

SOLENOID FRINGE FIELD EFFECTS FOR THE NEUTRINO FACTORY LINAC - MAD-X INVESTIGATION*

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Abstract

International Design Study for the Neutrino Factory (IDS-NF) assumes the first stage of muon acceleration (up to 900 MeV) to be implemented with a solenoid based Linac. The Linac consists of three styles of cryo-modules, containing focusing solenoids and varying number of SRF cavities for acceleration. Fringe fields of the solenoids and the focusing effects in the SRF cavities have significant impact on the transverse beam dynamics. Using an analytical formula, the effects of fringe fields are studied in MAD-X. The resulting betatron functions are compared with the results of beam dynamics simulations using OptiM code.

INTRODUCTION

The first stage of muon acceleration is assumed to be carried out using a Linac [1]. The Linac consists of 25 solenoids in 3 sets, with 6, 8 and 11 solenoids in the first, middles and last sets, respectively. The Linac also has 66 SRF cavities which accelerate the muons from 244 MeV up to 0.9 GeV. The resonant frequency of the cavities is 201.25 MHz. The number of bunches is 88 with 5 ns bunch spacing. The beam optics is designed in OptiM code and cross-checked using MADX.

THEORETICAL BACKGROUND

Due to the high number of the solenoids in the Linac it is essential to consider the effects of fringe fields on beam dynamics. The transfer matrix of an ideal hard edge solenoid is written as

$$M_{\text{sol}} = \begin{bmatrix} \cos^2\theta & \frac{1}{g}\sin\theta\cos\theta & -\sin\theta\cos\theta & -\frac{2}{g}\sin^2\theta \\ -g\sin\theta\cos\theta & \cos^2\theta & g\sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \frac{1}{g}\sin^2\theta & \cos^2\theta & \frac{1}{g}\sin\theta\cos\theta \\ -g\sin^2\theta & \sin\theta\cos\theta & -g\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} \quad (1)$$

where $\theta = gL = \frac{qB_0}{2p}L$ is the angle by which the beam is rotated. The focusing strength is equal to g^2 [2,3]. For a realistic solenoid, one should consider a correction because of the finite length of the solenoid edge. Although, the issue of the soft edge solenoids for MADX has been addressed before [4], we follow the analytical model presented in [5]. For a realistic soft edge solenoid, we define the solenoid length of the $L = \frac{1}{B_0} \int_{-\infty}^{\infty} B_z(s) ds$

The correction due to the edge focusing at each solenoid end is determined by the following formula

$$\Phi_{\text{edge}} = \left(\frac{q}{2p}\right)^2 \left(\int_{-\infty}^{\infty} B_z^2(s) ds - B_0^2 L \right) \quad (2)$$

The reason for using B^2 rather than just B in the equation (2) can be explained as follows. In the cylindrical coordinates with z axis parallel to the beam motion, we suppose that the solenoid lies along z axis and a particle is traveling along z , off the axis. The fields from the solenoid are in the z and r directions only. There would be no θ direction, so cross product of z with (z, r) results in θ direction and to 1st order in B there is no focusing. The beam only is rotated about the z axis. Now, if the particle obtains some θ velocity, then the cross product of (z, θ) with (z, r) results in r focusing and we have a focusing in 2nd order in B . In fact the transfer matrix of the solenoid can be written as the multiplication of a rotation matrix and quadruple focusing in the both transverse planes with g^2 as the focusing constant. The formula presented here is derived for the net effect far from the solenoid. For too close distances, the approximation presented here fails to give the right values for the fringe fields. The first term in equation (1) shows the total focusing function, while the second term results from a hard edge model. We approximate $B_z(s) = \frac{B_0}{2}(1 \pm \tanh x)$, in which $x = \frac{s \pm L/2}{a}$ and a is the radius of the aperture. The plus and minus signs correspond to the fringe fields at the left and the right end of the solenoid. In figure 1, we have shown the above idea for a typical one meter solenoid with a 0.25 cm aperture. We restrict our calculations to the minus sign as due to the symmetry the plus sign gives the same result. Mathematically, we have shifted the origin toward the right end of the solenoid where the magnetic field falls to half of its value in the flat top (i.e. $B_z = \frac{B_0}{2}$ at $x = 0$). In this way, for the magnetic fields at $x = -\infty$ and $x = \infty$, we obtain B_0 and 0, respectively, as expected. For zero radius aperture ($a=0$), x goes to infinity and $B_z(s) = 0$, for $s \geq L/2$ and $B_z(s) = B_0$ for $s < L/2$ as expected for a hard edge solenoid. Carrying out the integration, we obtain $\Phi_{\text{edge}} = -\frac{g^2 a^2}{2}$. The fringe fields of the solenoid decrease the total focusing in comparison with the ideal hard edge solenoid of the same length.

