_1$. At the second step, we determine the form factor G_3 from the leading-twist sum rule

$$\mathbb{B}[\mathcal{B}^{\text{QCD}}](M_B^2) = -\frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2 M_B^2} \frac{1}{3m_\Delta^2} [-2m_P G_1 + [m_\Delta(m_\Delta - m_P) - Q^2]G_2 + 2Q^2 G_3]. \quad (4.2)$$

The rationale for choosing this structure is that in the sum rule for \mathcal{A} the contribution of interest is multiplied by a small factor $m_\Delta - m_P$, cf. (2.27). Finally, we rewrite our results in terms of G_M , G_E and G_C .

A. Asymptotic expansion

As an example, consider the contribution of the leading-twist nucleon distribution amplitudes to the LCSR for $G_2(Q^2)$, the first equation in (4.1). Putting everything together, one obtains to this accuracy

$$\begin{aligned} \mathcal{G}^{\text{QCD}} - \mathcal{J}^{\text{QCD}} + Q^2 \mathcal{B}^{\text{QCD}} &= (e_d - e_u)Q^2 \int_0^1 dx_3 \left\{ -\frac{4m_P}{(x_3 P - q)^4} \tilde{T}_1 + \frac{4x_3 m_P}{(x_3 P - q)^4} (\tilde{T}_1 - \tilde{V}_1) + \frac{x_3 \bar{x}_3 m_P^3}{(x_3 P - q)^6} (\tilde{T}_1 - \tilde{V}_1) \right\} \\ &+ \mathcal{O}(\text{twist-4}). \end{aligned} \quad (4.3)$$

The main effect of the continuum subtraction and/or Borel transformation is that the integration over the momentum fraction x_3 gets restricted to a narrow interval of $1 > x_3 > x_0$ with $1 - x_0 = s_0/Q^2 \ll 1$. As the result, power counting of $1 - x_3$ factors in the integrand translates into the power counting in $1/Q^2$. The behavior of the nucleon distribution amplitudes close to the end point $x_3 \rightarrow 1$ is governed by conformal symmetry [96]. To the next-to-leading order accuracy in the expansion over the conformal spin one obtains [17,87]

$$\begin{aligned} V_1(x_1, x_2, x_3) &= 120f_N x_1 x_2 x_3 [1 + \frac{7}{2}(1 - 3V_1^d)(1 - 3x_3) + \dots], \\ T_1(x_1, x_2, x_3) &= 120f_N x_1 x_2 x_3 [1 + \frac{7}{4}(3A_1^u + 3V_1^d - 1)(1 - 3x_3) + \dots]. \end{aligned} \quad (4.4)$$

Here A_1^u and V_1^d are scale-dependent parameters that characterize the deviation of the distribution amplitude from its asymptotic form at $Q^2 \rightarrow \infty$. For the asymptotic distribution amplitude $V_1^d = 1/3$ and $A_1^u = 0$; the Chernyak-Zhitnitsky (CZ) model [17] corresponds to $V_1^d = 0.23$ and $A_1^u = 0.38$ at a low scale of a few hundred MeV. A simple calculation shows that in the endpoint region $\tilde{V}_1 \sim \tilde{T}_1 \sim (1 - x_3)^4$ and $\tilde{\tilde{V}}_1 \sim \tilde{\tilde{T}}_1 \sim (1 - x_3)^5$.

After some algebra we obtain the contribution of leading-twist nucleon distribution amplitudes to the light-cone sum rule (4.1) for $G_2(Q^2)$ in the $Q^2 \rightarrow \infty$ limit:

$$\begin{aligned} \frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2} G_2^{\text{tw-3}}(Q^2) &= \frac{4(e_u - e_d)m_P}{Q^4} \int_0^{s_0} ds e^{-s/M_B^2} \int_0^{s/Q^2} dx_1 V_1(x_1, s/Q^2 - x_1, 1 - s/Q^2) \\ &= \frac{80f_N m_P}{Q^{10}} [1 + 7(3V_1^d - 1)] \int_0^{s_0} ds s^3 e^{-s/M_B^2} + \mathcal{O}\left(\frac{1}{Q^{12}}\right). \end{aligned} \quad (4.5)$$

The term in $\sim(3V_1^d - 1)$ corresponds to the contribution of subleading conformal spin, cf. (4.4). For the CZ model this correction is very large, $[1 + 7(3V_1^d - 1)]_{\text{CZ}} \simeq -1.17$. It turns out, however, that the contribution of the leading-twist distribution amplitudes is only subleading for large Q^2 , and the true large- Q^2 asymptotics of the sum rules is determined by the contribution of higher-twist amplitudes, in particular, those corresponding to different helicity structures (compared to leading twist). Using full expres-

sions, i.e. taking into account the terms $\sim V_2 \dots V_6$ and $\sim T_2 \dots T_6$, we obtain

$$\begin{aligned} \frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2} G_2(Q^2) &= \frac{8m_P}{Q^8} [5f_N + 3\lambda_1] \int_0^{s_0} ds s^2 e^{-s/M_B^2} \\ &+ \mathcal{O}\left(\frac{1}{Q^{10}}\right), \end{aligned} \quad (4.6)$$

$$\begin{aligned}
& \frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2} [2G_1(Q^2) + (m_\Delta - m_p)G_2(Q^2)] \\
&= \frac{8}{Q^8} \left\{ 10f_N \int_0^{s_0} ds s^3 e^{-s/M_B^2} - m_p^2 \left[\frac{89}{3} f_N + 7\lambda_1 \right] \right. \\
&\quad \left. \times \int_0^{s_0} ds s^2 e^{-s/M_B^2} \right\} + \mathcal{O}\left(\frac{1}{Q^{10}}\right), \tag{4.7}
\end{aligned}$$

