Chiral Anomalies and their rôle in the $\pi^0$, $\eta$ and $\eta'$ Mesons

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PHP 2008 Workshop
Outline

• Chiral symmetries and anomalies in QCD
• Interplay of Goldstone Bosons and anomalies
• The $\pi^0$, $\eta$ and $\eta'$ trio and their $\gamma\gamma$ decays
• Why more precision is useful
• Other processes
• Summary
Chiral Symmetries and Anomalies

QCD Lagrangian

\[ L_{QCD} = -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} + i\bar{q} D^\mu (G) q + \bar{q} \mathcal{M}_q q \]

Global Symmetries for \( \mathcal{M}_q = 0 \)

Scale/Conformal \( \times SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1) \)

Anomalies

Scale Anomaly

\[ \Theta^\mu_\mu = \frac{\beta(g)}{2g} G^{\mu\nu} G_{\mu\nu} \]

Axial Anomaly

\[ \partial^\mu A_\mu = \frac{N_f \alpha_s}{2\pi} G^{\mu\nu} \tilde{G}_{\mu\nu} \]
UV origin of anomalies

Both diagrams linearly UV divergent; cancellation up to finite term, identified with anomaly

EM Chiral Anomalies
Photon couplings to quarks generate anomalies for the three axial currents $\bar{q}\gamma_\mu\gamma_5 q$, $\bar{q}\gamma_\mu\gamma_5\lambda_3 q$, and $\bar{q}\gamma_\mu\gamma_5\lambda_8 q$

$$\\partial^\mu A^a_\mu = \frac{N_c \alpha}{2\pi} \text{Tr}(\hat{Q}^2 \lambda_a) F^{\mu\nu} \tilde{F}_{\mu\nu}$$
General properties of the axial and chiral anomalies

- They are preserved under loop corrections

- They must be the same at the quark-gluon level as at the hadronic level: anomaly matching

- Goldstone Bosons coupling to anomalous currents are the key to the matching of chiral anomalies

- The axial anomaly is responsible for the mass of the $\eta'$ meson in the chiral limit
The fate of Chiral Symmetry in QCD

\[ SU_L(3) \times SU_R(3) \times U_A(1) \times U_B(1) \]

- \[ \langle \bar{q}q \rangle \neq 0 \]
- \[ U_A(1) \text{ anomaly} \]
- \[ M_q \neq 0 \]

\[ SU_V(3) \times U_B(1) \]
- 9 GBs

\[ SU_V(3) \times SU_R(3) \times U_B(1) \]

\[ U_u(1) \times U_d(1) \times U_s(1) \]

QCD Lite, 8 GBs, \( \eta' \) massive

GBs become massive

\[ \chi PT \times 1/N_c \]
Chiral Perturbation Theory

Low energy effective theory for GBs and the QuasiGB $\eta'$.

$$\mathcal{L}_\chi = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \cdots$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4} \text{Tr}(\partial_\mu U \partial_\mu U^\dagger) + \frac{2B_0F_0^2}{4} \text{Tr}(\mathcal{M}_q U + h.c)$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}_{GL} + \mathcal{L}^{(4)}_{WZW}$$

$$U = \exp\left(\frac{i}{F_0} \pi^a \lambda_a\right)$$

$\mathcal{L}^{(4)}_{WZW}$ provides the anomaly matching for chiral anomalies: coefficients fixed!

To include $\eta_1$ explicitly need $1/N_c$ expansion: count $1/N_c$ on same footing as chiral counting order $p^2$

Pascual et al; Kaiser & Leutwyler
**Singlet** \( \eta \)

\( \eta_1 \) acquires mass because of axial anomaly. In chiral limit:

\[
M_{\eta_1}^2 = M_0^2 = \frac{4N_f}{F_0^2} \chi
\]

\[
\chi = \left(\frac{\alpha_s}{4\pi}\right)^2 \langle G \tilde{G} \ G \tilde{G} \rangle = O(N_c^0)
\]

Topological susceptibility \( \chi \) has been evaluated in lattice Smith & Teper

\[
M_{\eta_1}^2 = O(N_f/N_c)
\]

In large \( N_c \) \( \eta_1 \) becomes GB: what happens at \( N_c = 3? \)
Matching anomalies

\[ \kappa \text{ fixed by matching anomalies in chiral limit: contained in WZW term} \]

This gives **chiral limit predictions** for the amplitudes

\[ \pi^0 = \pi_3 \to \gamma\gamma, \quad \eta_8 \to \gamma\gamma \text{ and } \eta_1 \to \gamma\gamma \]

\[ A(\pi_a \to \gamma\gamma) = i\alpha \frac{N_c}{12\pi} \frac{C_a}{F_0} \tilde{F} \]

\[ C_3 = 1, \quad C_8 = 1/\sqrt{3}, \quad C_0 = \sqrt{8/3} \]

\( \pi^0 \) width consistent with PDG values: precise test of anomaly

No anomaly: width would be \( \sim 10^{-4} \) times smaller!