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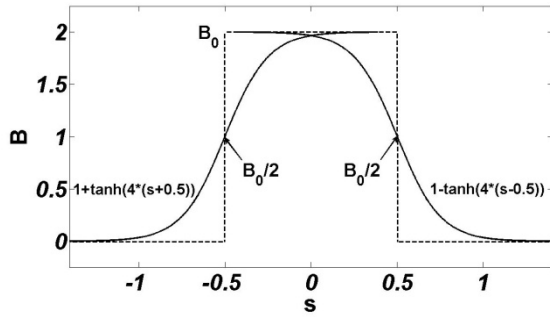


Figure 1: Magnetic field for a solenoid.

Therefore, the edge focusing of a solenoid can be described by an axially symmetric multipole ($m=-1$) through the following transfer matrix

$$M_{\text{edge}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\Phi_{\text{edge}} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\Phi_{\text{edge}} & 1 \end{bmatrix} \quad (3)$$

And thus, the transfer matrix of a soft edge solenoid can be expressed as [4]

$$M_{\text{soft sol}} = M_{\text{edge}} M_{\text{sol}} M_{\text{edge}} \quad (4)$$

METHOD

The first stage of muon acceleration is assumed to be carried out by the Linac. Due to relatively low energy acceleration from 244 MeV up to 900 MeV, the Linac design consists of 25 one meter long solenoids. The solenoids are distributed along the full length of the Linac in 3 different cell sizes. A matched solution for the beam optics of the Linac is shown in Figure 2. Longer cells are required to accommodate larger accelerating cryo-modules. Therefore, beta functions increase while passing from the first to the last sets, as shown in the Fig. 1. The transverse beta functions are produced using the usual built in function commands for the hard edge solenoids in MADX. The $\beta_{x,y} = 2.2151$ meters, $\alpha_{x,y} = -0.2630$ initial conditions were used which are different from the design parameters in OptiM code. Also we had to use different solenoid constants for the matching solenoids. The matching solenoids are the neighbouring solenoids at the transitions between the sets. The situation becomes completely different, as shown in figure 3 if we use the initial conditions parameters $\beta_{x,y} = 2.5862$ meters, $\alpha_{x,y} = -0.2532$ and also the designed values for the matching solenoids used in OptiM. Differences between the two simulations mainly stem from different modelling of the solenoids in MADX and in OptiM. While, the fringe field effects are included in OptiM, in MADX hard edge solenoids are used. This is a Linac focusing without a periodic boundary condition so that a small mismatch in optics at the beginning ends up with large beta beating, which we see as a result of MADX calculation. In order

to improve the situation, we include the fringe fields of solenoid into the lattice. We implement the M_{edge}

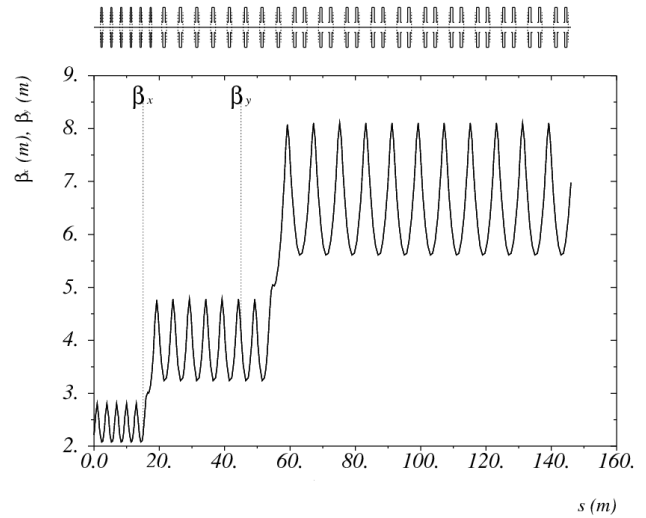


Figure 2: Beta functions of the Linac, using 25 hard edge solenoids in madx with matched initial conditions.

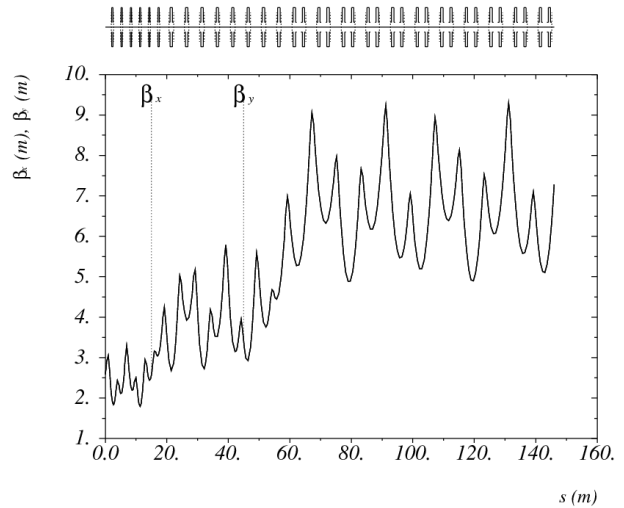


Figure 3: Beta functions of the Linac using 25 hard edge solenoids in madx with the required initial conditions.

matrix into the lattice using the MADX.