$$\begin{aligned}
& \frac{\lambda_\Delta e^{-m_\Delta^2/M_B^2}}{(2\pi)^2} [2G_3(Q^2) - G_2(Q^2)] \\
&= \frac{8m_\Delta^2 m_p}{Q^{10}} \left\{ [75f_N + 9\lambda_1] \int_0^{s_0} ds s^2 e^{-s/M_B^2} \right. \\
&\quad \left. + m_p^2 \left[34f_N + \frac{24}{5}\lambda_1 \right] \int_0^{s_0} ds s e^{-s/M_B^2} \right\} + \mathcal{O}\left(\frac{1}{Q^{12}}\right), \tag{4.8}
\end{aligned}$$

from the three sum rules in Eqs. (4.1) and (4.2), respectively. For simplicity, only the contributions of asymptotic distribution amplitudes are shown, see Appendix B.

To avoid misunderstanding, note that these expressions correspond to the soft contribution to the form factors that does not involve any hard gluon exchanges and correspond, loosely speaking, to the first term in the expansion in Eq. (1.1). In order to observe the true asymptotic behavior one has to calculate radiative corrections to the sum rules, cf. [56]. In this way, the hard-gluon-exchange contribution considered in [63] appears to be a part of the two-loop $\mathcal{O}(\alpha_s^2)$ correction. The corresponding calculation goes beyond the scope of the present work. In spite of being enhanced asymptotically by powers of the momentum transfer, the radiative corrections are accompanied by increasing powers of the (small) QCD coupling at a scale at least that of the Borel parameter, and we expect that for moderate momentum transfers in the range $1 < Q^2 < 10 \text{ GeV}^2$ their contribution is numerically less important.

We conclude that the leading-order LCSR calculation predicts a universal $1/Q^8$ falloff of soft contributions to all the three form factors G_1 , G_2 , G_3 , and to this accuracy $G_2 = 2G_3$. This translates to the asymptotic behavior of the soft terms $G_M \sim 1/Q^6$, $G_M \sim 1/Q^6$ and $G_Q \sim 1/Q^8$, for the magnetic, electric and quadrupole form factors defined in Eq. (2.4), respectively. In agreement with the common wisdom, soft contributions in the light-cone sum rules arise from the integration regions where the quark interacting with the virtual photon carries almost all hadron momentum, alias the corresponding momentum fraction $x \rightarrow 1$. To illustrate this feature, we have plotted in Fig. 3 the average value $\langle x \rangle$ of the momentum fraction x_3 in the integrals in the sum rules (4.1) and (4.2). The main effect here is that the integration region in the momentum fraction gets restricted to a narrow interval $x_0 < x < 1$ where x_0 is given by Eq. (3.16). For asymptotically large Q^2 one finds $x_0 \simeq 1 - s_0/Q^2$ and the integration region shrinks to

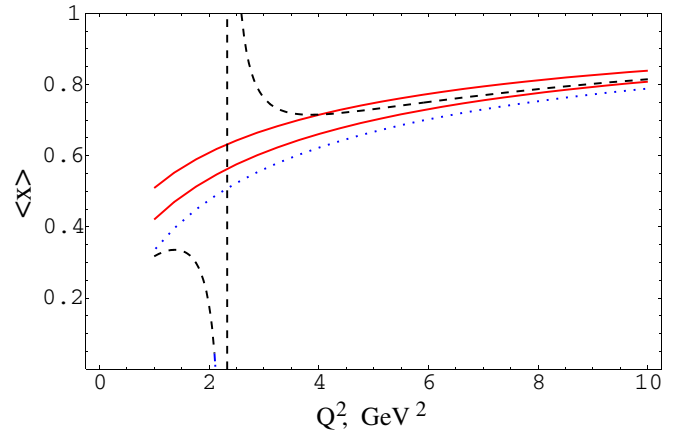


FIG. 3 (color online). The average value $\langle x \rangle$ of the momentum fraction x_3 in the sum rules (4.1) and (4.2) as a function of the momentum transfer Q^2 . The lower and the upper solid curves correspond to the first and the second sum rule in (4.1), respectively. The dashed curve corresponds to (4.2). The value of the threshold x_0 (3.16) is plotted by dots for comparison.

the endpoint. For realistic values of Q^2 in the range $1\text{--}10 \text{ GeV}^2$ the average $\langle x \rangle$ grows slowly and one finds very similar values for all the three sum rules in question. [The irregular behavior that is seen for the sum rule (4.2) around $Q^2 \sim 2 \text{ GeV}^2$ is due to accidental vanishing of the denominator involved in taking the average.]

B. Numerical analysis

The numerical results presented below are obtained using two models of the nucleon distribution amplitudes [87]. The first model corresponds to asymptotic distribution amplitudes of all twists. The corresponding expressions are collected in Appendix B. The second model corresponds to taking into account the corrections to the asymptotic distribution amplitudes that are due to the next-to-leading conformal spin (“ P -waves”) with parameters estimated using QCD sum rules. For the leading twist distribution amplitude this choice corresponds to a simplified version of the Chernyak-Zhitnitsky (CZ) model: we have truncated the original CZ expressions [17] leaving out contributions of the next-to-next-to-leading conformal spin operators (“ D -waves”) in order to simplify the calculation of nucleon mass corrections, see [87] for details. The explicit expressions of the distribution amplitudes including P -wave corrections are rather cumbersome, they can be found in [87,97].