Bad for \( \eta \) and \( \eta' \)

Chiral symmetry breaking must be important
Chiral symmetry breaking
Quark masses break chiral and $SU(3)$ symmetries
Two main sources of corrections to $\gamma\gamma$ amplitudes

- State mixing:

$$\pi^0 = (1 - \epsilon^2 - \tilde{\epsilon}^2) \pi_3 + \epsilon \eta_8 + \tilde{\epsilon} \eta_1$$

$$\eta = \cos \theta \eta_8 - \sin \theta \eta_1$$

$$\eta' = \cos \theta \eta_1 + \sin \theta \eta_8$$

- Decay constants

Order of mixings

$$\epsilon = \mathcal{O}\left(\frac{m_d - m_u}{m_s}\right)$$

$$\tilde{\epsilon} = \mathcal{O}\left(\frac{B_0(m_d - m_u)}{M_0^2}\right) = \mathcal{O}(p^2 N_c)$$

$$\tan \theta = \mathcal{O}\left(\frac{B_0 m_s}{M_0^2}\right)$$
$$\pi^0, \eta, \eta' \rightarrow 2\gamma: \text{Empirical status}$$

**PRIMEX: more in Ashot’s talk**
Decays @ LO

Widths predicted in terms of $F_0$, $M_0$, and quark mass ratios $2m_s/(m_u + m_d)$ and $R = m_s/(m_d - m_u)$

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_{\gamma\gamma}$ LO</th>
<th>No Mixing</th>
<th>Exp. PDG Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$</td>
<td>8.08 eV</td>
<td>7.73 eV</td>
<td>7.7 ± 0.6 eV</td>
</tr>
<tr>
<td>$\eta$</td>
<td>613 eV</td>
<td>170 eV</td>
<td>464 ± 45 eV</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>4.86 keV</td>
<td>7.36 keV</td>
<td>4.3 ± 0.4 keV</td>
</tr>
</tbody>
</table>

$\epsilon = 1^o$, $\bar{\epsilon} = 0.5^o$, $\theta = -20^o$, $M_0 = 850$ MeV

$2m_s/(m_u + m_d) = 26$, $R = 45$ (1)

Main effects correctly described at LO:

- Mixing is essential in $\eta$ and $\eta'$ decays: good agreement at LO.
- $\pi^0$ width enhanced by 5% as result of mixing: mixing with $\eta_8$ alone gives $\sim 3\%$ enhancement.

Moussallam; Bernstein, JG & Holstein
Decays @ NLO
Bernstein, JG & Holstein; Ananthanarayan & Moussallam

NLO corrections needed:

- Corrections to decay constants
- Better fit to masses: $\eta$ mass is 50 MeV too low at LO
- Test stability of enhancement of $\Gamma_{\pi^0 \rightarrow \gamma \gamma}$ (NLO corrections $\propto m_s$)

NLO analysis

- Inputs: $\Gamma_{\eta \rightarrow \gamma \gamma}$, $\Gamma_{\eta' \rightarrow \gamma \gamma}$, quark mass ratios (using NLO corrections to Dashen’s theorem), LECs: $L_5$ and $L_8$
- Fit parameters: LECs: $M_0$, $\Lambda_1$, $\Lambda_2$
- Estimated inputs (cannot be fitted): LECs: $t_1$, $K_1$
  (insignificant for $\pi^0$, but possibly relevant for $\eta'$)
NLO results $\pi^0 \to \gamma\gamma$

\[
\begin{align*}
\epsilon &= 0.85 \pm 0.10^\circ \\
\tilde{\epsilon} &= 0.30 \pm 0.03^\circ \\
\theta &= -10 \pm 2^\circ \\
\Gamma_{\pi^0 \to \gamma\gamma} &= 8.12 \pm 0.07 \text{ eV} \\
\text{With EM corr.} \quad \Gamma_{\pi^0 \to \gamma\gamma} &= 8.08 \pm 0.07 \text{ eV}
\end{align*}
\]

EM corrections Ananthanarayan & Moussallam: only enter through corrections to decay constants: $F_{\pi^+}$ vs $F_{\pi^0}$.

Uncertainties in $F_{\pi^+}$, $R$ and $t_1 \to$ theoretical error is $< 1\%$. 
Partial decay widths of $\eta$ normalized to $\Gamma(\eta \rightarrow \gamma\gamma)$: ($BR_{\gamma\gamma} \sim 40\%$)

Empirical improvement of $\Gamma(\eta \rightarrow \gamma\gamma)$ needed

$e^+ - e^-$ experiments consistent, but inconsistent with Primakoff (Cornell)

Important width: $\eta \rightarrow 3\pi$

- Decay amplitude $\propto (m_d - m_s)$
- Needs NLO $\chi$PT (also done at NNLO!)
- $\pi\pi$ FSI: analyzed using dispersion relations
- Small EM corrections
- Competitive or even better way of obtaining ratio $R$

$$Q^2 = \frac{m_s^2 -(m_u+m_d)/2}{m_d^2 - m_u^2}$$

Leutwyler
Important question: how much of the GB qualities are kept by $\eta'$?

