Parametric Resonance Ionization Cooling of Muons

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1. Introduction

In the linear solenoid channel, studied here, a half integer resonance is induced such that the normal elliptical motion of particles in x-x' phase space becomes hyperbolic, with particles moving to smaller x and larger x' as they pass down the channel. Thin absorbers placed at the focal points of the channel then cool the angular divergence of the beam by the usual ionization cooling mechanism where each absorber is followed by RF cavities.

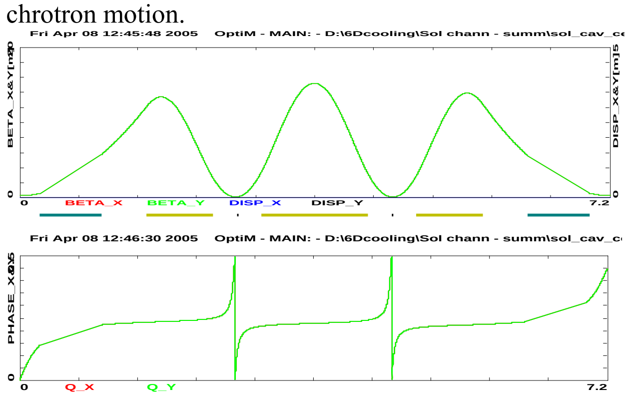
Chromatic aberration, the detuning effect considered here, is where the momentum-dependent betatron frequency causes off- momentum particles to be out of resonance with the focusing lattice. Choosing suitable synchrotron motion parameters, the resonance condition can be maintained. This paper reports the first simulation to test this prediction.

2. Solenoid Triplet Cell

An optimum lattice configuration for the cooling channel consists of alternating solenoid triplet cells. This provides strong focusing, so that the horizontal and vertical betatron phases advance by 3π across the cell and by π between the absorbers (indicated by markers). This configuration provides a periodic half-integer and integer resonant lattice.

The total length of the p = 286.8 MeV/c cell is 7.2 m with the Initial betas of 21 cm. In the transfer matrix formalism of OptiM the three “soft edge” solenoids have fields of B_0= -34.1, 32.4, and -34.1 kG with lengths L=80, 130, and 80 cm, respectively, and radius a=20 cm. The Larmor wave number is $k = eB_0/Pc$.

Figure 1 Beta functions and betatron phases for the solenoid triplet cell. Thin absorbers are placed at the two central focal points. 400 MHz RF cavities shown as blue bars replace the lost energy and provide synchrotron motion.



3. Thin Absorber and RF

Ionization cooling due to energy loss (-Δp) in a thin absorber followed by immediate re-acceleration (Δp) can be described as:

$$\Delta\theta_\perp = -\theta_\perp \frac{\Delta p}{p}$$

The corresponding canonical transfer matrix can be written as

$$M_{abs} = K \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\frac{\Delta p}{p} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\frac{\Delta p}{p} \end{bmatrix} K^{-1}$$

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k/2 & 0 \\ 0 & 0 & 1 & 0 \\ k/2 & 0 & 0 & 1 \end{bmatrix} \quad \hat{\mathbf{x}} \equiv \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} \quad \mathbf{x} \equiv \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}$$

where $k = eB_0/Pc$ and $\hat{\mathbf{x}} = K\mathbf{x}$.

In the simulation Δp/p of 0.05 was used to mock-up the effect of a thin (4 cm) Be plate followed by re-acceleration (Δp = 14 MeV/c).

To induce synchrotron oscillation into the channel dynamics two 400 MHz RF cavities at zero crossing are added symmetrically to each cell. The cavity gradient of 17.3 MeV/m was chosen to provide appropriate height of a stationary bucket with a synchrotron phase advance of about 2π/8 per cell

Table 1: Initial parameters for simulation studies.

5. Simulations

normalized emittance: $\varepsilon_x/\varepsilon_y$	mm	30
longitudinal emittance: ε_l ($\varepsilon_l = \sigma_{\Delta p}\,\sigma_z/m_\mu c$)	mm	0.8
momentum spread: $\sigma_{\Delta p/p}$		0.01
bunch length: σ_z	mm	30
momentum	MeV/c	286.8

The simulation involves tracking 5000 particles defined by a 6D Gaussian with the parameters from the Table 1, see Fig. 2, through a PIC channel consisting of 8 periodic cells (two absorbers per cell), or one synchrotron oscillation period. A sequence of transverse (*x, x'*) and longitudinal (ș, dp/p × 1000) phase space ‘snapshots’ starting with the initial distribution and followed by the ‘snapshots’ taken after passing through two cells is collected is shown in Fig. 3 and 4. In Fig. 3 the beam energy lost in the absorber is simply replaced as described in the previous section. In the

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simulations shown in Fig. 4, RF cavities are used to generate synchrotron motion as well as to replace the energy lost in the absorber.