In addition, for the numerical evaluation of the sum rules we have to specify the values of the Δ -coupling λ_Δ , the continuum threshold s_0 and the Borel parameter M_B^2 . The usual strategy is to determine all of them from the simplest QCD sum rule involving the Δ resonance [16], see Appendix C. We obtain from this sum rule $s_0 \simeq 3 \text{ GeV}^2$ and $\lambda_\Delta \simeq 2.0 \text{ GeV}^3$. Both numbers are somewhat lower than the ones obtained in [16] because we use the experi-

mental input value of the mass of the Δ -resonance $m_\Delta = 1.23$ GeV instead of trying to find it from the sum rule.

A suitable range of Borel parameters for the LCSR can be obtained as follows. On the one hand, M_B^2 has to be small enough in order to guarantee sufficient suppression of higher mass resonances and the continuum in the hadronic representation for the correlation function. This is the same criterium that is applied to the Belyaev-Ioffe sum rule (C1) for the coupling λ_Δ , hence we would want to take M_B^2 as close as possible to 1.5 GeV² which is the minimum value at which the stability in (C1) sets in. On the other hand, the Borel parameter in the LCSRs has to be large

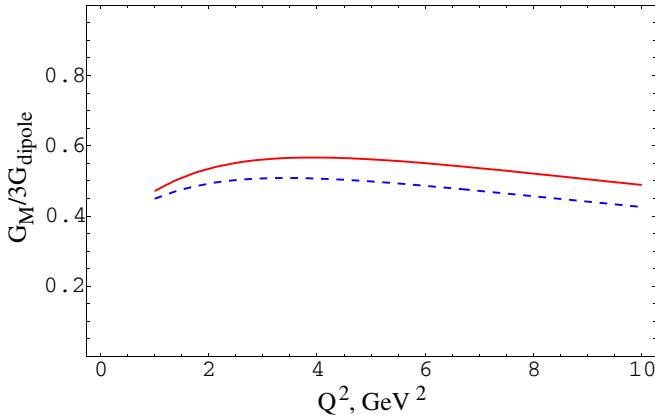


FIG. 4 (color online). The Borel parameter dependence of the ratio $G_M/3G_{\text{dipole}}$ where $G_{\text{dipole}} = 1/(1 + Q^2/0.71)^2$. The magnetic form factor G_M is defined in (2.4). The solid and the dashed curves correspond to the calculation using $M_B^2 = 2.5$ GeV² and $M_B^2 = 1.5$ GeV², respectively. In both cases the asymptotic nucleon distribution amplitudes of the leading and higher twists are used.

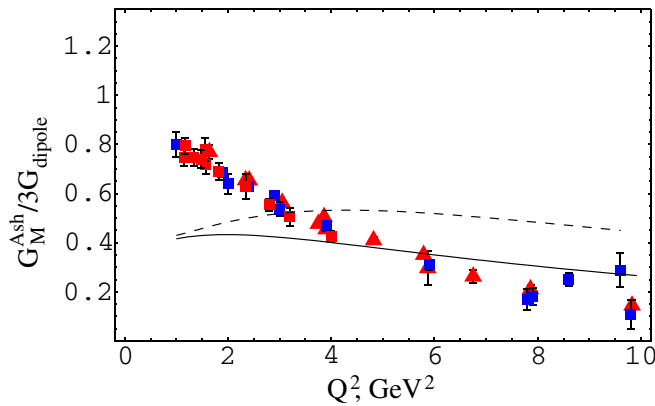


FIG. 5 (color online). The ratio $G_M^{\text{Ash}}/3G_{\text{dipole}}$ where $G_{\text{dipole}} = 1/(1 + Q^2/0.71)^2$. The form factor G_M^{Ash} is defined in (2.5). The solid and the dashed curves correspond to the calculation using the asymptotic and the QCD sum rules motivated nucleon distribution amplitudes of the leading and higher twists, respectively. The data points are from [82] (blue squares), [100] (red triangles), and [65,101–104] (red squares).

enough to guarantee convergence of the twist expansion in the QCD calculation. Note that for a fixed value of the momentum fraction x the light-cone expansion of the relevant correlation functions goes generically in powers of $1/(q - xP)^2$ which translates to the expansion in powers of $1/(xM_B^2)$ after the Borel transformation. The true expansion parameter is therefore of order $\sim 1/(\langle x \rangle M_B^2)$ where $\langle x \rangle$ is the average value of the momentum fraction in the corresponding integrals, rather than $1/M_B^2$ itself. In the range $1 < Q^2 < 10$ GeV² we find $0.4 < \langle x \rangle < 0.8$ (see Fig. 3) so that preferred values of M_B^2 in LCSRs appear from this side to be a factor 1.2–2.5 larger compared to the two-point sum rule Borel parameter M^2 . Note that for fixed M^2 there is effectively a bias towards using larger values of M_B^2 with decreasing Q^2 . All in all, it appears that the interval $1.5 < M_B^2 < 2.5$ GeV² presents a reasonable choice. The stability of the LCSRs in this range turns out

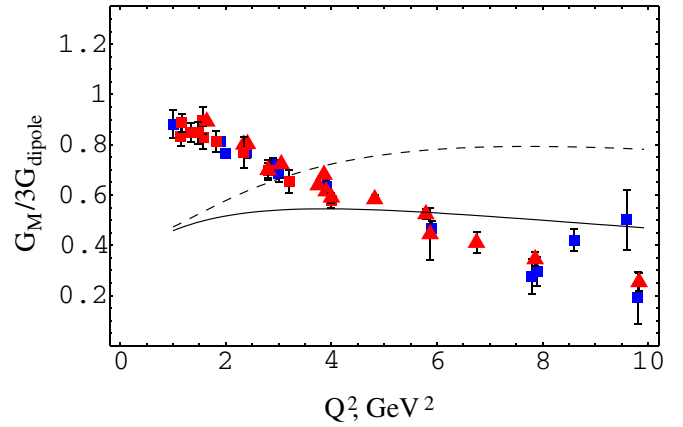


FIG. 6 (color online). The ratio $G_M/3G_{\text{dipole}}$ where $G_{\text{dipole}} = 1/(1 + Q^2/0.71)^2$. The magnetic form factor G_M is defined in (2.4). The identification of the curves and data points is the same as in Fig. 5.