A list of suggestive facts:

- Witten’s mass formula vis-á-vis determination of $M_0$:
  \[
  LO : \quad M_0 = 850 \text{MeV} \implies \chi = (0.151 \text{ GeV})^4
  \]
  \[
  \chi_{\text{Lattice}} = (0.187 \pm 0.022 \text{ GeV})^4 \quad \text{Smith & Teper}
  \]

- LO $\eta - \eta'$ mixing predicted from quark mass ratio $m_s/(m_u + m_d)$, receives large NLO correction

- Coupling $\eta_1 - \gamma \gamma$ from WZW term $\oplus$ mixing: widths OK with exp.

- $F_{\eta'}$ similar to $F_\pi$: consistent with chiral and $1/N_c$ counting

- Precision $\pi^0 \to \gamma \gamma$ width (1.5%) would show relevance or not of $\pi^0 - \eta'$ mixing
\( \eta - \eta' \) Mixing

- From the \( \gamma\gamma \) decays: \( \theta = -20^\circ \) at LO and \( -10 \pm 2^\circ \) at NLO. Both \( \frac{1}{N_c} \) and the corrections \( O(m_s) \) add up to big correction.

- Other analyses: Feldman et al.; Escribano & Frère

\[
\frac{\Gamma(J/\psi \to \eta'\gamma)}{(J/\psi \to \eta\gamma)} \quad \to \quad \theta = -15 \pm 3^\circ
\]
\[
\frac{\Gamma(J/\psi \to \eta'V)}{(J/\psi \to \eta V)} \quad \to \quad \theta = -17 \pm 2^\circ
\]

\( SU(3) \) breaking in amplitudes not included in these analyses.
Form Factors
Experimental status

Radii determined by one LEC of $\mathcal{L}^{(6)}$:

\[
t_2 = \begin{cases} 
\frac{3}{12\pi^2\Lambda^2} & \text{VMD} \\
\frac{3}{192\pi^2\tilde{m}_q} & \text{NJL}
\end{cases}
\]

Dipole fits $\Lambda_{\pi^0} = 0.75 \pm 0.03 \text{GeV}, \quad \Lambda_{\eta} = 0.65 - 0.90 \text{GeV}$
\[ \gamma p \rightarrow \pi^+ \pi^0 n \]

Important process mediated by WZW term

\[
A(\gamma \pi^+ \rightarrow \pi^+ \pi^0) = -iF^3\pi \epsilon_{\mu\nu\rho\sigma} \epsilon_{\mu p_1 \nu p_2 \rho p_3 \sigma}
\]

\[
F^3\pi = \frac{eN_c}{12\pi^2 F^3\pi} + O(p^2) = 9.72 \text{ GeV}^{-3} + \cdots
\]

\[
F_{\text{Exp}}^{3\pi} = 10.7 \pm 1.2 \text{ GeV}^{-3} \quad \text{Serpukhov}(1987)
\]

Hall B Miskimen et al; no final analysis published
\[ \eta \rightarrow \pi^0\gamma\gamma \]

\[ O(p^6) = O(m_d - m_u) \times O(p^4) \]: very suppressed.
Experimental values kept changing: newest from Crystal Ball and KLOE:

\[ \Gamma = 0.29 \pm 0.06 \pm 0.02 \text{ eV} \quad \text{MAMI} \]
\[ \Gamma = 0.11 \pm 0.04 \pm 0.02 \text{ eV} \quad \text{KLOE (Prelim.)} \]

Important corrections (unitarization) beyond \( O(p^6) \ \chi\text{PT} \)

Oset&Pelaez

The whole story in Liping’s talk
Summary

- $\pi^0 \rightarrow \gamma\gamma$ has tested chiral anomaly
- Increased precision by PRIMEX exposes quark mass effects: mixing with $\eta$ and perhaps $\eta'$. Theory predicts 5% enhancement
- $\eta$ partial rates are normalized to the $\gamma\gamma$ rates
- Need Primakoff for $\eta \rightarrow \gamma\gamma$ to cross check $e^+ - e^-$ results
- Improve $\Gamma(\eta \rightarrow 3\pi)$ to have better determination of $R = m_s/(m_d - m_u)$
- Still understanding what the GB traits of $\eta'$ are
- Other interesting processes with $\eta$ and $\eta'$ may be accessible in HALL D