6. Conclusions

Comparing Fig. 3 and 4 of the betatron and synchrotron phase space evolution through 8 solenoid triplet cells, it is easy to see that the synchrotron motion makes a significant difference. In Figure 3 the tails of the betatron distribution are larger and particles are lost. In Fig. 4, the final $x-x'$ plot shows the promise of PIC: the x distribution has narrowed from the initial one, and the x' distribution shows the effects of ionization cooling. However, the evaluation of the actual cooling of the beam awaits a proper simulation using all the power of the most realistic codes.

The simulation model used here has provided a first confirmation that our understanding of PIC is correct and that compensation for chromatic aberrations can be made. We will soon include multiple scattering and energy straggling effects using G4Beamline [2].

Figure 2 Initial phasespace of $x-x'$ (left) and $s-\Delta p/p$ (right)

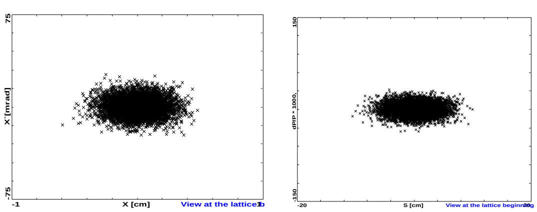
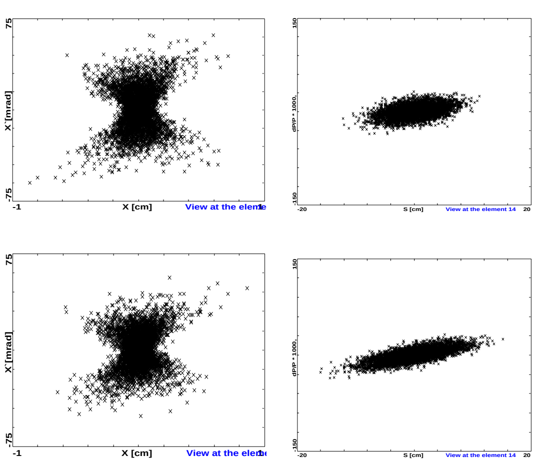


Figure 3 'Snapshots' of $x-x'$ (left) and $s-\Delta p/p$ (right) phase space without synchrotron motion to correct chromatic aberration. Each snapshot corresponds to passage through two of the cells shown in Fig. 1.



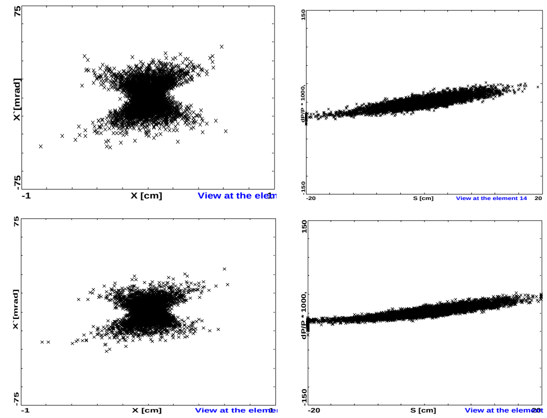
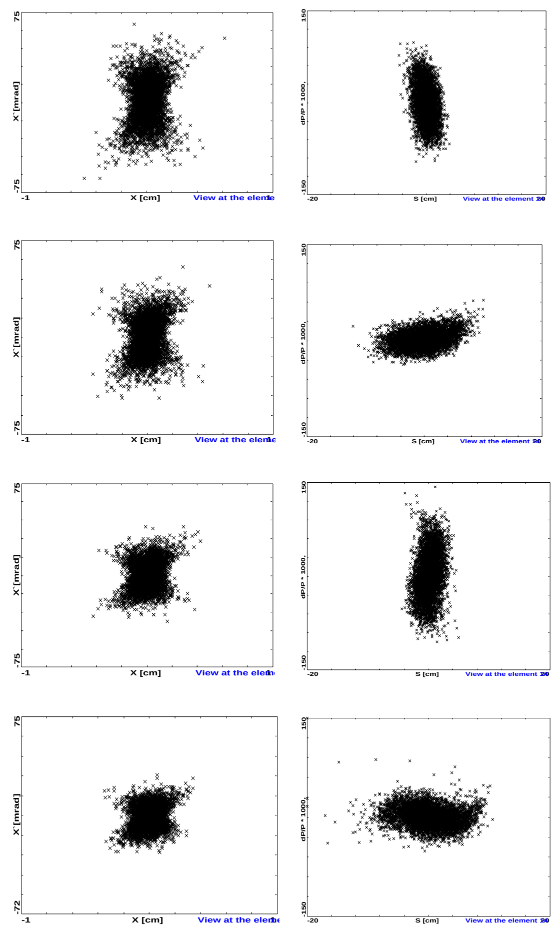


Figure 4 Phase space 'snapshots' with synchrotron motion compensation of chromatic aberration. The transverse phase space with synchrotron motion is seen to be smaller than in Fig. 3. The RF bunch rotation of one synchrotron period is seen on the right plots.



References

[1]Y. Derbenev and R. P. Johnson, Phys. Rev. ST Accel. Beams **8,** 041002 (2005)

[2]T.Roberts,http://www.muonsinc.com/g4beamline.html

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