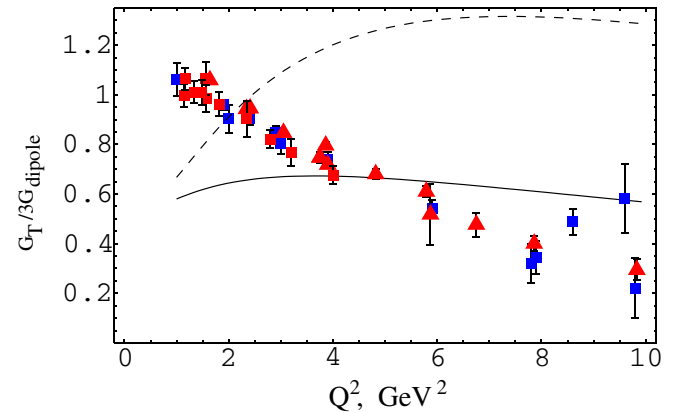


FIG. 7 (color online). The ratio $G_T/3G_{\text{dipole}}$ where $G_{\text{dipole}} = 1/(1 + Q^2/0.71)^2$. The form factor G_T is defined in (2.5). The identification of the curves and data points is the same as in Fig. 5.

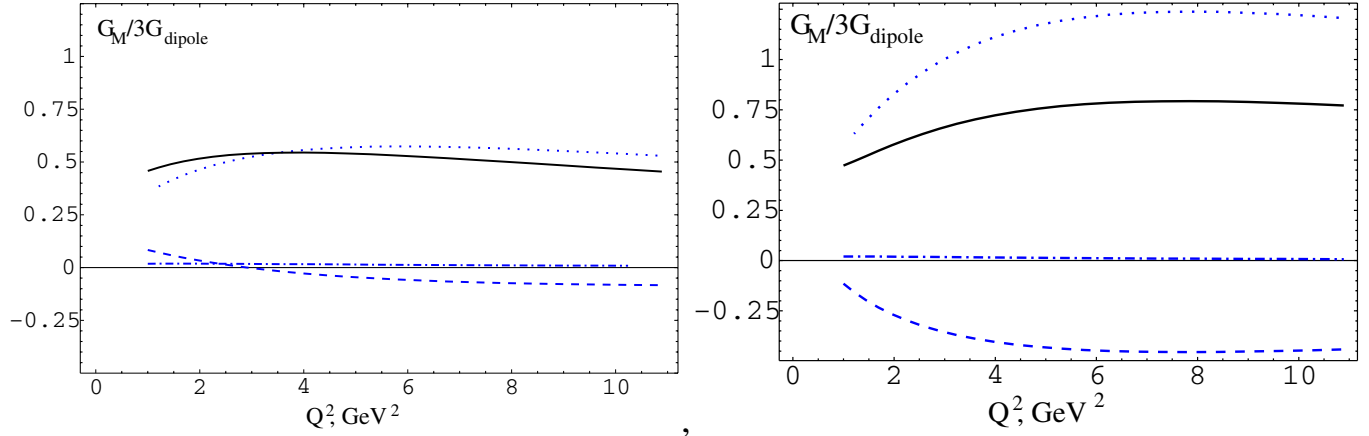


FIG. 8 (color online). Separate contributions to the light-cone sum rule results (solid curves) for the ratio $G_M/3G_{\text{dipole}}$ where $G_{\text{dipole}} = 1/(1 + Q^2/0.71)^2$. The terms in f_N , λ_1 and λ_2 are shown by the dashed, dotted and dash-dotted curves, respectively. The results on the left panel correspond to the asymptotic distribution amplitudes and the ones on the right panel are obtained including a CZ-like model, see text.

to be better than 10%–15%, see Fig. 4 for an example. In what follows we use the fixed value of the Borel parameter $M_B^2 = 2 \text{ GeV}^2$ in the center of this fiducial interval.

The results for the magnetic transition form factor are shown in Figs. 5–7 using three different ways to present the data (in terms of G_M^{Ash} , G_M , and G_T) accepted in the experimental papers. The difference between G_M^{Ash} and G_M is in a kinematical factor only, while G_T includes in addition the contribution of the electric form factor G_E which is negligible. We observe that the calculation with asymptotic distribution amplitudes (solid curves) is much closer to the data so that large deviations from the asymptotic shape of distribution amplitudes suggested by QCD sum rule calculations are disfavored. A similar conclusion was reached in Refs. [59,60] from the LCSR analysis of the electromagnetic nucleon form factors.