Command" MATRIX" which permits the definition of an arbitrary transfer matrix [6].

Label: MATRIX, TYPE=name, L=real,

KICK1=real,..., KICK6=real,

RM11=real,...,RM66=real,

TM111=real,...,TM666=real,

L is the length of the element, which may be zero.

RMik and TMikl define the linear transform matrix (6*6) and the second-order terms (6*6*6) of the element, respectively. In our case, there are 25 solenoids requiring 25 pairs of matrices of the following type

Fringe: MATRIX, TYPE=name, L=0,

RM21= RM43 = $\frac{g^2 a}{2}$,

The on diagonal elements and the other off diagonal elements are equal to one and zero, respectively, by default. The second order terms are also neglected. For the solenoids aperture, we consider $a=0.195$ meters. g for each solenoid should be defined separately.

RESULTS AND DISCUSSION

The resulting beta functions are shown in Figure 3. The initial conditions used are $\beta_{x,y} = 2.400$ meters, $\alpha_{x,y} = -0.2000$ which are much closer to the design parameters used in OptiM code.

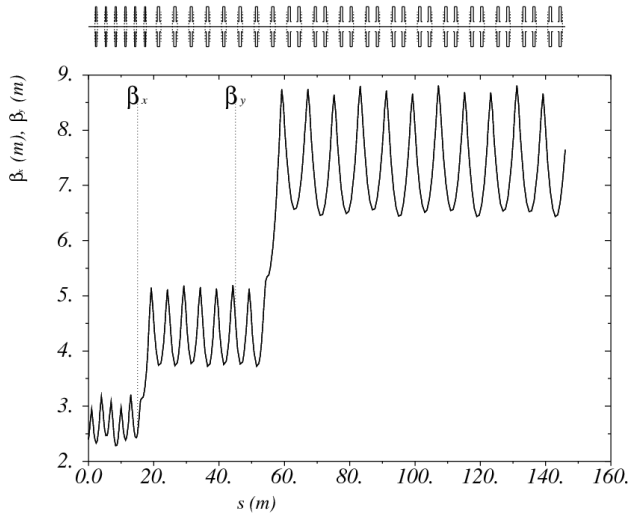


Figure 4: Beta functions of the Linac, using 25 soft edge solenoid in madx.

Although there is still difference in the initial conditions, but this time the values of matching solenoids implemented in the MADX are identical to the design values used in the OptiM Code. The result obtained by OptiM is shown in figure 5. One advantage of the theoretical model for the fringe fields and the soft edge transfer matrix presented here is that they can easily be implemented in the MADX and similar accelerator codes.

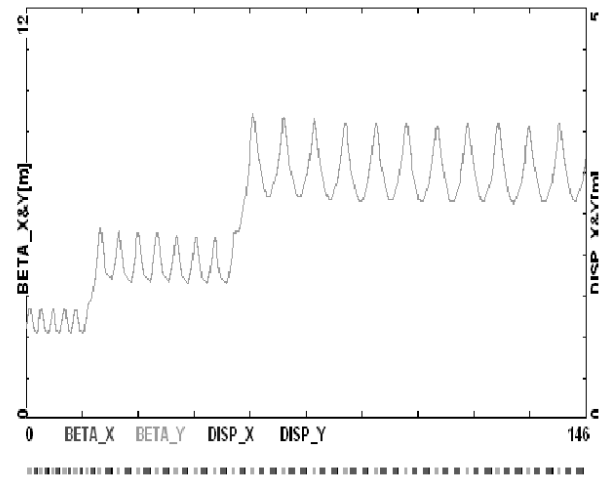


Figure 5 : Beta functions of the Linac, using 25 soft edge solenoid in Optim.

REFERENCES

- [1] <http://www.ids-nf.org>
- [2] S.Y.Lee, Accelerator Physics, World Scientific.1996.
- [3] M. Conte, W. W. MacKay, An introduction to the physics of Particle Accelerators, world scientific, 2008.
- [4] A. Koschik et al, "On the implementation of experimental solenoids in madx and their effects on coupling in the LHC", EPAC 2006, Edinburgh, Scotland.
- [5] K. Beard et al "Simulation of parametric resonance ionization cooling of muon beams ", PAC'05, Knoxville, Tennessee.
- [6] MAD-X web page <http://mad.web.cern.ch/mad/>.