It turns out that the sum rules for G_M are dominated by contributions of subleading twist-4. To illustrate this issue, we have plotted in Fig. 8 separate contributions to the sum rule for the ratio $G_M/(3G_{\text{dipole}})$ that come from the non-perturbative matrix element of twist three $\sim f_N$ (dashed curves) and the two existing matrix elements of twist four: terms in λ_1 (dotted curves) and λ_2 (dash-dotted curves), cf. Appendix B. The full result is given by the sum of all three contributions and is shown by the solid curves. We see that contributions $\sim \lambda_2$ are numerically very small, the contributions $\sim \lambda_1$ are the dominant ones and remain roughly constant in the considered Q^2 range, whereas the contributions $\sim f_N$ have a stronger Q^2 dependence and enter with an opposite sign. QCD sum rule motivated corrections to the asymptotic distribution amplitudes generally tend to increase both leading- and higher-twist contributions, so that the cancellation becomes more pronounced. Note that the uncertainties in nonperturbative parameters f_N , λ_1 , λ_2 presently are at the level of 20%–30%. Larger leading twist contributions $\sim f_N$ would yield a steeper falloff of

the form factor which is favored by the data. The same observation was made in [59] for the case of the electromagnetic form factors of the nucleon. Note that the dominant contributions $\sim \lambda_1$ correspond to the operators that include a minus component of one of the valence quark fields. They do not have a simple partonic interpretation in terms of quark parton amplitudes at small transverse separation but rather involve the orbital angular momentum, see [85,86] for the relevant formalism and discussion.

The results for the electric form factor G_E and quadrupole form factor G_C are shown in Figs. 9 and 10, respectively. In both cases we plot the experimentally measured quantities R_{EM} and R_{SM} (2.5) that are related to the form factor ratios, normalized to the magnetic form factor G_M .

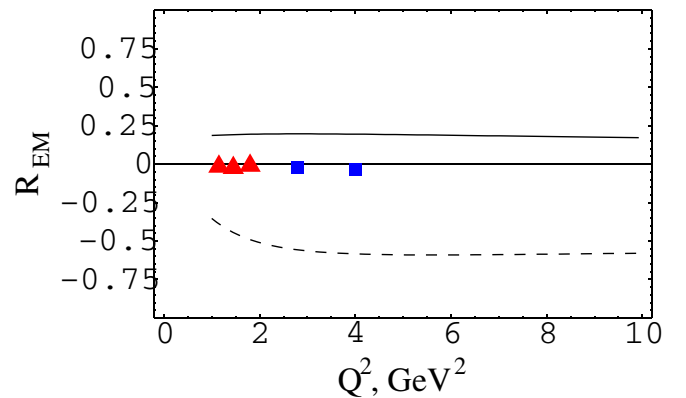


FIG. 9 (color online). The ratio $R_{\text{EM}}(Q^2) = -G_E/G_M$ of the electric and the magnetic form factors, cf. (2.5). The solid and the dashed curves correspond to the calculation using the asymptotic and the QCD sum rules motivated nucleon distribution amplitudes of the leading and higher twists, respectively. The data points are from [105] (red triangles) and [65] (blue squares).

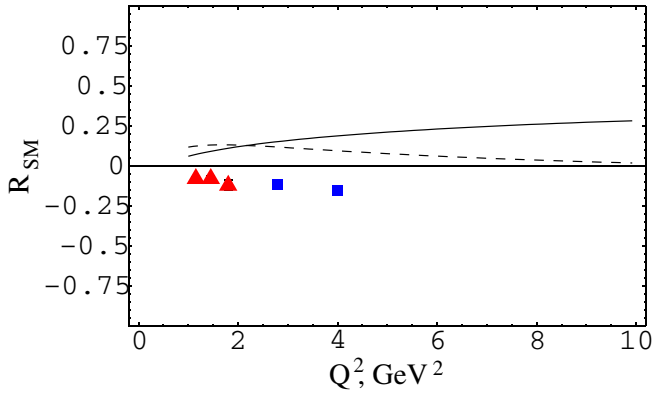


FIG. 10 (color online). The ratio $R_{SM} \sim -G_C/G_M$ of the quadrupole and the magnetic form factors, cf. (2.5). The solid and the dashed curves correspond to the calculation using the asymptotic and the QCD sum rules motivated nucleon distribution amplitudes of the leading and higher twists, respectively. The data points are from [105] (red triangles) and [65] (blue squares).

Here, again, the asymptotic distribution amplitudes tend to give a better description.

In the future, one should try to constrain the parameters in the distribution amplitudes by making a combined fit to the LCSR for the electromagnetic and weak form factors of the nucleon, including Δ -resonance production etc. In order to make this program fully quantitative one first has to calculate radiative corrections to the LCSRs, similar as this has been done for the pion form factor [56] and B -meson decays [98]. The corresponding task goes beyond the scope of this paper.

V. SUMMARY AND CONCLUSIONS

In this paper, we incorporated light-cone QCD sum rules approach to calculate the purely nonperturbative soft contribution to the $\gamma^*p \rightarrow \Delta^+$ transition. The soft contribution corresponds to the so-called Feynman mechanism of the large momentum transfer, which is suppressed asymptotically by power(s) of $1/Q^2$ compared to the perturbative contribution of hard rescattering. We argued from the very beginning that, due to two hard gluon exchanges required in the pQCD contribution, the latter is numerically suppressed by a factor of $1/100$ compared to the soft term. Indeed, our results for the dominant magnetic form factor $G_M(Q^2)$ are rather close to the experimental data in the region above $Q^2 \sim 2 \text{ GeV}^2$. This confirms the general wisdom that the soft (endpoint) contribution is dominant at the experimentally accessible momentum transfers. Moreover, the inspection shows that the soft contribution is dominated by quark configurations involving a minus light-cone projections of one of the quark field operators, which can be reinterpreted as importance of the orbital angular momentum (cf. [85,86]).

In the region of low $Q^2 < 2 \text{ GeV}^2$ our results for the magnetic form factor appear to be factor two below the

data. One reason for that are most likely the so-called ‘‘bilocal power corrections’’ that correspond to long-distance propagation in the Q^2 channel. Such terms were studied in case of the pion [99] and nucleon [37] form factors, and they provided a sizable enhancement of form factors at low momentum transfers as compared to the extrapolation of the large- Q^2 results. Another possibility is that the interpolating current for the Δ -particle is not good enough and couples strongly to the excited states. This can be checked by applying the light-cone QCD sum rules to the correlation function $\langle \Delta | J \eta_N | 0 \rangle$ of the electromagnetic current and a local current η_N with nucleon quantum numbers, while the Δ is left explicitly in the final state $\langle \Delta |$. To pursue such a program, one needs a systematic analysis of the distribution amplitudes of the Δ as the first step, the task which is interesting in its own.

Our calculations are in agreement with the experimental observation that the ratios $E2/M1$ and $C2/M1$ are small. The charge form factor G_E appears to be very sensitive to the shape of the nucleon distribution amplitudes so that the experimental data can easily be fitted by assuming moderate corrections to the asymptotic form, of the same sign but much smaller as compared to the CZ model. However, both the $G_E(Q^2)$ and $G_C(Q^2)$ form factors in our calculations appear as small differences between expressions dominated by the much larger form factor $G_M(Q^2)$, a situation similar to that encountered in the approach [78] based on the analysis of the 3-point correlator. For this reason, at this stage we restrict ourselves to a conservative statement that $G_E(Q^2)$ and $G_C(Q^2)$ are small compared to $G_M(Q^2)$ without insisting on a specific curve (or even sign) for them.

The inclusion of QCD radiative corrections to the sum rules presents another important issue, and is needed to make the theoretical studies of the $\gamma^*p \rightarrow \Delta^+$ transition fully quantitative. Such corrections contain both purely soft contributions and also hard gluon exchanges, with nontrivial interplay and connections between these two types of dynamics. For example, for the pion form factor it was found [56] that there exists a partial cancellation between soft contributions and hard contributions of higher twist. A big advantage of the light-cone sum rule technique is that it is free from double counting: soft and hard contributions can be separated rigorously. In the present case, such a separation becomes necessary starting at two-loop $O(\alpha_s^2)$ corrections to the sum rules.

To summarize, we believe that the light-cone sum rule approach currently offers the best compromise between theoretical rigor and the applicability to present and planned experiments involving elastic and transition form factors for baryons. This approach is rigorous as far as the separation of hard and soft dynamics is concerned, and provides one with a useful tool for the study of the transition region between hard perturbative and soft nonperturbative QCD dynamics. One goal of such studies is to determine nucleon distribution amplitudes from the data on

form factors, similar as parton distributions are extracted from the measured deep inelastic structure functions. Our work presents a step in this direction.

ACKNOWLEDGMENTS

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APPENDIX A: LORENTZ-INVARIANT DECOMPOSITION

The correlation function $T_{\mu\nu}(P, q)$ has to satisfy two conditions

$$\gamma^\mu T_{\mu\nu}(P, q) = 0, \quad q^\nu T_{\mu\nu}(P, q) = 0, \quad (\text{A1})$$

that follow from the property of the current η_μ and electromagnetic current conservation, respectively. In addition, $T_{\mu\nu}(P, q)$ has to be proportional to the nucleon spinor. This suggests to define

$$T_{\mu\nu}(P, q) = \sum_{k=1}^N T_k(P'^2, q^2) L_{\mu\nu}^{(k)}(P, q) \gamma_5 N(P) \quad (\text{A2})$$

where $T_k(P'^2, q^2)$ are scalar functions in front of the Lorentz structures $L_{\mu\nu}^{(k)}(P, q)$ that satisfy the conditions in Eq. (A1) themselves, i.e. $\gamma^\mu L_{\mu\nu}^{(k)}(P, q) N(P) = 0$ and $q^\nu L_{\mu\nu}^{(k)}(P, q) N(P) = 0$. The complete basis contains ten different Lorentz structures which can be chosen as

$$\begin{aligned} L_{\mu\nu}^{(1)} &= 2(q_\mu \gamma_\nu \not{\epsilon} - g_{\mu\nu} q^2) + \gamma_\mu (\gamma_\nu q^2 - q_\nu \not{\epsilon}), \\ L_{\mu\nu}^{(2)} &= 4(q_\mu P'_\nu - g_{\mu\nu} P' q) + \gamma_\mu (\gamma_\nu P' q - P'_\nu \not{\epsilon}), \\ L_{\mu\nu}^{(3)} &= P'_\mu (\gamma_\nu \not{\epsilon} - q_\nu) - \frac{1}{4} m_P (\gamma_\mu \not{\epsilon} \gamma_\nu - q_\nu \gamma_\mu) \\ &\quad + \frac{1}{4} \gamma_\mu (\gamma_\nu q^2 - q_\nu \not{\epsilon}) + \frac{1}{2} \gamma_\mu (\gamma_\nu P' q - P'_\nu \not{\epsilon}), \\ L_{\mu\nu}^{(4)} &= P' q P'_\mu \gamma_\nu \not{\epsilon} - P'_\mu P'_\nu q^2 + m_P (P' q P'_\mu \gamma_\nu - P'_\mu P'_\nu \not{\epsilon}) \\ &\quad + P'^2 (q_\mu P'_\nu - g_{\mu\nu} P' q), \\ L_{\mu\nu}^{(5)} &= q_\mu q_\nu - g_{\mu\nu} q^2 + \frac{1}{4} \gamma_\mu (\gamma_\nu q^2 - q_\nu \not{\epsilon}) \end{aligned} \quad (\text{A3})$$

and

$$\begin{aligned} L_{\mu\nu}^{(6)} &= 2(q_\mu \gamma_\nu - g_{\mu\nu} \not{\epsilon}) - (\gamma_\mu \not{\epsilon} \gamma_\nu - q_\nu \gamma_\mu), \\ L_{\mu\nu}^{(7)} &= 4(q_\mu P'_\nu \not{\epsilon} - g_{\mu\nu} P' q \not{\epsilon}) + 2P' q q_\nu \gamma_\mu - P' q \gamma_\mu \not{\epsilon} \gamma_\nu \\ &\quad - P'_\nu \gamma_\mu q^2, \\ L_{\mu\nu}^{(8)} &= P'_\mu (\gamma_\nu q^2 - q_\nu \not{\epsilon}) + P'_\mu m_P (\gamma_\nu \not{\epsilon} - q_\nu) \\ &\quad + \frac{1}{2} m_P (\gamma_\mu \gamma_\nu P' q - \gamma_\mu P'_\nu \not{\epsilon}) - \frac{1}{4} P'^2 (\gamma_\mu \not{\epsilon} \gamma_\nu - q_\nu \gamma_\mu) \\ &\quad + \frac{1}{2} (2P' q \gamma_\mu q_\nu - \gamma_\mu P' q \not{\epsilon} \gamma_\nu - P'_\nu q^2 \gamma_\mu), \\ L_{\mu\nu}^{(9)} &= q_\mu q_\nu \not{\epsilon} - g_{\mu\nu} q^2 \not{\epsilon} - \frac{1}{4} q^2 (\gamma_\mu \not{\epsilon} \gamma_\nu - \gamma_\mu q_\nu), \\ L_{\mu\nu}^{(10)} &= m_P P'_\mu (\gamma_\nu \not{\epsilon} - q_\nu) + P'_\mu (\gamma_\nu q^2 - q_\nu \not{\epsilon}) \\ &\quad + 2(P' q P'_\mu \gamma_\nu - P'_\mu P'_\nu \not{\epsilon}) - \frac{1}{4} P'^2 (\gamma_\mu \not{\epsilon} \gamma_\nu - q_\nu \gamma_\mu), \end{aligned} \quad (\text{A4})$$

where the two groups roughly correspond to contributions with even and odd number of gamma-matrices, respectively.

Taking the necessary light-cone projections we obtain for the amplitudes in (2.23):

$$\begin{aligned} \mathcal{A} &= Q^2 T_4, & \mathcal{B} &= -m_P T_4 - 2T_{10}, \\ m_P \mathcal{C} &= -Q^2 [m_P T_4 + 2T_7 - T_8 + 2T_{10}], \\ m_P \mathcal{D} &= -2T_2 + T_3 + (2P' q - m_P^2) T_4 + m_P T_8, \\ \mathcal{E} &= m_P (P' q) T_4 - Q^2 T_8 + (2P' q - Q^2) T_{10}, \\ \mathcal{F} &= T_3 + (P' q) T_4 + m_P T_8 + m_P T_{10}, \\ \mathcal{G} &= Q^2 T_7 + \frac{1}{2} Q^2 T_8, & \mathcal{H} &= -T_2 - \frac{1}{2} T_3 - \frac{1}{2} m_P T_8, \\ \mathcal{I} &= -Q^2 [3T_2 - \frac{1}{2} T_3 - (2P' q - m_P^2) T_4 - \frac{1}{2} m_P T_8], \\ \mathcal{J} &= -Q^2 [m_P T_4 + 3T_7 - \frac{1}{2} T_8 + 2T_{10}]. \end{aligned} \quad (\text{A5})$$

Note that four invariant amplitudes T_1, T_5, T_6, T_9 do not contribute to the next-to-leading twist accuracy. This implies that the invariant functions in (2.23) obey four relations which can be chosen as in Eqs. (2.25) and (2.26) in the text.

APPENDIX B: NUCLEON DISTRIBUTION AMPLITUDES

To make the paper self-contained, we collect here the necessary information on the nucleon distribution amplitudes that enter the sum rules. Our presentation follows Ref. [87].

A standard tool to study hadron distribution amplitudes is to use constraints on operator mixing and equation of motion relations that originate from the conformal symmetry of the QCD Lagrangian [96]. This approach suggests an expansion in contributions of increasing conformal spin. The leading-order contributions with the lowest conformal spin are usually referred to as asymptotic distribution amplitudes. To this accuracy there are only three nonperturbative parameters involved. One obtains [87] asymptotic twist-3 distribution amplitudes:

$$V_1(x_i) = 120x_1x_2x_3f_N, \quad T_1(x_i) = 120x_1x_2x_3f_N, \quad (\text{B1})$$

asymptotic twist-4 distribution amplitudes

$$\begin{aligned} V_2(x_i) &= 2x_1x_2[5(1+x_3)f_N + 6\lambda_1], \\ T_2(x_i) &= 4x_1x_2\lambda_2, \\ V_3(x_i) &= x_3[5(1+2x_1x_2-4x_3+3(x_1^2+x_2^2+x_3^2))f_N \\ &\quad - 6(1-x_3)\lambda_1], \\ T_3(x_i) &= x_3[5(1+2x_1x_2+2x_3-3(x_1^2+x_2^2+x_3^2))f_N \\ &\quad + (1-x_3)\lambda_2], \\ T_7(x_i) &= x_3[5(1+2x_1x_2+2x_3-3(x_1^2+x_2^2+x_3^2))f_N \\ &\quad - (1-x_3)\lambda_2], \end{aligned} \quad (\text{B2})$$

asymptotic twist-5 distribution amplitudes:

$$\begin{aligned} V_4(x_i) &= -\frac{1}{3}[28-65(x_1^2+x_2^2)-30x_1x_2-13x_3-15x_3^2]f_N \\ &\quad - [1+(x_1-x_2)^2-x_3^2]\lambda_1, \\ T_4(x_i) &= -\frac{1}{3}[28-65(x_1^2+x_2^2)+30x_1x_2-43x_3+15x_3^2]f_N \\ &\quad + \frac{1}{6}[1+x_1^2-6x_1x_2+x_2^2+2x_3-3x_3^2]\lambda_2, \\ T_8(x_i) &= -\frac{1}{3}[28-65(x_1^2+x_2^2)+30x_1x_2-43x_3+15x_3^2]f_N \\ &\quad - \frac{1}{6}[1+x_1^2-6x_1x_2+x_2^2+2x_3-3x_3^2]\lambda_2, \\ V_5(x_i) &= -\frac{2}{3}x_3(28-65x_3)f_N + 2x_3(1+x_3)\lambda_1, \\ T_5(x_i) &= \frac{2}{3}x_3(1+x_3)\lambda_2, \end{aligned} \quad (\text{B3})$$

asymptotic twist-6 distribution amplitudes:

$$\begin{aligned} V_6(x_i) &= \frac{1}{3}(5+3x_3)f_N - \frac{2}{3}(1-3x_3)\lambda_1, \\ T_6(x_i) &= \frac{1}{3}(8-6x_3)f_N, \end{aligned} \quad (\text{B4})$$

and, finally, the x^2 -corrections to the same accuracy:

$$\begin{aligned} V_1^{M(d)}(x_3) &= \frac{x_3^2}{24}[(1-x_3)((215-529x_3+427x_3^2-109x_3^3) \\ &\quad + 4\ln[x_3])f_N + 16(1-x_3)^3\lambda_1], \\ T_1^{M(d)}(x_3) &= \frac{x_3^2}{9}(1-x_3)^3[(76-39x_3)f_N - 6\lambda_1], \end{aligned} \quad (\text{B5})$$

where

$$\begin{aligned} \mathcal{V}_1^M(Pz): &= \int_0^1 dx_3 e^{-ix_3(P \cdot z)} V_1^{M(d)}(x_3), \\ \mathcal{T}_1^M(Pz): &= \int_0^1 dx_3 e^{-ix_3(P \cdot z)} T_1^{M(d)}(x_3), \end{aligned} \quad (\text{B6})$$

cf. (3.1) and (3.3).

The nonperturbative parameters f_N , λ_1 and λ_2 correspond to the vacuum-to-nucleon matrix elements of the three-quark local operators [87]. The existing estimates come from QCD sum rules. At the scale $\mu = 1$ GeV one gets

$$f_N = (5.0 \pm 0.3) \times 10^{-3} \text{ GeV}^2 \quad (\text{B7})$$

Calculations in this work are done also using a more sophisticated model for the distribution amplitudes, taking into account corrections that correspond to the contributions of operators with the next-to-leading conformal spin [96]. The corresponding expressions are given in [87] (see also [97]). They include five more parameters (the first two leading twist, and the rest twist-4) which we choose as

$$A_1^u = 0.38 \pm 0.15 \quad (\text{B8})$$

For a comparison, the asymptotic distribution amplitudes correspond in this more general parametrization to $A_1^u = 0$, $V_1^d = 1/3$, $f_1^d = 3/10$, $f_2^d = 4/15$ and $f_1^u = 1/10$, see [87,97] for details.

Note that there is a mismatch between the twist classification of the distribution amplitudes that implies counting powers of the large momentum p_+ , and the twist classification of the local operator matrix elements: e.g. the parameters of the twist-4 distribution amplitudes depend both on the leading twist-3 matrix element f_N and the twist-4 matrix elements λ_1, λ_2 . Such contributions of lower-twist matrix elements to higher-twist distribution amplitudes are well known and usually referred to as Wandzura-Wilczek contributions. In particular the distribution amplitudes of twist-5 and twist-6 are entirely of Wandzura-Wilczek type since there exist no genuine twist-5 and twist-6 operators to this order in the conformal expansion.

APPENDIX C: QCD SUM RULE FOR Δ -RESONANCE

The coupling λ_Δ (2.9) can be found from the Belyaev-Ioffe sum rule [16] for the correlation function of the two η_μ currents. The sum rule reads

$$\begin{aligned} \frac{1}{3} \lambda_\Delta^2 e^{-m_\Delta^2/M^2} &= \frac{1}{5} M^6 \left[1 - e^{-s_0/M^2} \left(1 + \frac{s_0}{M^2} + \frac{1}{2} \frac{s_0^2}{M^4} \right) \right] \\ &\quad - \frac{5b}{72} M^2 (1 - e^{-s_0/M^2}) + \frac{4a^2}{3} - \frac{7m_0^2 a^2}{9m_B^2}, \end{aligned} \quad (\text{C1})$$

where M^2 is the Borel parameter, s_0 the continuum threshold and

$$\begin{aligned} a &= -(2\pi)^2 \langle \bar{q}q \rangle = 0.55 \text{ GeV}^3, \\ b &= (2\pi)^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.47 \text{ GeV}^4, \quad m_0^2 = 0.8 \text{ GeV}^2. \end{aligned} \quad (\text{C2})$$

Using an experimental value $m_\Delta = 1232$ MeV we found that the sum rule (C1) is stable in a broad interval of Borel parameters $M^2 > 1.5$ GeV² for the choice of the continuum threshold $s_0 = 3.0 \pm 0.2$ GeV². The corresponding value of the coupling is equal to $\lambda_\Delta = 2.0 \pm 0.2$ GeV³.

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